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On separation of exchange term from the coefficient of the magnetoelectromechanical coupling

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Abstract. The purpose of this analysis is to introduce the separated exchange coefficient and to graphically investigate it. This coefficient, depending on the electromagnetic constant plus two coefficients of the electromechanical and magnetomechanical couplings, form the coefficient of magnetoelectromechanical coupling (CMEMC), a very important characteristic used for analysing magnetoelectroelastic smart (composite) materials. It was analytically and graphically demonstrated that the CMEMC can have a minimum due to the minimum of the exchange coefficient at a certain value of the electromagnetic constant. For graphical investigation, the frequently used transversely isotropic (6*mm*) composite materials such as $BaTiO_3$–$CoFe_2O_4$ and PZT–5H–Terfenol–D are exploited.

Keywords. Magnetoelectroelastics; exchange coefficient; magnetoelectric effect; piezoelectric effect; piezomagnetic effect.

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Magnetoelectroelastics, also known as piezoelectromagnetic smart materials, actually possess magnetoelectric, piezomagnetic, and piezoelectric effects. In this class of magnetoelectric materials, there exists the possibility to influence the electrical subsystem by changing the magnetic subsystem, and vice versa, via the mechanical subsystem [1–3]. This makes them highly competitive for utilization in various technical devices, for instance, for spintronics, that is electronics free of the electronic charges.

One of the main characteristics of such smart materials is the coefficient of magnetoelectromechanical coupling K_{em}^2 (CMEMC). The coefficient can be expressed as follows:

$$K_{\mathrm{em}}^2 = \frac{\mu e^2 + \varepsilon h^2 - 2\alpha e h}{C\left(\varepsilon\mu - \alpha^2\right)} = \frac{e(e\mu - h\alpha) - h(e\alpha - h\varepsilon)}{C\left(\varepsilon\mu - \alpha^2\right)}. \quad (1)$$

This coefficient depends on many material parameters such as the elastic stiffness constant C, piezoelectric constant e, piezomagnetic coefficient h, dielectric permittivity coefficient ε, magnetic permeability coefficient μ, and electromagnetic constant α [4–8]. However, the coefficient in expression (1) can be naturally written in another form consisting of three terms. It reads as

$$K_{\mathrm{em}}^2 = K_{\mathrm{e}}^2 + K_{\mathrm{m}}^2 + K_{\mathrm{ex}}^2. \tag{2}$$

The first and the second terms on the right-hand side of expression (2) are well-known. They also represent key characteristics for pure piezoelectrics and pure piezomagnetics, respectively. The first term is called the coefficient of electromechanical coupling K_{e}^2 (CEMC) and is defined as

$$K_{\mathrm{e}}^2 = \frac{e^2}{C\varepsilon}. \tag{3}$$

The second term is known as the coefficient of magnetomechanical coupling K_{m}^2 (CMMC). It is defined as

$$K_{\mathrm{m}}^2 = \frac{h^2}{C\mu}. \tag{4}$$

The explicit form of the third exchange term denoted by K_{ex}^2 on the right-hand side of expression (2) must be found in this short report, in order to compare with K_{em}^2. Analysing eq. (2), one can naturally write the following equation:

$$K_{\mathrm{ex}}^2 = K_{\mathrm{em}}^2 - K_{\mathrm{e}}^2 - K_{\mathrm{m}}^2. \tag{5}$$

It is worth noting that, for convenience, all the coefficients in equality (5) are made non-dimensional. Employing eqs (3) and (4), it is possible to obtain the following explicit form for the exchange coefficient:

$$K_{\mathrm{ex}}^2 = \frac{\alpha^2\left(\mu e^2 + \varepsilon h^2\right) - 2\alpha\varepsilon\mu e h}{C\varepsilon\mu\left(\varepsilon\mu - \alpha^2\right)}. \tag{6}$$

