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BRIEF NOTES AND DISCUSSIONS

On existence of eight new interfacial SH-waves in dissimilar piezoelectromagnetics of class 6 mm

A. A. Zakharenko

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Abstract This research report discovers eight new nondispersive interfacial shear-horizontal (SH) waves. The obtained theoretical results correspond to the combination of two sets of different eigenvector components in the propagation problems of the new interfacial SH-waves guided by the perfectly bonded interface between two dissimilar piezoelectromagnetics of symmetry class 6 mm. The explicit forms of the solutions for finding of the propagation velocities and the existence conditions for the new interfacial SH-waves were also obtained. The existence conditions are very important because they allow the existence of the new interfacial SH-waves. Sample calculation was performed for two dissimilar $BaTiO_3$–$CoFe_2O_4$ composites. This theoretical work and further developments can be also useful for design of various optical and microwave technical devices, as well as the nondestructive testing and evaluation of common interfaces between two suitable piezoelectromagnetic materials. These SH-polarized ultrasonic guided waves can be also useful for some considerations in order to infer changes in the adhesive properties at the interface located within an adhesive bond joining two PEM materials.

A. A. Zakharenko (✉)
International Institute of Zakharenko Waves (IIZWs), ul. Chaikovskogo, 20-304, 660014 Krasnoyarsk, Russia
e-mail: aazaaz@inbox.ru

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1 Introduction

1971 is the year when Maerfeld and Tournois [1] have introduced their collaborative theoretical work on new nondispersive interfacial shear-horizontal (SH) wave guided by the common interface of two dissimilar transversely isotropic piezoelectrics (semi-infinite media or half-spaces) belonging to class 6 mm. They have also found the existence condition for the new interfacial SH-waves later called the interfacial Maerfeld–Tournois (MT) wave. It is worth noting that the existence conditions are very important for the problems of wave propagation at the interface of two solids because they clarify some specific conditions of wave existence. They stated that the mechanical displacement amplitude of the MT-wave decreases with distance away from the common interface into both media. It is well-known that piezoelectrics (PEs) generally represent single-phase materials. This is also true for piezomagnetics (PMs). Piezoelectromagnetics (PEMs) frequently represent two-phase materials and they can possess piezoelectric, piezomagnetic, and magnetoelectric (ME) effects. As a result, twenty-two new nondispersive interfacial SH-waves can be revealed in the layered system consisting of two dissimilar PEM half-spaces of symmetry class 6 mm.

This was theoretically demonstrated in book [2] published in 2012.

The theory developed by Maerfeld and Tournois [1] can be used for the single-phase PE or PM materials and is not suitable for the two-phase PEM composites. Two-phase PEM laminated materials are multi-promising and cause a big interest among different research groups. Also, the problem of wave propagation is significantly complicated when one medium is a PEs and the second medium is a PMs. It is thought that this case represents the simplest laminated PEM composite. Soh and Liu [3] have theoretically treated propagation of interfacial SH-waves along the common interface in a PE–PM bi-material. The PE half-space is perfectly bonded with the PM one. Both the materials are hexagonal crystals of symmetry class 6 mm. For this case, they have obtained the phase velocity solution in an explicit form and discussed two existence conditions for the interfacial SH-waves. Concerning an imperfect interface, Huang et al. [4] have also developed a theory that describes interfacial SH-wave propagation in a two-phase PE/PM structure. They also treated the case when both the hexagonal materials pertain to class 6 mm. They have solidly obtained an exact solution and the existence condition for the interfacial SH-wave propagation in such bi-material and found that the interfacial imperfection can strongly affect the wave velocity. For the perfectly bonded interface, they stated that their result agrees with that derived in Ref. [3] when the interface is grounded. Book [2] stated that such nondispersive interfacial SH wave guided by the ungrounded interface can be called the Soh–Liu wave and the interfacial SH wave guided by the grounded interface can be called the Huang–Li–Lee wave. However, the treatment of the case when the wave propagation is guided by the common interface between two dissimilar PEMs is more complicated than the cases of propagation of the Maerfeld–Tournois, Soh–Liu, or Huang–Li–Lee waves. Therefore, this short report develops (but not finalizes) the theoretical results obtained in book [2].

It is also necessary to mention potential applications and review works on the ME effect and composites. Generally, a continuous interest occurs in a study of the ME effect in composites for development of smart materials in the microwave technology. This research arena is rapidly developed. Therefore, reviews are annually published on discussions of most recent advances in the physics of ME interactions in layered composites and nanostructures. The PEM composites are able to facilitate the conversion of energy between electric and magnetic fields and represent potential candidates for use as magnetic-field probes, in electronic packaging, as acoustic devices, hydrophones, in medical ultrasonic imaging, or as sensors and actuators [14]. Reviews [5–17] on the ME effect and composites are recommended for the reader to receive complete information on the subject, find the complete list of reviews in work [9]. Modern industry can have an increasing interest in the following possible applications of ME materials: solid state non-volatile memory, solid state memories based on spintronics, multi-state memory which can find application in quantum computing area, light computing, magnetic-electric energy converting components, electrical/optical polarization components which can find applications in communication. Therefore, to know acoustic wave characteristics can be important for nondestructive testing and evaluation of various interfaces between two dissimilar ME materials representing PEMs. Let's start the theoretical discovery of some wave properties for the problem of wave propagation along the common interface between two dissimilar PEMs.

2 Statement of the problem and fundamental formulae

Piezoelectromagnetic (composite) materials [18–25] possess the elastic stiffness constant C, piezoelectric constant e, piezomagnetic coefficient h, dielectric permittivity coefficient ε, magnetic permeability coefficient μ, and electromagnetic constant α. Also, it is natural that such two-phase composite can have an average mass density ρ. To study wave propagation guided by the common interface between two dissimilar piezoelectromagnetics (PEMs) denoted by PEM1 and PEM2, it is necessary to use the corresponding superscripts "I" and "II" to distinguish these transversely isotropic materials of class 6 mm from each other. Therefore, wave characteristics V_{tem}^{I} and V_{tem}^{II} stand for the corresponding bulk SH-waves coupled with both the electrical (φ) and magnetic (ψ) potentials. These velocities are very important in the theory of interfacial SH-wave propagation because they participate in the existence conditions. The rectangular coordinate system shown in Fig. 1 is utilized for the layered system consisting of two dissimilar PEM half-spaces perfectly bonded at the common interface when the x_3-axis is directed along the normal to the interface. For both the PEM1 and PEM2, the propagation direction is along the x_1-axis and perpendicular to the sixfold symmetry axis directed along the x_2-axis. The anti-plane polarized interfacial SH-waves can damp from the common interface towards the crystal depth, namely towards the positive values of the x_3-axis in half-space II (PEM2) and towards the negative values of the x_3-axis in half-space I (PEM1). It is worth noticing that this propagation direction allows one to study pure SH-waves coupled with both the electrical and magnetic potentials.

