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On cubic crystal anisotropy for waves with Rayleigh-wave polarization

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The anisotropy term $C^2 = [(C_{11} - C_{44})^2 - (C_{12} + C_{44})^2]/(C_{11}C_{44})$ for cubic crystals of the classes m3 and m3m, as well as the threshold velocity Vth, were calculated. It was found that the surface two-partial Rayleigh type waves (RTW2) cannot exist in propagation directions with values of the $C^2 < -4$. It was also found that for the cubic crystals, such as RbCl, RbBr, RbI, Li_2O and KCN, there is a corresponding great positive $C^2 > 5$. The cubic crystal NaCN (m3m class) possesses the gigantic anisotropy term $C^2 = 48.71$. It was discussed that crystals with $C^2 >> 1$ could be suitable for investigation of possible existence of new supersonic surface waves with the phase velocity $V > V_l$, because the velocity $V^{\mathrm{th}} \sim V_l$ of the bulk longitudinal wave: $V_l(\mathrm{RbI}) = 1.077V\mathrm{th}$ and $V_l(\mathrm{Li_2O}) = 1.07V\mathrm{th}$. The supersonic surface waves with $V > V_l$ are promising, for example, for mobile communication to increase work frequency in GHz-devices, such as surface acoustical wave (SAW) filters, etc. Also, the existence condition $V > V_l$ for the new supersonic SAW possessing the Rayleigh polarization does not obey the existence condition $V < V_{\mathrm{t}}$ for the Rayleigh SAW. The phase velocity range $V > V_l$, in which the new SAW can be found, is separated from the one $0 < V < V_{\mathrm{t}}$ for the Rayleigh SAW by the one $V_{\mathrm{t}} < V < V_l$ for leaky type waves. The universal existence condition $C^2 > -1 - C_{11}/C_{33} - 2\sqrt{C_{11}/C_{33}}$ for the RTW2-waves in both cubic and non-cubic crystals was also introduced. Possible applications are also discussed.

Keywords: Cubic crystal; Polarization; Rayleigh type waves (RTW2); Anisotropy

PACs: 43.35.Cg; 43.35.-c;

1. Introduction

Surface acoustical waves (SAW) polarized in the sagittal plane can propagate on the surface of a bulk isotropic medium that was initially discovered by Rayleigh (1885). The sagittal plane is formed by the vector **N** showing the propagation direction and the normal to the free surface directed along the Z-axis as shown in figure 1. The other unique wave type can propagate in a plate, consisting of an isotropic material, which represents the Lamb wave (Lamb 1917). The Lamb waves have polarization, like the one of the surface Rayleigh type waves. Both symmetric and anti-symmetric modes of the Lamb (type) waves can exist.

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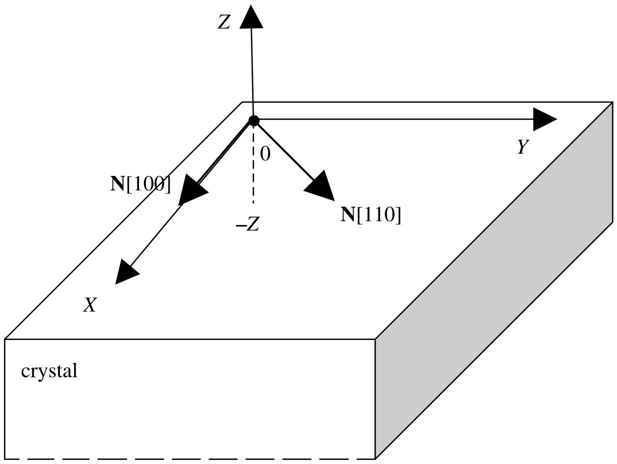

Figure 1. The coordinate system with two commonly used propagation directions shown by the vector **N**.

However, most of the materials are anisotropic, which represent crystals. Both two-partial Rayleigh type waves (RTW2) and two-partial Lamb type waves (LbTW2) can propagate in anisotropic media, too, only in the so-called highly-symmetric propagation directions, according to Lardat *et al.* (1971) and Farnell and Adler (1972). For instance, they can propagate in cases when the sagittal plane coincides with a symmetry plane of a crystal that was also discussed in Farnell (1978). The so-called "pure" RTW2-waves can propagate in these cases. Many known crystals have one of cubic symmetries, such as m3, 432, m3m, 23 and 43m. For the non-centrosymmetrical cubic classes 23 and 43m, both four-partial Rayleigh (RTW4) and Lamb (LbTW4) type waves can propagate, if the piezoelectric effect is taken into account. However, there are such propagation directions in the piezoelectric crystals, in which both RTW2 and LbTW2 waves can propagate in the corresponding systems. For example, this occurs in (001)-[100]-propagation direction for a cubic crystal. In Kadota *et al.* (1999), the attention was paid to studying the anisotropy influence on the RTW2-wave existence in cubic crystals.

Rayleigh (1885) has discovered that SAW polarized in the sagittal plane can exist in isotropic media with the condition for the phase velocity V such as $0 < V < V_\mathrm{t}$, where V_t is the speed of the bulk transverse wave. He has treated only isotropic materials. After that, the Rayleigh SAW existence with the condition $0 < V < V_\mathrm{t}$ was found in anisotropic and piezoelectric crystals. However, for mobile communication there is a requirement to increase work frequency for electronic devices. Therefore, attention was paid to the phase velocity range $V > V_\mathrm{t}$, where different leaky type waves, such as leaky, leaky pseudo, leaky surface and second leaky waves can be found, see for example, Kadota *et al.* (1999). The LSAWs allow usage of GHz and higher frequency range devices for mobile communication. The present work deals with some characteristics of cubic crystals, which differ cubic crystals from isotropic media and show similarity between cubic and non-cubic crystals. In particular, the present work is aimed to find some interesting cubic crystals, which can possess unique anisotropy properties resulting in SAW existence possibility with the other condition $V > V_l$, where V_l is the speed of the bulk longitudinal wave. It is noted that the present crystal anisotropy investigation represents an original work highlighting a problem of Rayleigh-polarized supersonic surface wave existence with the phase velocity $V > V_l$.

2. Theory and the anisotropy term

The influence of cubic crystal anisotropy on the Rayleigh type wave existence was studied in many works, see for example, Gazis *et al.* (1960), Tursunov (1967) and Farnell (1970), of which Farnell (1970) is the most famous. In order to characterize the cubic crystal anisotropy, which makes difference between cubic crystals and isotropic media, the anisotropy factor η for cubic crystals was introduced, see Gazis *et al.* (1960) and Farnell (1970):

$$\eta = \frac{2C_{44}}{C_{11} - C_{12}}. \tag{1}$$

The anisotropy factor η can be both $\eta > 1$ and $\eta < 1$ for a cubic crystal, but there is $\eta = 1$ for an isotropic medium. There are three independent elastic constants C_{11}, C_{12} and C_{44} in equation (1) for cubic crystals, see also Nye (1989). For an isotropic medium there is the well-known dependence:

$$C_{44} = (C_{11} - C_{12})/2. \tag{2}$$

The anisotropy factor η is suitable for studying wave propagation in cubic crystals only in the case of $C_{33} = C_{11}$. Probably, it is more convenient to evaluate other anisotropy characteristics studying SAW propagation in crystals, which will be further discussed.

