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Evidence of scattering of bulk elementary excitations in isotopically pure liquid helium-II at low temperatures

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This short report is concerned with experimental investigations of bulk elementary excitations (BEEs) in the isotopically pure liquid helium-II at low temperatures below 100 mK. The evidence of BEEs' scattering is introduced in this work. Two identical Au-heaters were used to generate BEEs. The first pulsed heater generates BEE beams to record them. The second heater serves to generate BEE beams in order to scatter the first beams, operating delay time between pulses of the heaters. Experimental signals were recorded by several bolometers situated both above and below the liquid surface: scattered BEEs are travelling in the liquid from the pulsed heater to the bolometer; scattered BEEs, reaching the liquid surface, evaporate ^{4}He-atoms detected by two bolometers positioned in a vacuum; scattered signals are reflected from the liquid surface back to the liquid and are detected by the other bolometer situated in the liquid. It is manifested that the experimental results showed a dramatic decrease in peaks of recorded signals. Also, signal losses for different heater powers were calculated.

1. Introduction

The subject of this work is the scattering of bulk elementary excitations (BEEs) of distinct types [1–3] propagating for macroscopic distances in isotopically pure liquid helium-II [4] at low temperatures below 2.15 K. The low-temperature experimental methods below 1 K are perfectly described in [5,6]. The theoretical description of BEE energy branches for the superfluid helium-II and the other phenomena can be read in classical books [1,7] for educational purpose. Mention should also be made of the book [8] by J.R. Waldram which represents the theory of thermodynamics in a new way. Recent experimental works [9,10] by W.G. Stirling dealt with the new high-resolution neutron scattering investigations of the BEE energy spectra in the liquid. In addition, an interesting work [11] on a density-functional study applied to the BEE spectra did not cover the backflow effect measured in [9,10].

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Note that the liquid helium-II at low temperatures can be treated as matter containing free quasi-particles, for instance, rotons in the phonon-roton energy spectra [1,2,7,12]. The well known dependence of free-particle energy $E(k)$ on the wavenumber k in quantum physics is commonly written in the well-known form of the de Broglie wave as follows: $E = \omega\hbar = +(k\hbar)^2/2m = -(ik\hbar)^2/2m = [a(k\hbar)^2 - b(ik\hbar)^2]/2m$ with $a + b = 1$, where ω is the angular frequency, $\hbar = 1.054571628(53) \times 10^{-34}$ [J × s] is the Dirac constant or reduced Planck constant, and m the quasi-particle (effective) mass. It is thought that it is the simplest and most informative example of exchange between the real and imaginary parts to support any wave propagation of free quasi-particles in a vacuum. Hence, it can be concluded that there is no any difference between Re(k) and Im(k) for nature. However, it is more convenient for researchers to deal with the real part.

Glyde and Griffin [13] studied zero sound and atomic-like excitations in superfluid ^{4}He, presenting a novel interpretation of the traditional Landau–Feynman picture of BEEs. Their detailed consideration of high-resolution neutron-scattering data has revealed a very complex temperature dependence of the dynamic structure factor. They suggested the condensate-induced hybridisation of two modes in the liquid: the phonon as a collective zero-sound mode and the maxon-roton as a mode of strongly renormalised single-particle excitation.

The BEE investigations [14–17] by A.F.G. Wyatt et al. suggested that low-energy ($\sim$1–2 K) and high-energy ($\sim$10 K) propagating phonons can be created in the same phonon energy branch of the BEE energy spectra (phonon branch, negative roton branch and positive roton branch) in the superfluid helium-II at low temperatures. They also concluded that the low-energy phonons create the high-energy phonons in a cascade effect. However, it is thought that reverse situation is energetically more preferable: the high-energy phonons create the low-energy phonons. Their conclusion is based on their observation of a single BEE peak for very short distances (several millimeters) and two BEE peaks for significantly longer distances. Note that the BEE velocities, $\sim$190 m/s for higher-energy BEEs and $\sim$250 m/s for lower-energy BEEs, are close to each other, and it is natural that the two different BEEs can start to be distinguished as two BEE peaks only for suitable long distances. However, these observations do not state solidly that the created BEE propagating phonons relate to the same phonon branch. The quantum evaporation effect was also studied by A.F.G. Wyatt et al. in [18,19,20]. Dalfovo et al. [21] also studied the BEEs and quantum evaporation from the superfluid ^{4}He, using the time-dependent density functional theory (TD-DFT).