For the used rectangular coordinate system, the corresponding coupled equations of motion are written in the well-known form of the homogeneous partial differential equations of the second order. With the fact that the light speed in a vacuum is five orders larger than the speed of sound in a solid, the composition of these coupled equations of motion is based on the quasi-static approximation: constitutive relations, electrostatics, and magnetostatics are written. In this short report, it is unnecessary to write down all the equations and their transformations to demonstrate the complete way leading to final results. The reader can actually read book [2] for the purpose. However, it is possible to write down the coupled equations of motion that correspond to the studied case of the SH-wave propagation coupled with both the electrical and magnetic potentials. These coupled equations of motion can be composed in the following differential form:

$$\rho \frac{\partial^2 U}{\partial t^2} = C\left(\frac{\partial^2 U}{\partial x_1^2} + \frac{\partial^2 U}{\partial x_3^2}\right) + e\left(\frac{\partial^2 \varphi}{\partial x_1^2} + \frac{\partial^2 \varphi}{\partial x_3^2}\right) + h\left(\frac{\partial^2 \psi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_3^2}\right) \quad (1)$$

$$0 = e\left(\frac{\partial^2 U}{\partial x_1^2} + \frac{\partial^2 U}{\partial x_3^2}\right) - \varepsilon\left(\frac{\partial^2 \varphi}{\partial x_1^2} + \frac{\partial^2 \varphi}{\partial x_3^2}\right) - \alpha\left(\frac{\partial^2 \psi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_3^2}\right) \quad (2)$$

$$0 = h\left(\frac{\partial^2 U}{\partial x_1^2} + \frac{\partial^2 U}{\partial x_3^2}\right) - \alpha\left(\frac{\partial^2 \varphi}{\partial x_1^2} + \frac{\partial^2 \varphi}{\partial x_3^2}\right) - \mu\left(\frac{\partial^2 \psi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_3^2}\right) \quad (3)$$

It is also necessary to state that for the treated case of the wave propagation with the anti-plane polarization, these homogeneous partial differential equations of the second order written above must have solutions in the following plane wave form:

$$U_2 = U = U^0 \exp[j(k_1x_1 + k_2x_2 + k_3x_3 - \omega t)] \quad (4)$$

$$U_4 = \varphi = \varphi^0 \exp[j(k_1x_1 + k_2x_2 + k_3x_3 - \omega t)] \quad (5)$$

$$U_5 = \psi = \psi^0 \exp[j(k_1x_1 + k_2x_2 + k_3x_3 - \omega t)] \quad (6)$$

In all the equations written above, U, φ, and ψ are the mechanical displacement component along the x_2-axis, electrical potential, and magnetic potential, respectively; U^0, φ^0, and ψ^0 are called the initial amplitudes that should be perfectly determined as eigenvector components. $j = (-1)^{1/2}$, t, and ω stand for the imaginary unity, time, and angular frequency, respectively; $\omega = 2\pi\nu$ where ν is the linear frequency. Also, the parameters k_1, k_2, and k_3 are the wavevector components: $(k_1, k_2, k_3) = k(n_1, n_2, n_3)$, where the directional cosines n_1, n_2, and n_3 are defined as follows: $n_1 = 1, n_2 = 0$, and $n_3 \equiv n_3$. The wavenumber k in the direction of wave propagation can be naturally normalized by the wavelength λ as follows: $k\lambda = 2\pi$.

It is convenient to deal with the coupled equations of motion transformed into the corresponding tensor

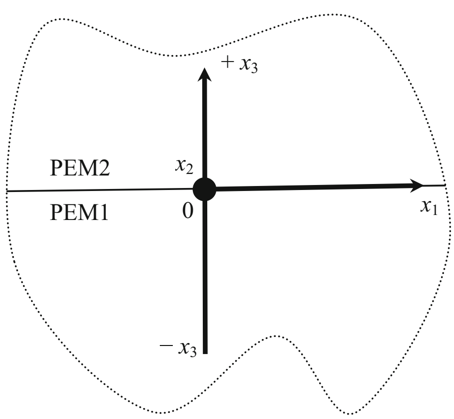

Fig. 1 The configuration of the problem of the SH-wave propagation guided by the interface between two dissimilar piezoelectromagnetics (PEM1 and PEM2) of the transversely isotropic 6 mm class. The x_2-axis is perpendicular to the figure plane and the coordinate beginning is denoted by "0"

form [2] known as the modified Green-Christoffel equation that can solidly reveal all the eigenvalues n_3 and the corresponding eigenvector components. Note that the differential and tensor forms of the coupled equations of motion are identical. With the tensor form, the following six eigenvalues can be obtained:

$$n_3^{(1,2)} = n_3^{(3,4)} = \pm \mathrm{j} \quad (7)$$

$$n_3^{(5,6)} = \pm \mathrm{j}\sqrt{1 - \left(V_{ph}/V_{tem}\right)^2} \quad (8)$$

It is clearly seen in Eq. (8) that the eigenvalues depend on the phase velocity $V_{ph} = \omega/k$ and the velocity V_{tem} called the bulk shear-horizontal (SH) wave coupled with both the electrical and magnetic potentials. The value of V_{tem} is defined by the following expression:

$$V_{tem} = \sqrt{C/\rho}\left(1 + K_{em}^2\right)^{1/2} \quad (9)$$

The bulk SH-wave velocity V_{tem} depends on the coefficient of magnetoelectromechanical coupling (CMEMC). This nondimensional parameter exposes the physical sense and is defined by

$$K_{em}^2 = \frac{\mu e^2 + \varepsilon h^2 - 2\alpha e h}{C(\varepsilon\mu - \alpha^2)} = \frac{e(e\mu - h\alpha) - h(e\alpha - h\varepsilon)}{C(\varepsilon\mu - \alpha^2)} = \frac{eM_2 - hM_1}{CM_3} \quad (10)$$