The SAW propagation treatment in a cubic crystal requires one to find both eigenvalues (the corresponding normal components $k_3^{(n)}$ of the wavevector $\mathbf{k}$) and eigenvectors (the corresponding displacements $u_1^{(n)}$ and $u_3^{(n)}$ for such waves polarised in the sagittal plane). For the coordinate system shown in figure 1, surface waves propagate along the *X*-axis damping towards negative values of the *Z*-axis. For waves polarized in the sagittal plane there are the following no-zero components of the Green-Christoffel tensor: GL_{11}, GL_{33} and $\mathrm{GL}_{13} = \mathrm{GL}_{31}$. Determinant for finding non-dimensional complex components $m_3 = k_3/k$ with $m_1 = 1$ and $m_2 = 0$ can be written in the following view:

$$\begin{vmatrix} C_{44}m_3^2 + C_{11}A_l^2 & (C_{12} + C_{44})m_3 \\ (C_{12} + C_{44})m_3 & C_{11}m_3^2 + C_{44}A_{\mathrm{t}}^2 \end{vmatrix} = 0, \tag{3}$$

where $C_{11} = C_{22} = C_{33}$, $C_{44} = C_{55} = C_{66}$ and $C_{12} = C_{21} = C_{13} = C_{31}$ are the corresponding non-zero components of the stress tensor C_{ijkl}. In equation (3) there are non-dimensional functions of the phase velocity $V = \omega/k$, such as $A_l^2 = 1 - (V/V_l)^2$ and $A_{\mathrm{t}}^2 = 1 - (V/V_{\mathrm{t}})^2$ with $V_l = [C_{11}/\rho]^{1/2}$ and $V_{\mathrm{t}} = [C_{44}/\rho]^{1/2}$ representing the bulk longitudinal and transverse waves, respectively. Expanding the equality (3), the secular equation appears after some transformations (Zakharenko 2005) in the following view:

$$m_3^4 + \left[A_l^2 + A_{\mathrm{t}}^2 + C^2\right]m_3^2 + A_l^2A_{\mathrm{t}}^2 = 0. \tag{4}$$

The fourth-order polynomial equation (4) gives the following four eigenvalues (polynomial roots m_3):

$$m_3^{1,2,3,4} = \pm\left[-\frac{1}{2}\left(A_l^2 + A_{\mathrm{t}}^2 + C^2\right) \pm \frac{1}{2}\left[\left(A_l^2 + A_{\mathrm{t}}^2 + C^2\right)^2 - 4A_l^2A_{\mathrm{t}}^2\right]^{1/2}\right]^{1/2}. \tag{5}$$

In the roots (5), the anisotropy term C^2 is equal to the following:

$$C^2 = \frac{\left[(C_{11} - C_{44})^2 - (C_{12} + C_{44})^2\right]}{C_{11}C_{44}}. \tag{6}$$

It is noted that the anisotropy term C^2 represents non-dimensional crystal characteristics, like the commonly used non-dimensional quantity, such as anisotropy factor η in equation (1) for cubic crystals. The anisotropy term C^2 depends on the values of $C_l^2 = (C_{11} - C_{44})^2$ and $C_t^2 = (C_{12} + C_{44})^2$. Indeed, it is possible to suggest that C_l^2 and C_t^2 relate to A_l^2 and A_t^2, respectively. Therefore, the usage of the symbol $C^2(C_l^2, C_t^2)$ manifests such square dependence on the elastic constants in contrast to the usual dependence of the η equation (1) on the elastic constants of cubic crystals. The C^2 with some modifications is suitable for the other propagation directions, for example in [110] direction, where so-called "pure" waves with polarization in the sagittal plane can propagate. Therefore, the anisotropy term C^2 can be an universal characteristics for cubic crystals, while the anisotropy factor equation (1) is suitable only in [100] propagation direction, where there is the equality $C_{11} = C_{33}$. In addition, A_l and A_t in equation (5), as well as the roots m_3, are non-dimensional that is convenient. It is clearly seen in equations (6) and (2) that there is $C^2 = 0$ for isotropic materials. Therefore, the C^2 manifests all difference between isotropic materials and cubic crystals, which results in wave characteristics.

The condition of the RTW2-wave existence for the coordinate system shown in figure 1 requires existence of all complex/imaginary roots (5), which is full-filled for the phase velocity V below the speed V_t. Only two roots with negative imaginary parts are suitable for the coordinate system, in order to have amplitude damping towards the depth of a bulk material giving surface wave solutions. However, there are no requirements for roots (5) studying Lamb (type) waves, which can be both real and complex/imaginary in the phase velocity range $0 < V < +\infty$. The roots (5) are completely imaginary for the zero anisotropy term C^2, as well as for classical surface RW2 wave in an isotropic medium with the condition for the phase velocity $0 < V < V_t$:

$$\begin{aligned} m_3^{1,2} &= \pm \mathrm{i}\sqrt{+A_l^2} \quad \text{for} \quad V < V_l, \quad \text{but} \quad m_3^{1,2} = \pm\sqrt{-A_l^2} \quad \text{for} \quad V > V_l, \\ m_3^{3,4} &= \pm \mathrm{i}\sqrt{+A_t^2} \quad \text{for} \quad V < V_t, \quad \text{but} \quad m_3^{3,4} = \pm\sqrt{-A_t^2} \quad \text{for} \quad V > V_t, \end{aligned} \tag{7}$$

where $\mathrm{i} = (-1)^{1/2}$ is the imaginary unity.

By analyzing the roots (5), it can be found that all roots (5) can be complex for the phase velocities $V < V_t$. Therefore, the surface RTW2-waves can there exist. On the other hand, all roots (5) could be complex for $V > V_l$ that allows the existence of new supersonic surface waves polarized in the sagittal plane. This can be the unique property of some suitable cubic crystals. This can also be shown by treating the possible case in equation (5) under square root:

$$A_l^2 + A_t^2 + C^2 = 0, \tag{8}$$

which gives either all complex or only two complex roots in equation (5) depending on the C^2:

$$m_3^{1,2,3,4} = \pm[\pm \mathrm{i} A_l A_t]^{1/2}. \tag{9}$$

The roots (9) should all be complex for SAW existence. Square root from the imaginary unity in equation (9) gives a complex number, using the well-known De Moivre's formula:

$$[R(\cos\varphi + \mathrm{i}\sin\varphi)]^p = R^p[\cos(p\varphi) + \mathrm{i}\sin(p\varphi)]. \tag{10}$$

Equality (8) is full-filled for the threshold phase velocity V^{th} (Zakharenko 2005):

$$V^{\mathrm{th}} = V_t V_l \left[\frac{(2 + C^2)}{(V_t^2 + V_l^2)}\right]^{1/2}. \tag{11}$$

It is clearly seen in equation (11) that the anisotropy term (6) should be $C^2 > -2$, in order to have a real velocity V^{th}. For the anisotropy term $C^2 < -2$, the velocity V^{th} in Equation (11) is imaginary.

There can be $C_{11} \neq C_{33}$ in the highly-symmetric propagation direction in cubic and non-cubic crystals. For instance, in [110] propagation direction for cubic crystals in figure 1 there is $C_{11} = C_{55} + (C_{33} + C_{13})/2$, according to Stoneley (1955) and Tursunov (1967). Therefore, in this case, the roots (5), the anisotropy term C^2 (6) and the velocity V^{th} (11) are as follows, respectively:

$$m_3^{1,2,3,4} = \pm\left[-\frac{1}{2}\left(\frac{C_{11}}{C_{33}}A_l^2 + A_{\mathrm{t}}^2 + C^2\right) \pm \frac{1}{2}\left[\left(\frac{C_{11}}{C_{33}}A_l^2 + A_{\mathrm{t}}^2 + C^2\right)^2 - 4\frac{C_{11}}{C_{33}}A_l^2 A_{\mathrm{t}}^2\right]^{1/2}\right]^{1/2}, \tag{12}$$

$$C^2 = \frac{\left[(C_{11} - C_{55})(C_{33} - C_{55}) - (C_{13} + C_{55})^2\right]}{C_{33}C_{55}}, \tag{13}$$

$$V^{\mathrm{th}} = V_{\mathrm{t}}V_l\left[\frac{1 + C_{11}/C_{33} + C^2}{(C_{11}/C_{33})V_{\mathrm{t}}^2 + V_l^2}\right]^{1/2}. \tag{14}$$