Original explanations of the propagating BEEs were presented in [22] by the author. In [23–25] the experimental investigation of the quantum phenomena of creation of BEEs by pulsed heaters and creation of ^{4}He-atomic beams by the BEEs to support and develop explanations introduced in the recent theoretical work [22] was carried out. The dependence of the BEE phase velocity $S_{ph}(k)$ on the wavenumber k in the BEE energy spectra can be naturally shown by the following way: the first derivative of S_{ph} with respect to the k is equal to zero ($\mathrm{d}S_{ph}/\mathrm{d}k = 0$). In addition to that, the second condition should be also fulfilled for non-dispersive waves showing independence on the angular frequency ω: $\mathrm{d}S_{ph}/\mathrm{d}\omega = 0$. It is obvious that both conditions are fulfilled when one of them is true owing to the following equality: $\mathrm{d}S_{ph}/\mathrm{d}k = S_g(\mathrm{d}S_{ph}/\mathrm{d}\omega) = 0$ for non-zero group velocity S_g. However, there can be $S_g = S_{ph} = 0(S_g = S_{ph} \to 0)$ for example, for the Bose–Einstein condensation giving dispersive waves because $\mathrm{d}S_{ph}/\mathrm{d}k \neq 0$. The non-dispersive Zakharenko waves [26] are recognised in any dispersion relation $S_{ph}(k)$: $\mathrm{d}S_{ph}/\mathrm{d}k = (1/k)(S_g - S_{ph})$. This equation shows that the non-dispersive Zakharenko waves correspond to extreme points of the $S_{ph}(k)$ with $S_g = S_{ph}$. The same is true for the dependence $S_{ph}(\omega)$ with $S_g = S_{ph}$: $\mathrm{d}S_{ph}/\mathrm{d}\omega = (S_{ph}/\omega)(1 - S_{ph}/S_g)$. The modern theory [22] shows that in each BEE energy zone (phonon, positive and negative roton branches) one corresponding non-dispersive Zakharenko wave should exist, because there are two possible dispersions in each energy branch: $S_g < S_{ph}$ and $S_g > S_{ph}$. Note that for a free quasi-particle (^{4}He-atom) propagating in a vacuum there is the constant relationship: $S_g = 2S_{ph}$.

A summary of this theory [22] can be given as postulates applied to each energy branch in quantum physics: (1) an energy branch of elementary excitations represents a mode of dispersive waves; (2) the energy branch can contain different dispersive waves with two possible dispersions, $S_g > S_{ph}$ and $S_g < S_{ph}$; (3) the energy branch can contain single corresponding non-dispersive Zakharenko wave [26]; (4) the non-dispersive Zakharenko wave divides the dispersive wave mode into two submodes with the different dispersions.

The topic of this study is experimental investigations of interaction (scattering) of different BEEs when complex experimental data on BEE propagation and reflection, as well as quantum evaporation by the propagating BEEs is available for the analysis.