This analysis illuminates that the CMEMC defined by expression (1) can be naturally decomposed into three parts: the piezoelectric effect part defined by expression (3), the piezomagnetic effect part defined by expression (4), and the exchange part (6) depending on the electromagnetic constant α. It is clearly seen in expression (6) that the exchange coefficient depends on both the electromagnetic constant α and α^2. In general, the value of α is very small, $\alpha^2 < \varepsilon\mu$ [2,3] and even frequently $\alpha^2 \ll \varepsilon\mu$. It is also obvious that the equality $K_{\mathrm{em}}^2 = K_{\mathrm{ex}}^2$ can never exist when $K_{\mathrm{e}}^2 > 0$ and $K_{\mathrm{m}}^2 > 0$. The case of $\alpha = 0$ results in $K_{\mathrm{ex}}^2 = 0$ and $K_{\mathrm{em}}^2 = K_{\mathrm{e}}^2 + K_{\mathrm{m}}^2$. According to expression (6) there is also non-zero value of α leading to $K_{\mathrm{ex}}^2 = 0$. This second value of α is given as

$$\alpha = 2\varepsilon\mu e h/\left(\mu e^2 + \varepsilon h^2\right). \tag{7}$$

Therefore, it is expected that the values of K_{em}^2 must be (significantly) larger than the values of K_{ex}^2. It is also well-known that coefficients (3) and (4) must satisfy the following limitation: $K_{\mathrm{e}}^2 < 1$ and $K_{\mathrm{m}}^2 < 1$. $K_{\mathrm{e}}^2 \ll 1$ and $K_{\mathrm{m}}^2 \ll 1$ for weak piezoelectrics and weak piezomagnetics, respectively. To graphically compare the coefficient K_{em}^2 with K_{ex}^2, two-phase magnetoelectroelastic composites such as $BaTiO_3$–$CoFe_2O_4$ and PZT–5H–Terfenol–D known as relatively weak and strong piezoelectromagnetics, respectively, were used. The material constants used for the transversely isotropic (6*mm*) composite materials are from [8–11] (see table 1).

Figure 1 shows that the coefficients $K_{\rm em}^2$ and $K_{\rm ex}^2$ can have extreme points depending on the dimensionless parameter $\alpha^2/\varepsilon\mu$. It is apparent that the minimum in $K_{\rm em}^2$ corresponds to the minimum in $K_{\rm ex}^2$ because the values of both $K_{\rm e}^2$ and $K_{\rm m}^2$ remain constant in the calculation. Also, the value of $K_{\rm ex}^2$ can be equal to zero. For $BaTiO_3$–$CoFe_2O_4$, $\alpha^2/\varepsilon\mu \sim 0.47$ leading to $K_{\rm ex}^2 = 0$. The value of $\alpha^2/\varepsilon\mu \sim 0.68$ results in $K_{\rm ex}^2 = 0$ for PZT–5H–Terfenol–D.

To find the extreme points, it is necessary to treat the first partial derivative of the coefficient $K_{\rm ex}^2$ with respect to the electromagnetic constant α and to deal with the following inequality:

$$\frac{\partial K_{\rm ex}^2}{\partial \alpha} = \frac{2C\varepsilon^2\mu^2\left[\alpha\left(\mu e^2+\varepsilon h^2\right)-\varepsilon\mu eh-\alpha^2 eh\right]}{\left[C\varepsilon\mu\left(\varepsilon\mu-\alpha^2\right)\right]^2} = 0. \tag{8}$$