It is convenient to have all non-dimensional values in equation (3), because the elastic constants have dimension (N/m^2) with power 10^{10} (table 1). However, it is possible to omit the power in the constants for further calculations, because the velocity V will depend on a non-dimensional term, in which the constants will finally be combined. Two equations for determination of two mechanical displacement components u_1 and u_3 are written from equation (3) as follows:

$$\begin{aligned} &\left[C_{44}m_3^2 + C_{11}A_l^2\right]u_1 + (C_{12} + C_{44})m_3u_3 = 0,\\ &(C_{12} + C_{44})m_3u_1 + \left[C_{11}m_3^2 + C_{44}A_{\mathrm{t}}^2\right]u_3 = 0. \end{aligned} \tag{15}$$

The complete two displacements U_1 and U_3 are written in the following view for plane waves:

$$\begin{aligned} U_1 &= \sum_N f^{(n)}u_1^{(n)}\exp\left[\mathrm{i}k\left(m_1X + m_3^{(n)}Z - Vt\right)\right],\\ U_3 &= \sum_N f^{(n)}u_3^{(n)}\exp\left[\mathrm{i}k\left(m_1X + m_3^{(n)}Z - Vt\right)\right], \end{aligned} \tag{16}$$

where $f^{(n)}$ are the so-called weight factors, X and Z are two real space coordinates from figure 1, and t is time. Also, in equation (16) there are the corresponding two eigenvectors $u_1^{(n)}$ and $u_3^{(n)}$ as follows:

$$\begin{aligned} u_1^{(n)} &= \pm\left[C_{11}(m_3^{(n)})^2 + C_{44}A_{\mathrm{t}}^2\right]^{1/2} = -\frac{(C_{12} + C_{44})m_3^{(n)}}{u_3^{(n)}},\\ u_3^{(n)} &= \mp\left[C_{44}(m_3^{(n)})^2 + C_{11}A_l^2\right]^{1/2} = -\frac{(C_{12} + C_{44})m_3^{(n)}}{u_1^{(n)}}. \end{aligned} \tag{17}$$

The eigenvector components $u_1^{(n)}$ and $u_3^{(n)}$ in the view (17) are readily obtained using the determinant equation (3) formed from the corresponding components of the GL-tensor. Also, equation (17) shows the relationship between the components $u_1^{(n)}$ and $u_3^{(n)}$ that is

Table 1. Dependence of both the non-dimensional anisotropy term C^2 and the velocities V_t, V_l and V^{th} on the material constants of the cubic crystals in [100] propagation direction for (001)-cut.

No	*Cubic crystal*	*Structure type*	*Density ρ (kg/m³)*	*Elastic constants C_{ij}, 10^{10} (N/m²)*			*Anisotropy factor η*	*Anisotropy term C^2*	*Velocity V_t (m/s)*	*Velocity V_l (m/s)*	*Velocity V^{th} (m/s)*
				C_{11}	*C_{44}*	*C_{12}*					
Non-piezoelectric class m3m											
1.	Ge	Diamond	5327	13.000	6.700	4.900	1.65	−1.09	3547	4940	2749
2.	Si	Diamond	2328	16.570	7.960	6.390	1.56	−1.00	5847	8437	4806
3.	Diamond	Diamond	3507	107.600	57.500	12.500	1.21	−0.39	12804	17516	13116
4.	Pb	fcc	11381	4.660	1.443	3.920	3.90	−2.74	1126	2024	i846
5.	Cu	fcc	8936	16.840	7.520	12.140	3.20	−2.37	2901	4341	i1467
6.	Ag	fcc	10534	12.400	4.610	9.340	3.01	−2.34	2092	3431	i1042
7.	Au	fcc	19754	20.200	4.600	16.970	2.85	−2.39	1526	3198	i860
8.	Pt	fcc	21400	42.300	8.500	26.700	1.09	−0.27	1993	4446	2392
9.	Pd	fcc	12000	23.410	7.120	17.610	2.46	−2.08	2436	4417	i603
10.	Ni	fcc	8914	26.100	13.090	15.100	2.38	−1.83	3832	5411	1289
11.	Al	fcc	2700	10.730	2.830	6.090	1.22	−0.57	3238	6304	3444
12.	W	bcc	18711	51.300	15.273	20.600	0.995	−0.014	2857	5236	3534
13.	Mo	bcc	10204	46.000	11.005	17.600	0.78	0.80	3284	6714	4936
14.	Fe	bcc	7848	24.300	12.170	13.810	2.32	−1.79	3938	5565	1473
15.	Na	bcc	970	0.740	0.420	0.620	7.00	−3.15	2080	2762	i1782
16.	Li	bcc	530	1.381	1.013	1.216	12.28	−3.46	4372	5105	i4012
17.	K	bcc	860	0.366	0.277	0.307	9.39	−3.29	1795	2063	i1538
18.	Rb	bcc	1530	0.332	0.245	0.271	8.03	−3.18	1265	1473	i1043
19.	NaCl	NaCl	2168	4.911	1.284	1.285	0.71	1.04	2434	4759	3778
20.	NaF	NaCl	2809	9.710	2.800	2.430	0.77	0.75	3157	5879	4612
21.	NaI	NaCl	3655	3.040	0.720	0.900	0.67	1.26	1404	2884	2279
22.	NaBr	NaCl	3200	4.010	1.143	0.697	0.69	1.06	1890	3540	2917
23.	NaCN	NaCl	1599	2.534	0.033	1.444	0.06	48.71	454	3980	3212
24.	KCN	NaCl	1553	1.916	0.141	1.197	0.39	5.07	951	3513	2441
25.	LiCl	NaCl	2070	4.940	2.460	2.280	1.85	−1.34	3447	4885	2288
26.	LiF	NaCl	2640	11.200	6.320	4.560	1.99	−1.34	4893	6513	3178
27.	LiBr	NaCl	3640	4.721	2.052	1.590	1.31	−0.63	2374	3601	2320
28.	KCl	NaCl	1989	4.070	0.630	0.690	0.37	3.94	1780	4524	4037
29.	KBr	NaCl	2750	3.460	0.515	0.560	0.36	4.28	1369	3547	3201
30.	KI	NaCl	3129	2.750	0.369	0.450	0.32	4.91	1086	2965	2681
31.	KF	NaCl	2525	6.580	1.280	1.490	0.50	2.42	2252	5105	4334
32.	AgCl	NaCl	5571	6.010	0.625	3.620	0.52	2.92	1059	3285	2236