Three coupling mechanisms of the CMEMC such as $M_1 = e\alpha - h\varepsilon$, $M_2 = e\mu - h\alpha$, and $M_3 = \varepsilon\mu - \alpha^2$ are discussed in paper [24]. It is also clearly seen in definition (10) that the CMEMC can approach an infinity when $\varepsilon\mu = \alpha^2$ occurs. In general, one can find that $\varepsilon\mu \gg \alpha^2$ is fair. Paper [25] explains the reader that two suitable sets of the eigenvector components can exist for each eigenvalue. It also discusses the suitable forms of the eigenvectors among the other possible forms. Therefore, the first suitable set of the eigenvector components can be written in the following forms:

$$\left(U^{0(1)}, \varphi^{0(1)}, \psi^{0(1)}\right) = \left(U^{0(3)}, \varphi^{0(3)}, \psi^{0(3)}\right) = (0, \alpha, -\varepsilon) \quad (11)$$

$$\left(U^{0(5)}, \varphi^{0(5)}, \psi^{0(5)}\right) = \left(\frac{e\alpha - h\varepsilon}{CK_{em}^2}, -\frac{eh}{CK_{em}^2} + \alpha, \frac{e^2}{CK_{em}^2} - \varepsilon\right) \quad (12)$$

It is stated that the corresponding eigenvector components in expressions (11) and (12) are coupled via the CMEMC mechanism $e\alpha - h\varepsilon$ mentioned above. This is obvious because one can get the following equalities:

$$e\varphi^{0(3)} + h\psi^{0(3)} = e\varphi^{0(5)} + h\psi^{0(5)} = e\alpha - h\varepsilon = M_1 \quad (13)$$

It is possible to compose the second set of the eigenvector components for the same eigenvalues defined by expressions (7) and (8). The eigenvector components respectively read:

$$\left(U^{0(1)}, \varphi^{0(1)}, \psi^{0(1)}\right) = \left(U^{0(3)}, \varphi^{0(3)}, \psi^{0(3)}\right) = (0, \mu, -\alpha) \quad (14)$$

$$\left(U^{0(5)}, \varphi^{0(5)}, \psi^{0(5)}\right) = \left(\frac{e\mu - h\alpha}{CK_{em}^2}, -\frac{h^2}{CK_{em}^2} + \mu, \frac{eh}{CK_{em}^2} - \alpha\right) \quad (15)$$

It is blatant in expressions (14) and (15) that the corresponding eigenvector components are coupled through the other CMEMC mechanism $e\mu - h\alpha$ because

$$e\varphi^{0(3)} + h\psi^{0(3)} = e\varphi^{0(5)} + h\psi^{0(5)} = e\mu - h\alpha = M_2 \quad (16)$$

It is essential to state that the coupled equations of motion are separately written for each of the dissimilar PEM half-spaces. Therefore, each PEM half-space possesses its own eigenvalues and eigenvectors and it is convenient to utilize the corresponding superscripts "I" and "II" to distinguish them. The found suitable eigenvalues and eigenvectors are exploited for construction of complete mechanical displacement, complete electrical potential, and complete magnetic potential. It is natural to require equalities of the corresponding complete parameters at the interface. After this requirement, the suitable phase velocity V_{ph} can be determined by utilizing possible mechanical, electrical, and magnetic boundary conditions at the interface. For this purpose, corresponding determinants of the boundary conditions must be also constructed. Different electrical and magnetic boundary conditions can be used when the following mechanical boundary conditions are employed at the common interface between PEM1 and PEM2: $U^{\mathrm{I}} = U^{\mathrm{II}}$ and $(\sigma_{32})^{\mathrm{I}} = (\sigma_{32})^{\mathrm{II}}$ at $x_3 = 0$ (see Fig. 1) where

σ_{32} stands for the normal component of the stress tensor. The boundary conditions are perfectly described in theoretical work [26] by Al'shits, Darinskii, and Lothe. The following electrical boundary conditions can occur: the electrically closed case ($\varphi = 0$), electrically open case ($D_3 = 0$), and the continuity of both φ and D_3 at the common interface ($x_3 = 0$ in Fig. 1) between two dissimilar PEM half-spaces, i.e. $\varphi^{\mathrm{I}} = \varphi^{\mathrm{II}}$ and $(D_3)^{\mathrm{I}} = (D_3)^{\mathrm{II}}$, where D_3 is the normal component of the electrical displacements (electrical induction) and φ is the electrical scalar potential. Also, the magnetic boundary conditions are as follows: the magnetically closed case ($B_3 = 0$), magnetically open case ($\psi = 0$), and the continuity of both ψ and B_3 at $x_3 = 0$, i.e. $\psi^{\mathrm{I}} = \psi^{\mathrm{II}}$ and $(B_3)^{\mathrm{I}} = (B_3)^{\mathrm{II}}$, where B_3 is the normal component of the magnetic flux (magnetic displacement, magnetic induction) and ψ is the magnetic scalar potential.

In this short report there is no necessity to write down the complicated boundary conditions' determinants. By expanding the corresponding determinant, the explicit form for calculation of the appropriate velocity can be obtained. Therefore, the final results for the found new interfacial SH-wave velocities are demonstrated in the cases from (i-1) to (i-8) written below. These eight cases correspond to different combinations of the electrical and magnetic boundary conditions at the interface $x_3 = 0$ (see Fig. 1) discussed above. These eight cases represent the discovery of this report, namely the discovery of eight new nondispersive interfacial SH-waves. To avoid confusion, the numeration of the new wave velocities in the cases from (i-1) to (i-8) starts with number 23 because twenty-two new nondispersive interfacial SH-waves propagating along the common interface between two dissimilar PEM half-spaces were previously discovered in book [2]. This book has investigated three possibilities for $V^{\mathrm{I}}_{tem} < V^{\mathrm{II}}_{tem}$: (A) only the first eigenvector components (11) and (12) are used for both the dissimilar PEM half-spaces; (B) only the second eigenvectors (14) and (15) are utilized for both the dissimilar PEM half-spaces; (C) the first eigenvectors are used for PEM1 and the second eigenvectors are exploited for PEM2. The third possibility is called the eigenvectors' mixing that is possible because two dissimilar media are studied. Also, possibilities (A) and (B) can be also valid for the reverse configuration of $V^{\mathrm{I}}_{tem} < V^{\mathrm{II}}_{tem}$ because one can always rearrange the configuration, namely PEM1 → PEM2 and PEM2 → PEM1 in order to get $V^{\mathrm{I}}_{tem} < V^{\mathrm{II}}_{tem}$ anew. This is actually true for possibility (A) when only the first eigenvectors are utilized for both the PEM materials because such rearrangement of the configuration does not change the eigenvectors: the final configuration will also use the first eigenvectors. This is also suitable for possibility (B) when only the second eigenvectors are exploited. Therefore, it is likely to state that possibilities (A), (B), and (C) for $V^{\mathrm{I}}_{tem} < V^{\mathrm{II}}_{tem}$ were completely researched in book [2] that is not true in the case of $V^{\mathrm{I}}_{tem} < V^{\mathrm{II}}_{tem}$ for the third possibility of the eigenvectors' mixing.