2. Description of experiments

Experiments on BEE propagation in the liquid helium-II and quantum evaporation of ^{4}He atomic beams in a vacuum were carried out within a small-size experimental cell filled with the liquid helium at low temperatures reached with the dilution refrigerator techniques [5,6]. By changing the liquid level in the cell it is possible to use the same bolometer $\mathbf{B_1}$ in Figure 1(a) and 1(b) for detection of propagating BEEs and ^{4}He atomic beams, respectively. Also, any information about reflected BEEs from the liquid surface towards the bolometer $\mathbf{B_3}$ in Figure 1(b) is very useful for complex analysis. The attenuating heater $\mathbf{H_a}$ is situated under 90°-angle to the figure plane to excite additional propagating BEEs in order to attenuate the main BEE beams created by the main heater **H** and propagating in the figure plane. The heaters **H** and $\mathbf{H_a}$ represent devices consisting of a thin gold film deposited on a glass substrate, and the bolometers represent a superconducting thin Zn-film deposited on a glass substrate. Such devices were widely used in many previous experiments, for example, see [27].

In Figure 1(a), the main heater **H** and main bolometer $\mathbf{B_1}$ are positioned at the same angle of 10° to the surface normal (point line in the figure). The distance between the **H** and $\mathbf{B_1}$ is approximately 22 mm with a possible error not more than ±1 mm. This distance is already quite long in low temperature experiments. Created propagating BEEs are collimated on the way from the **H** to the $\mathbf{B_1}$. More detailed description of the collimation method used in this work is done in [27] manifested that such collimation effect can result in almost complete separation of the two BEE types. The distance of (5.7 ± 0.5) mm was set between the **H** and the first collimator, (4.5 ± 0.5) mm between the collimators, and (11.7 ± 0.5) mm between the second

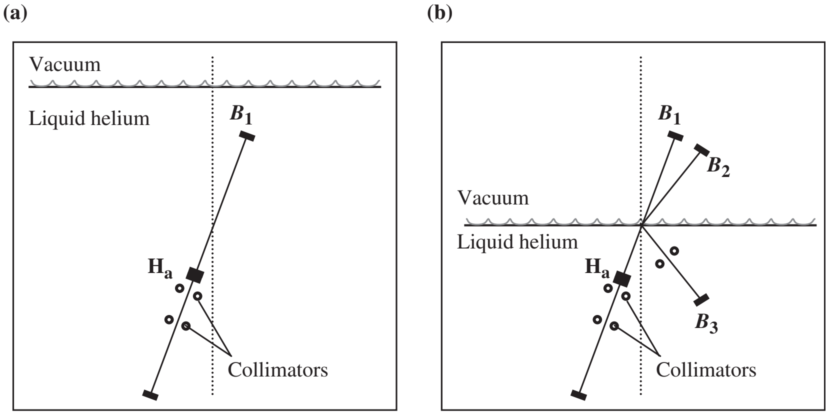

Figure 1. Co-positions of the heaters and bolometers in the experimental cell used in the low temperature experiments. **H** and $\mathbf{H_a}$ are the main and attenuating heaters, and $\mathbf{B_1}$, $\mathbf{B_2}$, and $\mathbf{B_3}$ are the bolometers for BEE detection. The attenuating heater ejects BEEs in direction perpendicular to the figure plane and the trajectory of BEEs generated by the main heater. (a) The liquid helium surface is above all the heaters and bolometers, and (b) the bolometers $\mathbf{B_1}$ and $\mathbf{B_2}$ are situated above the liquid surface.

collimator and the $\mathbf{B_1}$. It is thought that it is possible to take here a relatively large error of ±0.5 mm for the distance measurements, because they are for room temperatures, but not for low temperatures below 100 mK. Note that a direct measurement of the distances inside the cell at low temperatures is still not feasible.

In Figure 1(b), the liquid helium surface is below the bolometer $\mathbf{B_1}$. The positions of the **H**, $\mathbf{H_a}$, $\mathbf{B_1}$, and collimators are the same to those in the Figure 1(a) configuration. The additional bolometer $\mathbf{B_2}$ above the liquid surface was set under angle of 35° to the surface normal. Also, possible reflection of propagating BEEs can be readily recorded by the third bolometer $\mathbf{B_3}$ situated under angle of 40° to the surface normal and below the liquid surface level. Note that the co-position of the heater $\mathbf{H_a}$ and the bolometer $\mathbf{B_3}$ does not allow for any created BEEs by the heater to come to the bolometer to record some noisy signals.