Table 1 – *continued*

No	*Cubic crystal*	*Structure type*	*Density ρ* *(kg/m^3)*	*Elastic constants* *C_{ij}, 10^{10} (N/m^2)*			*Anisotropy factor* *η*	*Anisotropy term* *C^2*	*Velocity V_t* *(m/s)*	*Velocity V_l* *(m/s)*	*Velocity V^{th}* *(m/s)*
				C_{11}	*C_{44}*	*C_{12}*					
33.	AgBr	NaCl	6476	5.630	0.720	3.300	0.62	1.96	1054	2949	1975
34.	RbCl	NaCl	2800	3.640	0.470	0.630	0.31	5.17	1296	3606	3266
35.	RbBr	NaCl	3350	3.150	0.382	0.480	0.29	5.75	1068	3066	2808
36.	RbI	NaCl	2550	2.760	0.279	0.370	0.23	7.45	913	2788	2667
37.	RbF	NaCl	3200	5.530	0.930	1.400	0.45	3.06	1705	4157	3548
38.	MgO	NaCl	3576	29.590	15.390	9.540	1.54	− 0.92	6560	9097	5530
39.	NaO	NaCl	2805	13.890	4.030	4.050	0.82	− 0.57	3790	7037	3990
40.	CaO	NaCl	3300	22.350	9.870	9.870	1.58	− 1.06	5469	8230	4508
41.	SrO	NaCl	4900	17.400	5.600	4.700	0.88	0.34	3381	5959	4498
42.	BaO	NaCl	5720	19.400	6.440	6.440	0.99	0.02	3355	5824	4132
43.	NiO	NaCl	6700	27.000	10.500	12.500	1.45	− 0.91	3959	6348	3507
44.	CoO	NaCl	6400	25.600	8.000	14.400	1.43	− 0.94	3536	6325	3178
45.	MnO	NaCl	5390	22.300	7.900	12.000	1.53	− 1.07	3828	6432	3172
46.	PbS	NaCl	7470	12.690	2.469	2.970	0.51	2.39	1818	4122	3485
47.	CsCl	CsCl	3988	3.640	0.800	0.920	0.59	1.75	1416	3021	2483
48.	CsBr	CsCl	4455	3.072	0.725	0.800	0.64	1.43	1276	2626	2126
49.	CsI	CsCl	4537	2.457	0.629	0.647	0.70	1.11	1178	2327	1853
50.	CsCN	CsCl	3410	1.880	0.300	1.080	0.75	1.05	938	2348	1521
51.	TlCl	CsCl	7000	4.010	0.760	1.530	0.61	1.75	1042	2393	1850
52.	TlBr	CsCl	7560	3.783	0.758	1.521	0.67	1.37	1001	2237	1676
53.	NH_4Cl	CsCl	1530	3.795	0.838	0.955	0.59	1.34	2340	4980	4096
54.	NH_4Br	CsCl	2440	3.413	0.697	0.879	0.55	2.06	1690	3740	3103
55.	TlBr−TlI, KRS-5	CsCl	7371	3.600	0.555	1.500	0.53	2.53	868	2210	1720
56.	TlBr−TlCl, KRS-6	CsCl	7192	4.200	0.760	1.350	0.53	2.31	1028	2417	1964
57.	β-brass	CsCl	8590	12.410	8.090	10.420	8.13	− 3.23	3069	3801	i2648
58.	$SrTiO_3$	Perovskite	5122	31.760	12.350	10.250	1.15	− 0.34	4910	7875	5368
59.	$RbMnF_3$	Perovskite	4317	11.740	3.190	4.210	0.85	0.49	2718	5215	3803
60.	Er_2O_3	–	8640	25.640	7.520	14.680	1.37	− 0.86	2950	5448	2770
61.	$Y_3Al_5O_{12}$	YAG	4552	33.320	11.500	11.070	1.03	− 0.09	5026	8556	5989
62.	$Y_3Fe_5O_{12}$	YIG	5188	26.900	7.640	10.800	0.95	0.15	3838	7201	4966
63.	$Y_3Ga_5O_{12}$	YGG	5790	29.030	9.547	11.730	1.10	− 0.27	4061	7081	4634
64.	CaF_2	Fluorite	3180	16.420	3.370	4.398	0.56	1.99	3255	7186	5923
65.	BaF_2	Fluorite	4893	9.040	2.530	4.060	1.02	− 0.05	2274	4298	2807
66.	SrF_2	Fluorite	4240	12.500	3.200	4.500	0.80	0.68	2747	5430	4013

Table 1 – *continued*

No	*Cubic crystal*	*Structure type*	*Density ρ (kg/m³)*	*Elastic constants C_{ij}, 10^{10} (N/m²)*			*Anisotropy factor η*	*Anisotropy term C^2*	*Velocity V_t (m/s)*	*Velocity V_l (m/s)*	*Velocity V^{th} (m/s)*
				C_{11}	*C_{44}*	*C_{12}*					
67.	Cu_2O	Antifluorite	6140	12.610	1.360	10.680	1.41	− 1.07	1488	4532	1363
68.	Li_2O	Antifluorite	2013	26.450	2.180	4.180	0.20	9.51	3291	11463	10734
69.	K_2O	Antifluorite	2350	8.450	2.870	3.250	1.10	− 0.26	3495	5997	3983
70.	$MgAl_2O_4$	Spinel	3585	28.131	15.459	15.544	2.46	− 1.84	6567	8858	2110
71.	$FeAl_2O_4$	Spinel	4280	26.600	13.350	18.250	3.20	− 2.32	5585	7884	i2578
72.	$FeCr_2O_4$	Spinel	4450	32.200	11.700	14.400	1.32	− 0.69	5128	8506	5027
73.	$MnFe_2O_4$	Spinel	5000	21.300	8.600	13.500	2.21	− 1.79	4147	6527	1604
74.	γ-Mg_2SiO_4	Spinel	3559	32.700	13.100	11.400	1.23	− 0.50	6067	9585	6279
75.	Fe_2TiO_4	Antispinel	4836	13.900	3.960	11.200	2.93	− 2.38	2862	5361	i1556
76.	Fe_3O_4	Antispinel	5163	26.760	9.530	10.560	1.18	− 0.42	4296	7199	4637
77.	$Ca_3Al_2Si_3O_{12}$	Garnet	3594	31.700	10.200	9.500	0.92	0.23	5327	9392	6919
78.	$Ca_3Cr_2Si_3O_{12}$	Garnet	3850	30.400	8.400	9.100	0.79	0.70	4671	8886	6794
79.	$Fe_3Al_2Si_3O_{12}$	Garnet	4318	30.900	9.700	10.900	0.97	0.08	4740	8518	5969
80.	$Mn_3Al_2Si_3O_{12}$	Garnet	4190	30.400	9.400	11.200	0.98	0.06	4737	8518	5942
81	$Mg_3Al_2Si_3O_{12}$	Garnet	3582	29.400	9.100	11.600	1.02	− 0.06	5040	9060	6135
Non-piezoelectric class m3											
82.	FeS_2	Pyrite	4890	34.480	10.810	3.120	0.69	0.98	4702	8397	7082

clearly seen:

$$u_1^{(n)}u_3^{(n)} = -(C_{12} + C_{44})m_3^{(n)}. \tag{18}$$

It is natural that the displacements components from equation (18) could be taken in the following view $u_1^{(n)} = 1$ and $u_3^{(n)} = -(C_{12} + C_{44})m_3^{(n)}/u_1^{(n)}$ for convenience. However, taking $u_1^{(n)} = 1$ or $u_3^{(n)} = 1$ in equation (18) is not suitable for equation (15). Indeed, for $u_1^{(n)} = 1$ there is suitable value of $u_3^{(n)} = -[C_{44}(m_3^{(n)})^2 + C_{11}A_1^2]/[(C_{12} + C_{44})m_3^{(n)}]$. There is also one useful function $A_t^2(A_l^2)$ from Zakharenko (2005):

$$C_{44}A_t^2 \equiv C_{11}A_l^2 + (C_{44} - C_{11}), \tag{19}$$

Two boundary conditions at $Z = 0$ (figure 1) for stress tensor components, such as $\mathrm{ST}_{31} = 0$ and $\mathrm{ST}_{33} = 0$, give the following second-order boundary conditions determinant (BCD2) for two-partial waves:

$$\begin{vmatrix} u_3^{(1)} + m_3^{(1)}u_1^{(1)} & u_3^{(2)} + m_3^{(2)}u_1^{(2)} \\ C_{12}u_1^{(1)} + C_{11}m_3^{(1)}u_3^{(1)} & C_{12}u_1^{(2)} + C_{11}m_3^{(2)}u_3^{(2)} \end{vmatrix} = 0. \tag{20}$$

Applying some transformations to both the first and second columns separately, such as the relationship in equation (17) between $u_1^{(n)}$ and $u_3^{(n)}$, it is possible to obtain a more convenient BCD2 for further simplifications. For the first and second columns in equation (20), taking the corresponding normalized displacement components $u_3^{(1)} = -(C_{12} + C_{44})m_3^{(1)}/u_1^{(1)}$ with $m_3^{(1)} = (A_lA_t/2)^{1/2}(1 - \mathrm{i})$ and $u_3^{(2)} = -(C_{12} + C_{44})m_3^{(2)}/u_1^{(2)}$ with $m_3^{(2)} = (A_lA_t/2)^{1/2}$ $(-1 - \mathrm{i})$, according to equations (9), (10) and (18), one gets:

$$\begin{vmatrix} \left[u_3^{(1)}\right]^2 - (C_{12} + C_{44})\left[m_3^{(1)}\right]^2 & \left[u_3^{(2)}\right]^2 - (C_{12} + C_{44})\left[m_3^{(2)}\right]^2 \\ -C_{12}(C_{12} + C_{44})m_3^{(1)} + C_{11}m_3^{(1)}\left[u_3^{(1)}\right]^2 & -C_{12}(C_{12} + C_{44})m_3^{(2)} + C_{11}m_3^{(2)}\left[u_3^{(2)}\right]^2 \end{vmatrix} = 0. \tag{21}$$