Let's discuss possibility (C) of the eigenvectors' mixing and explain the results of the rearrangement for $V^{\mathrm{I}}_{tem} < V^{\mathrm{II}}_{tem}$ given below in the cases from (i-1) to (i-8). Indeed, in possibility (C) the first and second eigenvectors are used for PEM1 and PEM2, respectively. It is necessary to state that the following rearrangements can be carried out for the case of the eigenvectors' mixing: the first is for $V^{\mathrm{I}}_{tem} < V^{\mathrm{II}}_{tem}$ and the second is for $V^{\mathrm{I}}_{tem} < V^{\mathrm{II}}_{tem}$. For $V^{\mathrm{I}}_{tem} < V^{\mathrm{II}}_{tem}$, it is obvious that it is possible to apply a rearrangement that actually results in the situation when the second eigenvectors instead of the first ones are used for PEM1 and the first eigenvectors instead of the second ones are used for PEM2. This rearrangement requires complicated calculations. The case of $V^{\mathrm{I}}_{tem} < V^{\mathrm{II}}_{tem}$ does not require any change in the exploitation of the eigenvectors: the first eigenvectors are used for PEM1 and the second eigenvectors are utilized for PEM2. It is thought that it is more suitable case realized below in points from (i-1) to (i-8) for various electrical and magnetic boundary conditions. To circumvent any confusion, the numeration of the new interfacial SH-wave velocities obtained below starts with number 23 to follow the results obtained in book [2].

(i-1) The case of $\varphi^{I} = \varphi^{II} = 0$ and $\psi^{I} = \psi^{II} = 0$

$$\sqrt{1-\left(\frac{V_{new23}}{V^{\mathrm{I}}_{tem}}\right)^2} + \frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{A^{\mathrm{II}}}{A^{\mathrm{I}}}\sqrt{1-\left(\frac{V_{new23}}{V^{\mathrm{II}}_{tem}}\right)^2} = \frac{\left(K^{\mathrm{I}}_{em}\right)^2}{A^{\mathrm{I}}} + \frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{\left(K^{\mathrm{II}}_{em}\right)^2}{A^{\mathrm{I}}} = \frac{M^{\mathrm{II}}_3\left(e^{\mathrm{I}}M^{\mathrm{I}}_2 - h^{\mathrm{I}}M^{\mathrm{I}}_1\right) + M^{\mathrm{I}}_3\left(e^{\mathrm{II}}M^{\mathrm{II}}_2 - h^{\mathrm{II}}M^{\mathrm{II}}_1\right)}{M^{\mathrm{II}}_3 D^{\mathrm{I}}} \tag{17}$$

(i-2) The case of $\varphi^I = \varphi^{II} = 0$ *and* $B_3^I = B_3^{II} = 0$

$$\sqrt{1-\left(\frac{V_{new24}}{V_{tem}^{\mathrm{I}}}\right)^2}+\frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{A^{\mathrm{II}}}{A^{\mathrm{I}}}\sqrt{1-\left(\frac{V_{new24}}{V_{tem}^{\mathrm{II}}}\right)^2} = \frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}}{M_1^{\mathrm{I}}}\frac{K_A^{\mathrm{I}}}{A^{\mathrm{I}}}+\frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{K_M^{\mathrm{II}}}{A^{\mathrm{I}}} = \frac{e^{\mathrm{I}}\mu^{\mathrm{II}}M_3^{\mathrm{II}}M_2^{\mathrm{I}}+M_3^{\mathrm{I}}\left(M_2^{\mathrm{II}}\right)^2}{\mu^{\mathrm{II}}M_3^{\mathrm{II}}D^{\mathrm{I}}} \tag{18}$$

(i-3) The case of $D_3^I = D_3^{II} = 0$ *and* $\psi^I = \psi^{II} = 0$

$$\sqrt{1-\left(\frac{V_{new25}}{V_{tem}^{\mathrm{I}}}\right)^2}+\frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{A^{\mathrm{II}}}{A^{\mathrm{I}}}\sqrt{1-\left(\frac{V_{new25}}{V_{tem}^{\mathrm{II}}}\right)^2} = \frac{K_E^{\mathrm{I}}}{A^{\mathrm{I}}}-\frac{h^{\mathrm{II}}\alpha^{\mathrm{II}}}{M_2^{\mathrm{II}}}\frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{K_A^{\mathrm{II}}}{A^{\mathrm{I}}} = \frac{M_3^{\mathrm{II}}\left(M_1^{\mathrm{I}}\right)^2-h^{\mathrm{II}}\varepsilon^{\mathrm{I}}M_3^{\mathrm{I}}M_1^{\mathrm{II}}}{\varepsilon^{\mathrm{I}}M_3^{\mathrm{II}}D^{\mathrm{I}}} \tag{19}$$

(i-4) The case of $\varphi^I = \varphi^{II}$, $D_3^I = D_3^{II}$, $\psi^I = \psi^{II}$, *and* $B_3^I = B_3^{II}$