3. Evidence of BEE scattering and discussions

The 90°-angle for the co-position of the heaters allows one to study propagating-BEE interaction with crossing trajectories perpendicular to each other for both the propagating BEEs to record BEE scattering effect. The experiments of this paper for such heaters' co-position are preliminary because experimental data about BEE scattering with other crossing trajectories are still not available. Some experimental records presented here were taken from [28]. Forbes and Wyatt [29,30] give the first evidence on the scattering of BEEs. Their work [29] uses the technique crossed ballistic BEE beams to investigate the BEE scattering under the right angle, for which some details of employments of the new method were published earlier in [30]. Note that Wyatt et al.'s explanation [14–20] of their experimental results forbids any propagation for the first non-dispersive Zakharenko wave with energy ~7 K in the BEE phonon branch owing to the phonon backflow effect measured by

W.G. Stirling [9,10]. Note that the experimental data [9,10] illuminate the phonon backflow effect, but not that any wave motion cannot exist within the backflow region. Also, the works like in [14–20] do not allow one to conclude that the Cooper pairing phenomenon (see the Bardeen–Cooper–Schrieffer theory (BCS) [31]) can be recognised by studying the propagating BEEs in coupe with the quantum evaporation phenomenon, because the superfluidity phenomenon is akin to the superconductivity phenomenon mentioned by L.D. Landau half a century ago.

Therefore, the explanations of the experimental results introduced in this short report are based on the recent works [22–26]. It is possible that these three BEE energy branches can be also called those of phonons (first sound), thermal phonons (second sound) and supra-thermal phonons (fourth sound). It is thought that they all can obey the phonon definition applied to each energy branch: weakly-dispersive waves with (relatively) small energies and momenta near boundaries of each energy branch. The Cooper pairing phenomenon in the BCS theory [31] can be readily recognised when studying the quantum evaporation process in the low temperature experiments. That was recently discussed in [23] since two propagating BEEs with their doubled total energy $\sim 2 \times 17\,\mathrm{K}$ ($17\,\mathrm{K} =$ kinetic energy plus potential energy) in the positive roton branch can liberate a single ^{4}He-atom with the kinetic energy $\sim 35\,\mathrm{K}$.

Figure 2 shows the BEE signals created by the main heater **H**, attenuated with the heater $\mathbf{H_a}$ by BEE scattering, and detected by the bolometer $\mathbf{B_1}$ in the liquid helium. It is clearly seen that the signal magnitudes depend on the delay time between the heaters. Two curves correspond to the both types of propagating BEEs: the first non-dispersive Zakharenko waves in the phonon branch and the third non-dispersive Zakharenko waves (grey line) in the positive roton branch. For the short heater pulses 0.2 μs, the signal magnitudes in the figure for the slower propagating BEEs, ~190 m/s, is significantly larger than that for the fastest propagating BEEs, ~250 m/s.

The maximum attenuation of BEE signals is observed in Figure 2 at ~28 μs. That corresponds to a distance of approximately 6 mm. Delay times towards the negative delay time scale from the maximum attenuation means that the first arrival point of the long pulses 4 μs from the $\mathbf{H_a}$ came first through the crossing point comparing with the second short heater pulse 0.2 μs of the **H**. Significant decrease in the magnitude of the slower BEEs manifests presence of a long 150 μs tail belonging to the long 4 μs $\mathbf{H_a}$-pulses (see some typical signals for long heater pulses in [24]). That can occur owing to the heating regime of the devices consisting of a thin gold film on the glass substrate. Note that the devices are commonly utilised in the experiments to excite the propagating BEEs. It is assumed that in the heating devices, a several-microsecond impulse heats a thin Au-film, which heats the glass substrate deeply and the liquid helium exciting the propagating BEEs. The substrate with thickness of ~1 mm can obviously heat the Au-film during a long relaxation time and present the long BEE tail when the long $\mathbf{H_a}$-pulses are applied. When the short 0.2 μs **H**-impulse with no tail came first through the crossing point, the signal magnitudes (grey line) return right away to the non-attenuated level of 1.3 mV for the delay times larger than 30 μs.