The further transformations include corresponding expression choice under square roots in (17) for both $u_3^{(1)}$ and $u_3^{(2)}$ and multiplying the last row in equation (21) by the root $m_3^{(1)}$ for convenience. That results in the following intermediate state of the BCD2:

$$\begin{vmatrix} C_{11}A_l^2 - C_{12}\left[m_3^{(1)}\right]^2 & C_{11}A_l^2 + C_{12}\left[m_3^{(1)}\right]^2 \\ \left[m_3^{(1)}\right]^2\left(-a + C_{11}^2A_l^2 + C_{11}C_{44}\left[m_3^{(1)}\right]^2\right) & m_3^{(1)}m_3^{(2)}\left(-a + C_{11}^2A_l^2 - C_{11}C_{44}[m_3^{(1)}]^2\right) \end{vmatrix} = 0, \tag{22}$$

where the following useful relationship $[m_3^{(1)}]^2 = -[m_3^{(2)}]^2$ was used for complex roots in view (9) and the constant $a = C_{12}(C_{12} + C_{44})$ was introduced. The following root property is also applicable for both roots (5) and (9), as well as for the roots (7) in the case of $C^2 \equiv 0$:

$$m_3^{(1)}m_3^{(2)} \equiv -A_lA_t. \tag{23}$$

Finally, using the root property (23) and the roots (9), the BCD2 in equation (22) transforms into the BCD:

$$\begin{vmatrix} -C_{12}A_t + \mathrm{i}C_{11}A_l & -\mathrm{i}C_{12}A_t + C_{11}A_l \\ a + \mathrm{i}C_{11}C_{44}A_lA_t - C_{11}^2A_l^2 & -a + \mathrm{i}C_{11}C_{44}A_lA_t + C_{11}^2A_l^2 \end{vmatrix} = 0, \tag{24}$$

which already shows clear dependence on both A_l and A_t. Expanding the equation (24), a secular equation appears as the result, which consists of both imaginary and real parts, Abs{Re(BCD2)} = Abs{Im(BCD2)}. Therefore, it is possible to treat only one real part for simplicity. Hence, for evaluation of the phase velocity below the speed V_t, one can obtain:

$$aC_{12}A_\mathrm{t} - aC_{11}A_l - (C_{12} + C_{44})A_\mathrm{t}C_{11}^2A_l^2 - C_{12}C_{44}A_\mathrm{t}^2C_{11}A_l + C_{11}^3A_l^3 = 0. \qquad (25)$$

Probably, the sixth-order polynomial (25) for the phase velocity $V < V_\mathrm{t}$ represents the exact dependence $V_R = F(V_\mathrm{t}, V_l, C_{11}, C_{12}, C_{44})$ for Rayleigh SAW in the velocity range $0 < V_R < V_\mathrm{t}$. It is noted that there is the phase velocity $V \to 0$ for $C^2 \to -4 + \delta_c$ with $\delta_c \to +0$. It was found that there can be $V < 100\,\mathrm{ms}^{-1}$ for $C^2 \sim -3.9999$, but such crystals do not exist, see table 1. The same there is for the threshold velocity V^th with $C^2 \sim -2$. The slowest real velocity V^th there is for Cu_2O on line 67 in table 1 with $V^\mathrm{th} \sim 1363\,\mathrm{ms}^{-1}$. For the case of $V^\mathrm{th} \to V_\mathrm{t} - \delta\,(\delta \ll V_\mathrm{t})$ there are suitable cubic crystals in the table, such as LiBr on line 37 with $V^\mathrm{th} \sim 2320\,\mathrm{ms}^{-1}$ and $V_\mathrm{t} \sim 2374\,\mathrm{ms}^{-1}$ and $FeCr_2O_4$ on line 72 with $V^\mathrm{th} \sim 5027\,\mathrm{ms}^{-1}$ and $V_\mathrm{t} \sim 5128\,\mathrm{ms}^{-1}$. It is noted that there are relatively the same values of negative $C^2 \sim -0.63$ and $C^2 \sim -0.69$ for LiBr and $FeCr_2O_4$, respectively. This shows that they are alike. Cubic crystals in table 1 will be further discussed in the next Sections. It is noted that for the case of $C^2 \equiv 0$ there is $V_\mathrm{t} < V^\mathrm{th} < V_l$. The crystals W and BaO with $C^2 \to 0$ on lines 12 and 42 of table 1 support this.

Now it is possible to leave the case of $0 < V < V_\mathrm{t}$ for Rayleigh SAW, as well as the case of $V_\mathrm{t} < V < V_l$ for leaky type waves, and to teat the possible case of $V > V_l$ for possible existence of new surface wave caused by unique anisotropy properties of some suitable crystals with $C^2 \geqslant 0$. In this case, it is convenient to use $B_\mathrm{l}^2 = (V/V_l)^2 - 1$ and $B_\mathrm{t}^2 = (V/V_\mathrm{t})^2 - 1$ instead of $A_l^2 = 1 - (V/V_l)^2$ and $A_\mathrm{t}^2 = 1 - (V/V_\mathrm{t})^2$, respectively. It is noted that there can be readily done $A_l \to \mathrm{i}B_l$ and $A_\mathrm{t} \to \mathrm{i}B_\mathrm{t}$ in the secular equation (25) that results in the other secular equation for the surface waves with $V > V_l$:

$$aC_{12}B_\mathrm{t} - aC_{11}B_l + (C_{12} + C_{44})B_\mathrm{t}C_{11}^2B_l^2 + C_{12}C_{44}B_\mathrm{t}^2C_{11}B_l - C_{11}^3B_l^3 = 0. \qquad (26)$$

Also, there is the following useful relationship between B_l and B_t from equation (19):

$$C_{44}B_\mathrm{t}^2 \equiv C_{11}B_l^2 + (C_{11} - C_{44}). \qquad (27)$$

It is noted that for leaky type waves confined in the phase velocity range $V_\mathrm{t} < V < V_l$, the relationship (27), using equation (19), becomes as $-C_{11}A_l^2 \equiv C_{44}B_\mathrm{t}^2 + (C_{44} - C_{11})$. In this case of $V > V_l$, the root property (23) for such surface waves transforms into the following view:

$$m_3^{(1)}m_3^{(2)} \equiv -B_lB_\mathrm{t} \qquad (28)$$

for the suitable roots $m_3^{(1)} = (B_lB_\mathrm{t}/2)^{1/2}(1 - \mathrm{i})$ and $m_3^{(2)} = (B_lB_\mathrm{t}/2)^{1/2}(-1 - \mathrm{i})$. Comparising equations (19), (23) and (25) with equations (26)–(28), it is obvious to do a statement that such the problem to find the phase velocity for new SAWs in the V-range $V > V_l$ is different from the problem to find the one for Rayleigh SAWs in the V-range $0 < V < V_\mathrm{t}$.