$$\begin{aligned}&\sqrt{1-\left(\frac{V_{new26}}{V_{tem}^{\mathrm{I}}}\right)^2}+\frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{A^{\mathrm{II}}}{A^{\mathrm{I}}}\sqrt{1-\left(\frac{V_{new26}}{V_{tem}^{\mathrm{II}}}\right)^2}\\ &= \left(1+\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{M_1^{\mathrm{I}}}\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{M_2^{\mathrm{II}}}\right)^{-1}\times\left\{\left(\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}}{M_1^{\mathrm{I}}}\frac{K_A^{\mathrm{I}}}{A^{\mathrm{I}}}-\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{M_2^{\mathrm{II}}}\frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{K_M^{\mathrm{II}}}{A^{\mathrm{I}}}\right)\right.\\ &\left.\left(1-\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{M_1^{\mathrm{I}}}\right)+\left(1+\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{M_2^{\mathrm{II}}}\right)\left(\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{M_1^{\mathrm{I}}}\frac{K_E^{\mathrm{I}}}{A^{\mathrm{I}}}-\frac{h^{\mathrm{II}}\alpha^{\mathrm{II}}}{M_2^{\mathrm{II}}}\frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{K_A^{\mathrm{II}}}{A^{\mathrm{I}}}\right)\right\}\\ &= \frac{M_1^{\mathrm{I}}M_2^{\mathrm{II}}}{M_1^{\mathrm{I}}M_2^{\mathrm{II}}+e^{\mathrm{I}}h^{\mathrm{II}}\varepsilon^{\mathrm{I}}\mu^{\mathrm{II}}}\left\{\frac{e^{\mathrm{I}}\left(M_2^{\mathrm{I}}M_3^{\mathrm{II}}-M_2^{\mathrm{II}}M_3^{\mathrm{I}}\right)}{M_3^{\mathrm{II}}D^{\mathrm{I}}}\frac{M_1^{\mathrm{I}}-h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{M_1^{\mathrm{I}}}\right.\\ &\left.+\frac{h^{\mathrm{II}}\left(M_1^{\mathrm{I}}M_3^{\mathrm{II}}-M_1^{\mathrm{II}}M_3^{\mathrm{I}}\right)}{M_3^{\mathrm{II}}D^{\mathrm{I}}}\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}+M_2^{\mathrm{II}}}{M_2^{\mathrm{II}}}\right\}\end{aligned} \tag{20}$$

(i-5) The case of $\varphi^I = \varphi^{II}$, $D_3^I = D_3^{II}$, *and* $B_3^I = B_3^{II} = 0$

$$\sqrt{1-\left(\frac{V_{new27}}{V_{tem}^{\mathrm{I}}}\right)^2}+\frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{A^{\mathrm{II}}}{A^{\mathrm{I}}}\sqrt{1-\left(\frac{V_{new27}}{V_{tem}^{\mathrm{II}}}\right)^2} = \frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}}{M_1^{\mathrm{I}}}\frac{K_A^{\mathrm{I}}}{A^{\mathrm{I}}}-\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{M_2^{\mathrm{II}}}\frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{K_M^{\mathrm{II}}}{A^{\mathrm{I}}} = \frac{e^{\mathrm{I}}\left(M_3^{\mathrm{II}}M_2^{\mathrm{I}}-M_3^{\mathrm{I}}M_2^{\mathrm{II}}\right)}{M_3^{\mathrm{II}}D^{\mathrm{I}}} \tag{21}$$

(i-6) The case of $\varphi^I = \varphi^{II}$, $D_3^I = D_3^{II}$, *and* $\psi^I = \psi^{II} = 0$

$$\begin{aligned}&\sqrt{1-\left(\frac{V_{new28}}{V_{tem}^{\mathrm{I}}}\right)^2}+\frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{A^{\mathrm{II}}}{A^{\mathrm{I}}}\sqrt{1-\left(\frac{V_{new28}}{V_{tem}^{\mathrm{II}}}\right)^2}\\ &= \frac{\left(K_{em}^{\mathrm{I}}\right)^2}{A^{\mathrm{I}}}-\left(1+\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{M_2^{\mathrm{II}}}\right)\frac{h^{\mathrm{II}}\alpha^{\mathrm{II}}}{M_2^{\mathrm{II}}}\frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{K_A^{\mathrm{II}}}{A^{\mathrm{I}}}-\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{M_2^{\mathrm{II}}}\frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{K_M^{\mathrm{II}}}{A^{\mathrm{I}}}\\ &= \frac{\left(K_{em}^{\mathrm{I}}\right)^2}{A^{\mathrm{I}}}-\frac{M_3^{\mathrm{I}}}{M_3^{\mathrm{II}}}\frac{h^{\mathrm{II}}M_1^{\mathrm{II}}\left(e^{\mathrm{I}}\mu^{\mathrm{II}}+M_2^{\mathrm{II}}\right)+e^{\mathrm{I}}\left(M_2^{\mathrm{II}}\right)^2}{M_2^{\mathrm{II}}D^{\mathrm{I}}}\end{aligned} \tag{22}$$

(i-7) The case of $D_3^I = D_3^{II} = 0$, $\psi^I = \psi^{II}$, *and* $B_3^I = B_3^{II}$

$$\sqrt{1-\left(\frac{V_{new29}}{V_{tem}^{\mathrm{I}}}\right)^2}+\frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{A^{\mathrm{II}}}{A^{\mathrm{I}}}\sqrt{1-\left(\frac{V_{new29}}{V_{tem}^{\mathrm{II}}}\right)^2} = \frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{M_1^{\mathrm{I}}}\frac{K_E^{\mathrm{I}}}{A^{\mathrm{I}}}-\frac{h^{\mathrm{II}}\alpha^{\mathrm{II}}}{M_2^{\mathrm{II}}}\frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{K_A^{\mathrm{II}}}{A^{\mathrm{I}}} = \frac{h^{\mathrm{II}}\left(M_3^{\mathrm{II}}M_1^{\mathrm{I}}-M_3^{\mathrm{I}}M_1^{\mathrm{II}}\right)}{M_3^{\mathrm{II}}D^{\mathrm{I}}} \tag{23}$$

(i-8) The case of $\varphi^I = \varphi^{II} = 0$, $\psi^I = \psi^{II}$, *and* $B_3^I = B_3^{II}$

$$\sqrt{1-\left(\frac{V_{new30}}{V_{tem}^{\mathrm{I}}}\right)^2}+\frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{A^{\mathrm{II}}}{A^{\mathrm{I}}}\sqrt{1-\left(\frac{V_{new30}}{V_{tem}^{\mathrm{II}}}\right)^2} = \frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{A^{\mathrm{II}}}{A^{\mathrm{I}}}+\left(1-\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{M_1^{\mathrm{I}}}\right)\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}}{M_1^{\mathrm{I}}}\frac{K_A^{\mathrm{I}}}{A^{\mathrm{I}}}+\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{M_1^{\mathrm{I}}}\frac{K_E^{\mathrm{I}}}{A^{\mathrm{I}}} = \frac{C^{\mathrm{II}}}{C^{\mathrm{I}}}\frac{A^{\mathrm{II}}}{A^{\mathrm{I}}}+\frac{e^{\mathrm{I}}M_2^{\mathrm{I}}\left(M_1^{\mathrm{I}}-h^{\mathrm{II}}\varepsilon^{\mathrm{I}}\right)+h^{\mathrm{II}}\left(M_1^{\mathrm{I}}\right)^2}{M_1^{\mathrm{I}}D^{\mathrm{I}}} \tag{24}$$