Analysing the numerator in eq. (8), one can readily find two values of the electromagnetic constant α that correspond to the extreme points. They can be written as follows:

$$\alpha_{01} = \mu e^2/eh = \mu e/h, \tag{9}$$

$$\alpha_{02} = \varepsilon h^2/eh = \varepsilon h/e. \tag{10}$$

The calculated values are $\alpha_{01}^2/\varepsilon\mu \sim 3.580379469$ and $\alpha_{02}^2/\varepsilon\mu \sim 0.279300004$ for PZT–5H–Terfenol–D, $\alpha_{01}^2/\varepsilon\mu = 6.388464920$ and $\alpha_{02}^2/\varepsilon\mu = 0.156532127$ for $BaTiO_3$–$CoFe_2O_4$. The values of $\alpha_{02}^2/\varepsilon\mu$ are also listed in the table and they are significantly smaller than the values of $\alpha_{01}^2/\varepsilon\mu$. These large values are not listed in the table because $\alpha^2/\varepsilon\mu < 1$ [2,3] is used. It is clearly seen in the figure that the minima are smooth. These smooth minima allow the utilization of α_{02} in computer simulation of various composites when average properties of the piezoelectric and piezomagnetic phases are exploited.

Also, one can see in the following formula which defines the bulk wave velocity $V_{\rm tem}$ that this velocity also has a minimum as soon as the coefficient $K_{\rm em}^2$ reaches the minimum:

$$V_{\rm tem} = \sqrt{C/\rho}\left(1+K_{\rm em}^2\right)^{1/2}. \tag{11}$$

Table 1. The material parameter [8–11] for the $BaTiO_3$–$CoFe_2O_4$ and PZT–5H–Terfenol–D. The normalized value of the electromagnetic constant α_{02} (s/m) is calculated for minimum values of both $K_{\rm em}^2$ and $K_{\rm ex}^2$.

Composite material	C, 10^{10} (N/m^2)	h (T)	e (C/m^2)	μ, 10^{-6} (N/A^2)	ε, 10^{-10} (F/m)	ρ (kg/m^3)
$BaTiO_3$–$CoFe_2O_4$	4.40	275	5.80	81.0	56.4	5730
PZT–5H–Terfenol–D	1.45	83.8	8.50	2.61	75.0	8500
Composite material	$\varepsilon\mu$, 10^{-16}	$\alpha_{02}^2/\varepsilon\mu$	$K_{\rm m}^2$	$K_{\rm e}^2$	$K_{\rm em}^2$	$K_{\rm ex}^2$
$BaTiO_3$–$CoFe_2O_4$	4568.4	0.15653	0.0212	0.1356	0.13556	−0.02122
PZT–5H–Terfenol–D	195.75	0.27930	0.1856	0.6644	0.66437	−0.18555

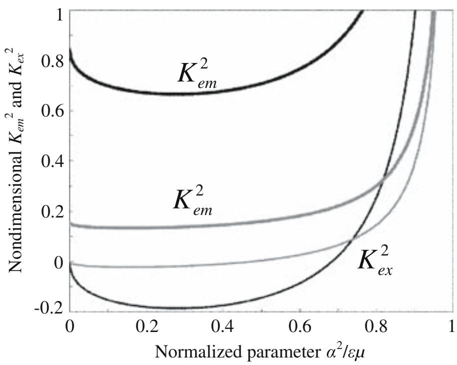

Figure 1. The comparison of the coefficients K_{em}^2 (thick lines) and K_{ex}^2 (thinner lines) vs. the dimensionless parameter $\alpha^2/\varepsilon\mu$ for the $BaTiO_3$–$CoFe_2O_4$ (gray lines) and PZT–5H–Terfenol–D (black lines) piezoelectromagnetic composites.

In eq. (1), ρ is the mass density of the material that is given in the last column of the table and the velocity V_{tem} is well-known in the theory of wave propagation in a magnetoelectroelastic bulk material. It is called the shear-horizontal bulk acoustic wave velocity.

In conclusion, it is necessary to state that this theoretical study has found the exchange coefficient K_{ex}^2 extracted from the coefficient of the magnetoelectromechanical coupling K_{em}^2. The introduced exchange coefficient K_{ex}^2 was analytically investigated. Minimum value of K_{em}^2 due to the minimum value of K_{ex}^2 at $\alpha_{02}^2/\varepsilon\mu$ (see the table) can be the preferred choice which should be experimentally verified.

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