There are no cubic crystals in table 1 with the condition $V^\mathrm{th} > V_l$ for a great $C^2 \geqslant 0$. However, it is thought that such systems will be found (synthesized) in the future. On the other hand, there are cubic crystals, such as NaCN, RbI and Li_2O (see lines 23, 36 and 68 in the table), with a great $C^2 \geqslant 0$, for which the velocity V^th is several percents less than the speed V_l. Also, it is possible to take into account temperature dependence of the elastic constants $C_{11}(T)$, $C_{44}(T)$ and $C_{12}(T)$. It is natural to assume that there will be $V^{\text{th}} \approx V$, in general. Therefore, it is possible to write that the existence V-range for such surface waves will be as follows $V_l < V < V_l + 2(V^{\text{th}} - V_l)$ for $V^{\text{th}} > V_l$. It is also possible to introduce the case of $B_l \ll B_{\mathrm{t}}$ at $V = V_l + \delta$ with $\delta \ll V_l$ ($\delta \ll V_{\mathrm{t}}$). Moreover, it is necessary to require the following $C_{44}B_{\mathrm{t}} \gg C_{11}B_l$ for safety. Hence, the secular equation (26) can be rewritten in the following view:

$$aC_{12}B_{\mathrm{t}} = (a - C_{12}C_{44}B_{\mathrm{t}}^2)C_{11}B_l + \delta_0(B_l^2, B_l^3) \tag{29}$$

with $\delta_0(B_l^2, B_l^3) = -(C_{12} + C_{44})B_{\mathrm{t}}C_{11}^2B_l^2 + C_{11}^3B_l^3 \ll C_{11}B_{\mathrm{l}}$. Applying the expression (27) for $C_{44}B_{\mathrm{t}}^2$ and leaving only terms with B_l and B_{t}, one can get the following dependence $B_{\mathrm{t}}(B_l)$: $aC_{12}B_{\mathrm{t}} = [a - C_{12}(C_{11} - C_{44})]C_{11}B_l + \delta_{01}(B_l^2, B_l^3)$ with $\delta_{01}(B_l^2, B_l^3) = \delta_0(B_l^2, B_l^3) - C_{12}C_{11}^2B_l^3$. Indeed, it is natural that the further mathematical step will be as follows:

$$a^2C_{44}B_{\mathrm{t}}^2 = C_{44}[C_{11} - 2C_{44} - C_{12}]^2C_{11}^2B_{\mathrm{l}}^2 + \delta_{02}\left(B_{\mathrm{l}}^3, B_{\mathrm{l}}^4, B_{\mathrm{l}}^5, B_{\mathrm{l}}^6\right). \tag{30}$$

Finally, using the relationship (27) again and neglecting the term $\delta_{02}(B_l^3, B_l^4, B_l^5, B_l^6)$, the phase velocity for the surface waves with the case $V^{\text{th}} = V_l + \delta$ ($\delta \ll V_{\mathrm{l}}$ and $\delta \ll V_{\mathrm{t}}$), can be calculated with the following approximate formula:

$$V_n \sim V_l\left[1 + \frac{a^2(C_{11} - C_{44})}{[C_{11} - 2C_{44} - C_{12}]^2C_{44}C_{11}^2 - a^2C_{11}}\right]^{1/2}. \tag{31}$$

It is natural that the phase velocity V_n of the new SAWs is introduced as the function $V_n(V_l)$, because there is $V^{\text{th}} \sim V_l$. Probably, this formula (31) could be used as a first approximation of the phase velocity for new supersonic surface waves even in piezoelectric and other possible cases. The formula (31) can be also simplified for the case of $C_{11}B_l \ll C_{44}B_{\mathrm{t}}$. The conditions $C_{11} \gg C_{44}$ and $C_{11} \gg C_{12}$ allow doing neglections, such as $(C_{11} - C_{44}) \rightarrow C_{11}$ and $(C_{11} - 2C_{44} - \mathrm{C}_{12}) \rightarrow C_{11}$. It is obvious that the case $C_{44}C_{11}^3 \gg a^2$ gives:

$$V_n' \sim V_l\left[1 + \frac{C_{12}^2(C_{12} + C_{44})^2}{C_{44}C_{11}^3}\right]^{1/2}. \tag{32}$$

If $C_{12} \sim C_{44}$ is taken into account in equation (32), the phase velocity can be written as

$$V_n' \sim V_l[1 + 4(C_{44}/C_{11})^3]^{1/2}. \tag{33}$$

The cubic crystals, such as RbI$\{C_{11} = 2.760, C_{44} = 0.279, C_{12} = 0.370\}$ with $C^2 \sim 7.45$ on line 36 of table 1 and Li$_2$O$\{26.450, 2.180, 4.180\}$ with $C^2 \sim 9.51$ on line 68 of the table, represent such examples for the conditions $C_{11} \gg C_{44}$ and $C_{11} \gg C_{12}$. However, there are cubic crystals with a great $C^2 \gg 0$ in the table, such as NaCN $\{2.534, 0.033, 1.444\}$ with $C^2 \sim 48.71$ on line 23, for which there is $C_{11} \gg C_{44}$ with $C_{11}/C_{12} \sim 3 - 4$. There is also one interesting crystal in the table on line 82, such as pyrite FeS$_2\{34.480, 10.810, 3.112\}$ with a relatively small $C^2 \sim 0.98$, but with velocity $V^{\text{th}} \sim 0.84V_{\mathrm{l}}$ that is close to V_l, for which there is $C_{11} \gg C_{12}$ with $C_{11}/C_{44} \sim 3$. It is obvious that there are two limit cases caused by the unique crystals anisotropy. In the first case with $C^2 < -4$, it is a problem to find a real phase velocity for Rayleigh SAW in the V-range $0 < V < V_{\mathrm{t}}$. The second already represents the other limit situation with a great $C^2 \gg 0$, which can result in existence of the new supersonic SAWs with the phase velocity $V > V_l$.

3. Acoustic cubic crystals

The material constants of cubic crystals were taken from Anderson (1965), Farnell (1970), Ashcroft and Mermin (1976), Blistanov *et al.* (1982), Landolt-Boernstein International Tables (1985), Nikonorov and Kardashov (1985), Belomestnych (1998) and Aleksandrov and Prodaivoda (2000), such as densities ρ and the elastic constants C_{11}, C_{12} and C_{44}, which are listed in table 1. The anisotropy term C^2 was calculated with formula (6), and the velocity V^{th} in the last column of the table was calculated using formula (11), which depends on both the speeds V_{t} and V_l, as well as on the C^2. The material constants of both NaCN and KCN were taken from Haussuehl (1977) and Haussuehl and Michaelis (1979), and the material constants of CsCN were taken from Loidl *et al.* (1983). In addition, perovskites were taken from the review book (Aleksandrov and Beznosikov 2004) on perovskites. According to table 1, there is no cubic crystal symmetry for which the anisotropy terms C^2 were all either negative or positive. Examination of table 1 makes this statement obvious. Probably, cubic crystals of the piezoelectric class 23 possess only positive values of the anisotropy term C^2. However, there are no many known crystals of the class for Statistics. This can be clarified in the future, if a 23-symmetry cubic crystal with a negative C^2 will be discovered. It is also possible to discuss the anisotropy term C^2 as the crystal characteristics for cubic crystals within each symmetry class. For example, for m3m-symmetry class, the cubic crystals Ge, Si and Diamond with the diamond structure type have only positive values of the C^2. However, the cubic crystals with both bcc- and fcc-lattices, as well as with both the NaCl and CsCl structure types, can have both positive and negative C^2. Probably, cubic crystals with the same structure type, such as Diamond, can have only either positive or negative values of the C^2. The "sign" of the anisotropy term C^2 for suitable structures could also be the structure characteristics. It is noted that all structure types of cubic crystals are described in Crystallography, see the book (Kleber 1971).

Indeed, it is clearly seen in the formulae (1), (2) and (6), as well as in table 1, that the anisotropy terms $C^2 > 0$ and $C^2 < 0$ correspond to the anisotropy factors $\eta < 1$ and $\eta > 1$, respectively. Also, zero anisotropy term $C^2 = 0$ corresponds to $\eta = 1$. However, for W on line 12 of the table there are $C^2 \sim -0.014 < 0$ with $\eta \sim 0.995 < 1$ that could be explained by the way that this caused by square dependence (6) of the C^2 resulting in such invalid correspondence between C^2 and η about $C^2 \sim 0$. On the other hand, this could be excluded, if the elastic constants in equation (6) will be measured with a greater accuracy. In addition, the C^2 can be significantly different from crystal to crystal for the same η. For example, Diamond on line 3 of the table with $C^2 \sim -0.39$ and Al on line 11 with $C^2 \sim -0.57$ have the same $\eta \sim 1.21$–1.22. The other instance is for the crystals NH_4Br, KRS-5 and -6 on lines 54–56 of the table, respectively. This can be explained by the complicated dependence C^2 on the elastic constants. LiCl and LiF with the same $C^2 \sim -1.34$ possessing $\eta \sim 1.85$ and $\eta \sim 1.99$ on lines 25 and 26, respectively, give also one example.