In the expressions from (17) to (24), one can also find that the following equalities were used:

$$A = 1 + K_{em}^2 \tag{25}$$

$$D = C\left(\varepsilon\mu-\alpha^2\right)+e(e\mu-h\alpha)-h(e\alpha-h\varepsilon) = CM_3+eM_2-hM_1 \tag{26}$$

$$K_E = K_{em}^2 - K_e^2 = \frac{M_1^2}{C\varepsilon M_3} = \frac{(e\alpha-h\varepsilon)^2}{C\varepsilon(\varepsilon\mu-\alpha^2)} \tag{27}$$

$$K_M = K_{em}^2 - K_m^2 = \frac{M_2^2}{C\mu M_3} = \frac{(e\mu-h\alpha)^2}{C\mu(\varepsilon\mu-\alpha^2)} \tag{28}$$

$$K_A = K_{em}^2 - K_\alpha^2 = \frac{M_1M_2}{C\alpha M_3} = \frac{(e\alpha-h\varepsilon)(e\mu-h\alpha)}{C\alpha(\varepsilon\mu-\alpha^2)} \tag{29}$$

In expression (27), the material parameter K_e^2 is called the coefficient of the electromechanical coupling that illuminates the physical sense. This parameter represents an important characteristic for purely piezoelectric materials. This nondimensional parameter is defined by

$$K_e^2 = \frac{e^2}{C\varepsilon} \tag{30}$$

In equality (28), one can find the other material parameter K_m^2 called the coefficient of the magnetomechanical coupling that physically combines the magnetic and mechanical material parameters and represents an important characteristic of pure piezomagnetics. The following equality clarifies this nondimensional parameter:

$$K_m^2 = \frac{h^2}{C\mu} \tag{31}$$

The third material parameter denoted by K_α^2 in expression (29) is also nondimensional and combines two terms with the electromagnetic constant α in the CMEMC K_{em}^2 (10). Like the previous two material parameters, it is also an exchange parameter that reads:

$$K_\alpha^2 = \frac{eh}{C\alpha} = \frac{\alpha eh}{C\alpha^2} \tag{32}$$

Comparison of the left-hand side with the right-hand side in expression (27) can reveal the physical sense of the following equality: $e\alpha = h\varepsilon$. It is apparent that $e\alpha = h\varepsilon$ satisfies when $K_{em}^2 = K_e^2$ occurs. It can be assumed that the situation of $e\alpha = h\varepsilon$ relates to the case when the magnetic contribution to the CMEMC can be compensated because the CMEMC value becomes equal to the value of the purely piezoelectric characteristic defined by expression (30). Indeed, the CMEMC can be also introduced as a sum of several terms: $K_{em}^2 = K_e^2 + K_m^2 + K_{ex}^2$, where the last component stands for an exchange term. A complicated explicit form of this exchange term can be readily demonstrated by using definitions (10), (30), and (31). In the same manner, it is possible to compare the left-hand and right-hand sides in expression (28). It is flagrant that $e\mu = h\alpha$ occurs when $K_{em}^2 = K_m^2$ can exist. This can also mean that the condition of $e\mu = h\alpha$ belongs to the second possibility when the electric contribution to the CMEMC can be compensated for the reason that the CMEMC value becomes equal to the value of the purely piezomagnetic material parameter K_m^2 (31). In equality (29), one can also find that both the possibilities can exist: $K_{em}^2 - K_\alpha^2 = 0$ occurs when either $e\alpha$ - $h\varepsilon = 0$ or $e\mu$ - $h\alpha = 0$ is fulfilled. This can manifest that $K_{em}^2 = K_\alpha^2$ can occur when either electrical or magnetic properties can dominate. In addition, one can state that both $K_{em}^2 - K_e^2 = 0$ and $K_{em}^2 - K_\alpha^2 = 0$ as soon as the equality of $e\alpha - h\varepsilon = 0$ satisfies, and both $K_{em}^2 - K_m^2 = 0$ and $K_{em}^2 - K_\alpha^2 = 0$ as soon as $e\mu - h\alpha = 0$ occurs.

3 Discussion on the existence of the interfacial SH-waves

Next, one can also find that in the final expressions from (17) to (24), $\varepsilon^{II}\mu^{II} - \alpha^{II^2} = 0$ in the corresponding denominators on the left- and right-hand sides leads to the fact that the both sides of each expression approach to infinite values. Therefore, it is possible to multiply the both sides of each expression by the factor of $\varepsilon^{II}\mu^{II} - \alpha^{II^2}$. As a result, the first term on the left-hand side vanishes and one does not deal here with infinities on the both sides. However, $\varepsilon^{II}\mu^{II} - \alpha^{II^2} = 0$ sets an infinite number for the value of the bulk velocity V_{tem}^{II} (9) due to the following fact: $K_{em}^{II^2}\left(\varepsilon^{II}\mu^{II} - \alpha^{II^2} = 0\right) = \infty$, see definition (10) for the CMEMC. It is assumed that the value of V_{tem}^{II} cannot be larger than the well-known table value of the speed of light in a vacuum. Also, the used quasi-static approximation requires that the values of V_{tem}^{II} and the new interfacial SH-wave velocities must be significantly smaller than the value of the speed of light. As a rule, the values of V_{tem}^{I} and V_{tem}^{II} must be close to each other to have corresponding solutions for obtained expressions from (17) to (24). Therefore, it is essential to avoid the situation when $\varepsilon^{II}\mu^{II} - \alpha^{II^2} = 0$ occurs. In general, $\varepsilon^{I}\mu^{I} > > \alpha^{I^2}$ and $\varepsilon^{II}\mu^{II} > > \alpha^{II^2}$ occur because the value of the electromagnetic constant α is very small. So, it is possible to make a statement that one must deal with both the sides of the expressions when they represent finite numbers to find solutions. There is also the second peculiarity when the value of the right-hand side in all the expressions from (17) to (24) can approach to an infinite number. This is true when the following equality is satisfied:

$$D^{\mathrm{I}} = C^{\mathrm{I}}\left(\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}} - \alpha^{\mathrm{I}^2}\right) + e^{\mathrm{I}}\left(e^{\mathrm{I}}\mu^{\mathrm{I}} - h^{\mathrm{I}}\alpha^{\mathrm{I}}\right) - h^{\mathrm{I}}\left(e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right) = 0 \tag{33}$$

Also, one has to focus attention on the following three equalities for expression (20) to evade the discussed peculiarity:

$$e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}} = 0 \tag{34}$$

$$e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}} = 0 \tag{35}$$

$$\left(e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)\left(e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}\right) + e^{\mathrm{I}}h^{\mathrm{II}}\varepsilon^{\mathrm{I}}\mu^{\mathrm{II}} = 0 \tag{36}$$

It is worth noting that equality (34) also sets an infinite number for the right-hand side of expression (22) and equality (35) makes the same with the one of expression (24).