The other crystal characteristics could be the threshold velocity V^{th} (11). It is seen in table 1 that the velocity V^{th} can be imaginary for negative $C^2 < -2$. Also, the real velocity V^{th} can be below the speed V_{t}, between the speeds V_{t} and V_l, and, perhaps, even greater than the speed V_l. The "gigantic" positive anisotropy term $C^2 = 48.71$ for the cubic crystal NaCN due to the small constant $C_{44} = 0.033 \times 10^{10}$ [Nm^{-2}] at 293 K, according to Haussuehl 1977 and Haussuehl and Michaelis 1979, does not give the condition $V^{\mathrm{th}} > V_l$, because the speed V_t is very slow. In both NaCN and KCN there is a phase transition observed at temperatures 283 and 168 K, respectively. Table 1 lists only non-piezoelectric classes of cubic crystals. Crystal with $V^{th} \sim V_l$ can be convenient for different technical devices on SAWs. Also, it is well-known that values of the elastic constants C_{11}, C_{12} and C_{44} depend on temperature for all cubic crystals. It is assumed that the electrical field and/or pressure can be applied to a crystal with $C^2 \geqslant 0$ that could also result in appearance of the new supersonic SAWs. It is noted that application of an electrical field or a pressure to a cubic crystal can convert the cubic symmetry into monoclinic symmetry. This is the non-linear case, for which the corresponding third-order elastic constants C_{ijklmn}, as well as the fourth-order $C_{ijklmnpq}$ for some cases, should be considered in addition to the second-order C_{ijkl}.

For the other instance, numerical investigations of elastic instabilities in cubic crystals from stress–strain relations were carried out in Karki *et al.* (1997) for several cubic crystals. The elastic stability criteria for a cubic crystal are written in the following view, according to Wallace (1972) and Wang *et al.* (1995)):

$$C_{11} + 2C_{12} > 0 \quad \text{(spinodal criterion)}; \tag{34}$$

$$C_{44} > 0 \quad \text{(shear criterion)}; \tag{35}$$

$$C_{11} - C_{12} > 0 \quad \text{(Born criterion)}. \tag{36}$$

In the case when a cubic crystal undergoes hydrostatic pressure P representing an isotropic stress, according to Wallace (1972), there is:

$$c_{ijkl} = C_{ijkl} + 0.5P(2\delta_{ij}\delta_{kl} - \delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl}). \tag{37}$$

The generalized stability criteria from Wallace (1972) and Wang *et al.* (1995) for a cubic crystal under hydrostatic pressure can be analogically written, using equations (34)–(36):

$$c_{11} + 2c_{12} > 0; \quad c_{44} > 0; \quad c_{11} - c_{12} > 0. \tag{38}$$

The elastic constants c_{ij} representing the case of hydrostatic pressure are related to the C_{ij} defined with respect to the Eulerian strain variables by

$$c_{11} = C_{11}; \quad c_{12} = C_{12} + P; \quad c_{44} = C_{44} - \frac{P}{2}. \tag{39}$$

In the case of vanishing load, only the Born stability criterion can be treated for a cubic crystal in the finite-load stability conditions (38). It is also noted that the shear instability can be reached before the spinodal one. It is clearly seen in equation (39) that hydrostatic pressure can significantly change both the elastic constants C_{12} and C_{44}, but not C_{11}.

The existence of forbidden propagation directions for the RTW2-waves in cubic crystals was stated in Gazis *et al.* (1960), Buchwald (1961), Buchwald and Devis (1963) and Tursunov (1967). However, this is still not obvious as was mentioned in Farnell (1970). It is clearly seen in formula (5) that for negative $C^2 < -4$, the RTW2-waves cannot exist, because the waves are confined in the V-range $0 < V < V_t$, where both $A_t^2 \leq 1$ and $A_l^2 \leq 1$ give $(A_t^2 + A_l^2) \leq 2$ and $4A_l^2 A_t^2 \leq 4$. Therefore, there is $(A_t^2 + A_l^2 + C^2)^2 \geq 4$ for $C^2 < -4$ that does not give all complex roots in equation (5), two of which with negative imaginary parts should be taken for surface waves in order to have wave damping towards negative values of the Z-axis in figure 1. In table 1 there are no cubic crystals with such negative $C^2 < -4$, but there are Na, Pb, Ag, Au, Cu, Pd, Li, K, Rb, CuCl, CuBr, β-brass, $FeAl_2O_4$ and Fe_2TiO_4, for which there is the imaginary V^{th} for $-2 > C^2 > -4$. For cubic crystals with such negative C^2, the existence section of RTW2-waves with the real phase velocity V is divided into two sub-sections. Within the first, there is the case of all real roots. In the second, where there are complex roots in equation (5), the RTW2-waves can be found. It is noted that there are all imaginary roots in equation (5) for positive C^2 in the V-range $0 < V < V_t$. Figure 2 shows the displacement behavior for both complex and imaginary roots. Complex roots $m_3 = a - \mathrm{i}b$ in equation (16) result in amplitude oscillation along negative values of the Z-axis, because $U_{1,3}\ (-Z) \sim \exp[\mathrm{i}k(a - \mathrm{i}b)(-Z)] = (\cos aZ - \mathrm{i}\sin aZ)\exp(-bkZ)$. It is also noted that the RTW2-waves can exist in both cubic and non-cubic crystals, if the following condition is full-filled:

$$C^2 > -1 - C_{11}/C_{33} - 2\sqrt{C_{11}/C_{33}}. \tag{40}$$

It is clearly seen that the condition (40) of the RTW2 existence is converted into the condition $C^2 > -4$ for the case of $C_{11} = C_{33}$. It is noted that it can be $C^2 > \sim -2$ for crystals with the elastic constant $C_{11} \ll C_{33}$ and even $C^2 > \sim -10$ for crystals with $C_{11} \gg C_{33}$.

Also, there is a situation when solutions give the imaginary phase velocity iV with $\mathrm{i} = (-1)^{1/2}$. It is possible that in this case the imaginary phase velocity iV will result in the imaginary group velocity iV_g and, hence, in negative energy $E \sim (\mathrm{i}V_g)^2$. It is noted that the well-known relationship $V_g = \mathrm{d}(kV)\mathrm{d}k$ was verified in Rayleigh (1877) for running waves. There is an example for systems with negative energy, such as electron–positron pairs as neutral quasi-particles, consisting of electrons with positive energy and positrons with negative energy in physics of elementary quasi-particles. It is thought that imaginary iV would appear to mean an exponentially damped non-wave like disturbance for some cases. Solutions for the imaginary iV can be also found for real wavenumber k. For example, among the fcc-lattice crystals in the right hand column of table 1, all values of the velocity Vth are imaginary showing a finding problem of real V of the RTW2-waves for such materials, except for Pt, Ni and Al. Also, the imaginary $\mathrm{i}V = -\omega/\mathrm{i}k$ can exist for imaginary wavenumber ik. Therefore, the exponential law $\exp[\mathrm{i}k(m_1X + m_3Z - Vt)]$ for plane waves transforms into $\exp[\mathrm{i}(\mathrm{i}k)(m_1X + m_3Z + \mathrm{i}Vt)]$. Moreover, time t can be taken imaginary $t \rightarrow \mathrm{i}t$, like it is taken in theory of black holes. Therefore, it is possible to do the following careful transformations $k \rightarrow \mathrm{i}k$ and $V \rightarrow \mathrm{i}V$ that can result in negative energy. In addition, in homogeneous waves can have the same mathematical structure as pure plane waves, except that their wavevector is complex. The later results in an exponential decay along the wave front. Inhomogeneous waves representing a new kind of leaky surface waves on a