For the final expressions from (17) to (24), the following existence conditions must be also taken into account to obtain solutions:

$$Y > 0 \quad \text{and} \quad \left(V_{tem}^{\mathrm{II}}/V_{tem}^{\mathrm{I}}\right)^2 > 1 - Y^2 \quad \text{for } V_{tem}^{\mathrm{I}} > V_{tem}^{\mathrm{II}} \tag{37}$$

In inequalities (37), the parameter Y represents the corresponding right-hand sides of equations from (17) to (24). It is apparently seen in all the equations from (17) to (24) for the determination of the corresponding velocity of the new interfacial SH-wave that this parameter must be real and preferably have a positive sign. Also, the second inequality is for the case of $V_{tem}^{\mathrm{I}} > V_{tem}^{\mathrm{II}}$ and therefore, $V_{tem}^{\mathrm{II}}/V_{tem}^{\mathrm{I}}$ is smaller than unity. This means that values of the parameter Y must be smaller than unity, too. However, it is obvious that existence conditions (37) are the most simple because equations from (17) to (24) are quit complicated. Indeed, they can be suitable for the case of $C^{\mathrm{II}} \sim C^{\mathrm{I}}$. For the case of $C^{\mathrm{II}} \gg C^{\mathrm{I}}$, it is however possible that the right-hand side can be significantly larger than unity and one deals here with a more complicated case. As a result, a numerical simulation must be performed even in the cases when exact formulas are found.

It is also noted that the formulae from (17) to (24) can be significantly simplified when one deals with piezoelectromagnetic laminated composites consisting of a piezoelectric layer (half-space) on a piezomagnetic layer (half-space) or vice versa. This can be realized by applying $h^{\mathrm{I}} = 0$ and $e^{\mathrm{II}} = 0$ in the equations. It is also possible to use the electromagnetic constants such as $\alpha^{\mathrm{I}} = \alpha^{\mathrm{II}} = 0$ for simplicity. Similar laminated configurations can be also studied numerically. For instance, a meshless method based on the local Petrov–Galerkin approach was proposed in theoretical work [27] to solve static and dynamic problems of two-layered piezoelectromagnetic composites with specific properties. The authors of paper [27] have stated that the magnetoelectric effect is dependent on the ratio of the layer thicknesses and considered various boundary conditions and geometric parameters to analyze their influence on the value of the electromagnetic parameter. So, it is possible to discuss that the theoretical researches of this short report and book [2] represent the limit cases when the layer thicknesses have infinite values (the case of half-spaces) and the SH-waves are guided by the perfectly bonded interface. There is also book [28] on the SH-wave propagation in the transversely isotropic piezoelectromagnetic plates. It is expected that bi-plates consisting of two dissimilar piezoelectromagnetics perfectly coupled at the common interface can be analytically researched in the future. It is also well-known that suitable SH-waves can be used for nondestructive testing and evaluation of interfaces between two PEMs. SH-polarized ultrasonic guided waves are frequently considered in order to infer changes in the adhesive properties at interfaces located within an adhesive bond joining two materials. For instance, experimental work [29] suggests a promising use of SH-like guided modes for quantifying shear properties at adhesive interfaces, and shows that such waves can be employed for inferring adhesive and cohesive properties of bonds separately. Michel Castaings [29] has also discussed some improvements that can be applied to the process, and its potential for testing the interfacial adhesion of adhesively bonded composite components.

4 Results of calculation and further discussion

It is thought that for sample calculations, a frequently studied solid with the hexagonal (6 mm) symmetry can be chosen. For this purpose, the well-known $BaTiO_3$–$CoFe_2O_4$ composites can be utilized. For instance, the material parameters of two dissimilar $BaTiO_3$–$CoFe_2O_4$ composites can be borrowed from paper [30] or [31]. Let's use the material parameters given in paper [30] because they are more realistic in comparison with those given in paper [31]. This will be further discussed below. For the case of $V^{I}_{tem} > V^{II}_{tem}$, the following material parameters must be used for the second $BaTiO_3$–$CoFe_2O_4$ medium, namely composite II: $\rho^{II} = 5{,}730$ [kg/m^3], $C^{II} = 4.5 \times 10^{10}$ [N/m^2], $e^{II} = 0.1$ [C/m^2], $h^{II} = 340$ [T], $\varepsilon^{II} = 3.3 \times 10^{-10}$ [F/m], $\mu^{II} = -390 \times 10^{-6}$ [N/A^2], $\alpha^{II} = 2.8 \times 10^{-12}$ [s/m]. As a result, the SH-BAW velocity is $V^{II}_{tem} = 2794.094$ [m/s]. These material parameters roughly correspond to the 20 % volume fracture of $BaTiO_3$ in composite of $BaTiO_3$–$CoFe_2O_4$ [30]. The other dissimilar composite can roughly correspond to the 80 % volume fracture of $BaTiO_3$ in composite of $BaTiO_3$–$CoFe_2O_4$ [30]. So, the material parameters for medium I are as follows: $\rho^{I} = 5730$ [kg/m^3], $C^{I} = 5.0 \times 10^{10}$ [N/m^2], $e^{I} = 0.4$ [C/m^2], $h^{I} = 80$ [T], $\varepsilon^{I} = 10.0 \times 10^{-10}$ [F/m], $\mu^{I} = -80 \times 10^{-6}$ [N/A^2], $\alpha^{I} = 6.8 \times 10^{-12}$ [s/m], and $V^{I}_{tem} = 2956.343$ [m/s]. Thus, the reader can find that the condition of $V^{I}_{tem} > V^{II}_{tem}$ is satisfied. Besides, the reader can check that the limitation condition of $\varepsilon\mu > \alpha^2$ [13, 14] is also fulfilled for both the dissimilar composites.

However, one must be careful when the material parameters of paper [31] are used for calculations because the $BaTiO_3$–$CoFe_2O_4$ composite parameters [31] do not satisfy the obligatory condition of $\varepsilon\mu > \alpha^2$ [13, 14]. In addition, the dimension of the electromagnetic constant α in paper [31] is strange: [Ns/(Vm)] instead of [Ns/(VC)] or [s/m], namely $\alpha \sim -30 \times 10^{-6}$ [Ns/(Vm)]. Using the values of $\varepsilon \sim 30 \times 10^{-10}$ [F/m] and $\mu \sim 113 \times 10^{-6}$ [N/A^2] from paper [31], one can calculate that their value of $\alpha^2 \sim 0.90 \times 10^{-9}$ is significantly larger their value of $\varepsilon\mu \sim 3.39 \times 10^{-13}$. These facts lead to the necessity to exploit for calculations the material parameters [30] discussed above.

Sample calculations were performed with formula (24) representing case (i-8) for the following electrical and magnetic boundary conditions at the common interface: the electrical potential $\varphi^{I} = \varphi^{II} = 0$, continuity of the magnetic potential $\psi^{I} = \psi^{II}$, and the continuity of the normal component of the magnetic displacement $B^{I}_3 = B^{II}_3$. The result of the calculation for the case is shown in Fig. 2. The used function for the calculation was formed from Eq. (24): the left-hand side expression minus the right-hand side expression must be equal to zero to demonstrate the solution existence. For this purpose, the suitable value of the velocity V_{new30} of the thirtieth new interfacial SH-wave can be found when the function changes its sign, see in Fig. 2. According to the figure, the found value of the velocity is $V_{new30} \sim 2513.3$ [m/s]. It is clearly seen that the found speed is slower than both the SH-BAW speeds V^{I}_{tem} and V^{II}_{tem}. This is the requirement to deal with the interfacial SH-wave that are localized at the common interface and must damp towards the depth of either piezoelectromagnetic medium.

It is also possible to briefly discuss the theoretical method used in paper [31] concerning the same case of the interfacial SH-wave propagation in the hexagonal (6 mm) piezoelectromagnetics. For this aim, Otero et al. [31] have treated the same coupled equations of motion given in expressions (1), (2), and (3). It is well-known that to resolve these differential equations, they can be readily written in the tensor form [2] known as the modified Green-Christoffel equation. Indeed, the substitution of plane wave solutions (4), (5), and (6) into the differential form of the coupled equations of motion definitely leads to the aforementioned tensor form. It is indispensable to state here that both the differential and tensor forms of the coupled equations of motion are identical. To resolve the tensor form representing a set of three coupled homogeneous equations means to find the eigenvalues and the corresponding eigenvectors. This method is actual for

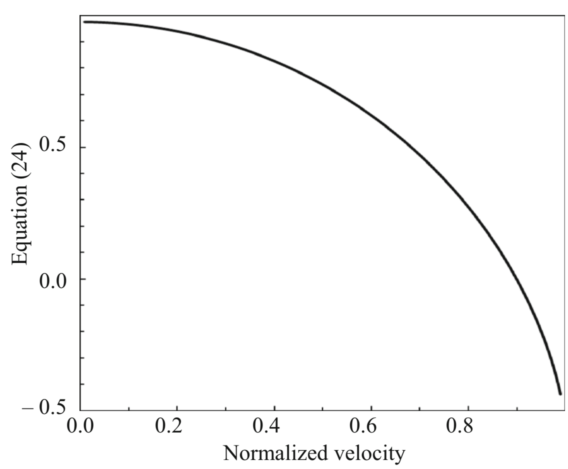

Fig. 2 The values of formula (24) versus the normalized velocity V_{new30}/V^{II}_{tem} because $V^{I}_{tem} > V^{II}_{tem}$, where the solution is found as soon as in Eq. (24) the expression on the left-hand side minus the expression on the right-hand side is equal to zero

wave propagation in any direction and any cut including the particularistic case treated in this report and paper [31]. However, Otero et al. [31] have used the other theoretical method to get solutions. Unfortunately, their strange method is developed to obtain solutions only in this particularistic case. Their method does not resolve the coupled equations of motion in any aforementioned form. Instead of that, they construct auxiliary equations that are already independent from each other and therefore, each of them can be independently resolved. To couple these independent equations anew, the boundary conditions are used, with which some coefficients that correspond to eigenvector components are found. However, the eigenvector components must be found when the equations of motion are used but not the boundary conditions because the eigenvector components certainly relate to the equations of motion. The boundary conditions are usually used to construct the corresponding determinant of the boundary conditions and to obtain the weight factors (but not the eigenvector components) in the theory of partial waves. So, it is necessary to state that the method by Otero et al. [31] can frequently lead to incorrect final results for the propagation velocity.

5 Conclusion

In the case of the perfectly bonded interface between two dissimilar PEM materials of class 6 mm, it was demonstrated in this report and book [2] that as many as thirty new nondispersive interfacial SH-waves can propagate. This short theoretical work has demonstrated eight new nondispersive interfacial SH-waves in addition to the twenty-two possibilities treated in book [2]. The discovery of this short report relate to the case called the eigenvectors' mixing when different forms of the eigenvectors are used for the dissimilar piezoelectromagnetics. The results of this theoretical work are the obtained explicit forms of formulae for the phase velocity determination of the new SH-waves and the existence conditions. This fact resulted from the discussed peculiarity such as the existence of the first and second eigenvector components are respectively used for the first and second piezoelectromagnetic half-spaces. For the results obtained in this short report, the PEM1-SH-BAW speed V_{tem}^{E} must be larger than the PEM2-SH-BAW speed V_{tem}^{II}, $V_{tem}^{\mathrm{I}} > V_{tem}^{\mathrm{II}}$. The performed calculations have illuminated that the thirtieth new interfacial SH-wave can propagate with the velocity $V_{new30} =$ m/s (Fig. 2) along the common interface between two dissimilar piezoelectromagnetic composites. It is thought that these theoretical findings can be useful for verification of some significantly more complicated theoretical patterns. It is also noted that these theoretical investigations can be also useful for design of optical and microwave technical devices when suitable layered structures are exploited. It is also well-known that interfacial waves can be used for the nondestructive testing and evaluation of appropriate interfaces between two dissimilar PEM materials.

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