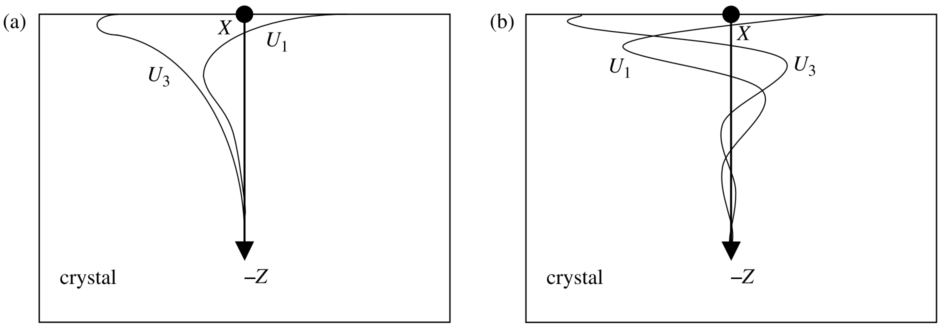

Figure 2. The displacement amplitude (U_1 and U_3) behaviour according to equation (16): (a) imaginary roots for crystals with positive C^2, as well as for isotropic materials; (b) complex roots for crystals with negative C^2, as well as for the possible case of $C^2 \geqslant 0$ and $V^{\mathrm{th}} > V_L$.

brass-water smooth interface were theoretically discovered in Declercq *et al.* (2003), using the Deschamps principle (Deschamps 1994). Also, inhomogeneous waves, such as dispersive Floquet waves and multi-layered Rayleigh waves were studied in Potel *et al.* (1999) in a lossy medium, which can be characterized by complex elastic constants C_{11}, C_{33}, C_{12}, C_{13} and C_{44} (Dransfeld and Salzmann 1970, Hosten and Castaings 1993). It is noted that the C^2 can be also complex for such complex elastic constants.

The dynamic characteristics from equations (4)–(5), such as $A_0 = A_l^2 + A_t^2 + C^2$ and $B_0 = A_l^2 A_t^2$ of the Li_2O and NaCN cubic crystals with $C^2 \gg 1$ are shown in figure 3. The dynamic characteristics become $A_0 = (C_{11}/C_{33})A_l^2 + A_t^2 + C^2$ and $B_0 = (C_{11}/C_{33})A_l^2 A_t^2$ from equation (12) in [110] propagation direction of cubic crystals, as well as for suitable non-cubic crystals. It is thought that further research requires evaluation of the C^2 for new materials already available, as well as crystal (system) synthesis with the suitable C^2 for SAW applications. The condition $V^{\text{th}} > V_l$ for crystals could be also one multi-promising problem already in Acoustics, as there is the room temperature superconductivity problem. The supersonic SAWs along with the RTW-waves could be convenient phenomena for measurements of thickness and other properties of superconducting surface layer of a bulk superconductor discussed in Collins and Grant (1972), because superconductivity can be kept in the layer at magnetic fields, at which superconductivity is destroyed. Also, Collins and Grant (1972) review many applications of the RTW-waves, for instance, in electronic devices (Ballato 2001). It is impossible to review all possible application for the SAW different types. The new supersonic SAWs can be studied in the same ways, which are already done for the RTW-waves, for example, interactions with the other phonons, photons and electrons, as well as different scattering. It is thought that phonon–electron interactions play a major role in superconductivity. In general, superconducting materials are treated as isotropic in theory of superconductivity. However, they are anisotropic with a great number of chemical elements.

In addition, both the anisotropy term C^2 in equations (6) and (13) and the velocity V^{th} in equations (11) and (14) can be applied for searching the supersonic SAWs with $V^{\text{th}} > V_l$ in non-cubic crystals and layered systems. All crystal classes can be found in the excellent and classical textbook by Nye (1989). In order that the C^2 can exist in the corresponding view in equations (6) and/or (13) for non-cubic crystals, the elastic constants C_{15} and C_{35} should be zero. This can be fulfilled for suitable classes of tetragonal, orthorhombic, trigonal and hexagonal symmetries, see for example Nye (1989). Both the piezoelectric and

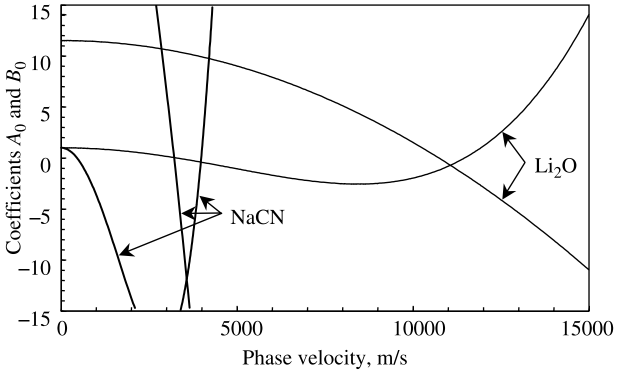

Figure 3. The coefficients $A_0 = A_l^2 + A_t^2 + C^2$ and $B_0 = A_l^2 A_t^2$ for the cubic crystals Li_2O (normal lines) and NaCN (thick lines), where there is $A_0(V^{\text{th}}) = 0$ and $B_0(V_t \text{ and } V_l) = 0$, as well as $B_0(V = 0) = 1$.

piezomagnetic effects can also result in the possible existence of the new SAWs in non-cubic crystals with $C^2 \gg 0$ and $V > V_l$. It is noted that the effects can broaden the V-range, in which the new supersonic SAWs can be found. It is also noted that for strong piezoelectrics, metallization of the surface of semi-infinite crystals can result in an unusual situation, namely, in increasing the phase velocity of Rayleigh type SAWs (Kakio *et al.* 1999). It is thought that electrical boundary conditions, such as the surface metallization, could also have an effect on the existence possibility of the new supersonic SAW with $V \sim V_l$.

4. Conclusions

In the present work, the focus was given to such crystal characteristics as both the anisotropy term C^2 and the threshold velocity V^{th}. Evaluation of the both for crystals can give information about the possible existence of new supersonic SAWs with $C^2 \gg 0$ and $V > V_l$, as well as about the RTW-wave missing with $C^2 \ll 0$ in the phase velocity range $0 < V < V_{\mathrm{t}}$ in some propagation directions. For instance, for the cubic crystals with $C^2 < -4$, the RTW2-waves cannot propagate below the speed V_{t} in [100] propagation direction. Of all the treated cubic crystals listed in the table, none with negative $C^2 < -4$ were found. Probably, there are no such cubic crystals, because there is the condition $C_{11} > C_{12}$. The cubic crystals Na, Pb, Ag, Au, Cu, Pd, Li, K, Rb, CuCl, CuBr, β-brass, $FeAl_2O_4$ and Fe_2TiO_4 have negative $C^2 < -2$, for which there is a sector with all four real roots for $V < V_{\mathrm{t}}$. Also, such negative $C^2 < -2$ results in the imaginary V^{th}. In the case of a positive C^2, the RTW2-waves can always exist with $V < V_{\mathrm{t}}$. It is noted that the RTW2-wave features in crystals with $C^2 > 0$ do not differ from the classical RW2-waves in isotropic media characterized by completely imaginary roots (5). However, in the case of $C^2 \gg 0$ there is the existence possibility of the new supersonic SAWs with $V > V_l$, for which all the roots (5) are already complex, similar to the case of $C^2 < 0$. The cubic crystals Li_2O, KRS-5, KRS-6, NH_4Br, PbS, RbF, RbCl, RbI, RbBr, AgCl, KF, KI, KCl, KCN and NaCN have a corresponding great anisotropy term $C^2 > 2$, of which there are crystals with even a "gigantic" $C^2 > 5$. However, evaluation of the velocity Vth did not show the possible existence of the new supersonic SAWs with the condition $V^{\mathrm{th}} > V_l$ in the crystals. Also, many effects were discussed, which can cause the appearance of the supersonic SAWs above the speed V_l with the same basis, namely, with a great $C^2 \gg 0$ as the limit case of such SAW existence.

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