Concerning the results for the fastest BEEs shown in Figure 2 by the solid line, no clear evidence for interaction between the fastest BEEs of the phonon branch with

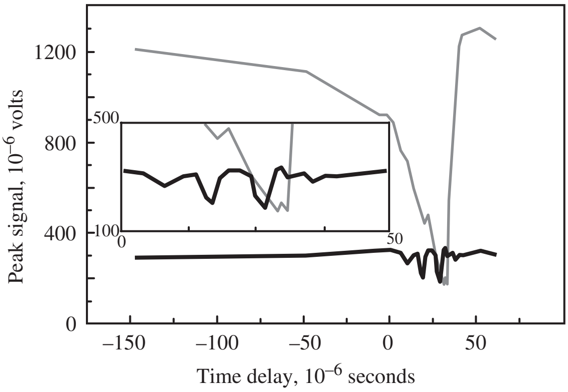

Figure 2. The BEE signals being attenuated by scattering and detected by the bolometer $\mathbf{B_1}$ in the liquid helium. The main heater power is constant (−20 dB and 0.2 µs). The $\mathbf{H_a}$-power is −20 dB with the heater pulse length of 4 µs. Both the BEE types corresponding to the first (thick black line) and third BEE-energy branches are shown. The insertion shows the signals in the range from 0 µs to 50 µs.

the long tail (the slower BEEs of the positive roton branch) of the 4 µs $\mathbf{H_a}$-pulses can be seen. However, the signal magnitude slightly increases with the delay time from −150 µs to 0 µs, after which several minima for the signal attenuation can be clearly observed. It is natural that for the fastest BEEs the minimum magnitude is observed at the delay time of ~26 µs that is several µs earlier than that for the slower BEEs (grey line). Note that the duplet-like main minimum for the magnitudes of the slower BEEs gives some hope for some possibility of interaction between the fastest and slower BEEs from the different energy branches. That is similar to an interaction between phonons and thermal phonons in solids at low temperatures. It is also noted that both propagating BEEs were observed for very short **H**-pulses of 0.01 µs (10 ns) with no attenuation in the heater power (even with the heater power attenuation of −13 dB) and without application of the $\mathbf{H_a}$-pulses (see [28]).

It is possible to evaluate the BEE lifetime. For the third non-dispersive Zakharenko wave with the energy of ~17 K in the positive roton branch, it is possible to take equal kinetic and potential energies to each other. The potential energy contribution in the 17 K equals to ~8.6 K that represents the roton minimum energy. Therefore, the kinetic energy can be taken ~8.6 K, too. Accounting the Cooper pairing phenomenon [23], one can find the doubled value of ~17.2 K = 2 × 8.6 K for the kinetic energy of the propagating BEEs. For the positive rotons with the kinetic energy of ~17.2 K, one can calculate the frequency $\omega = k_B \times (T = 17.2\,\mathrm{K})/\hbar \sim 2.252\,\mathrm{THz}$, using the well known energy-frequency relationship: $E/\omega = \hbar$. That corresponds to the lifetime t_{ps} value of approximately 0.444 ps. Here, the reduced Planck constant, also known as the Dirac constant, is $\hbar = 1.05457 \times 10^{-34}$ [J × s], and the Boltzmann constant is $k_B = 1.38065 \times 10^{-23}$ [J/K].

The very short lifetime t_{ps} for the propagating BEEs in the positive roton branch can mean in this case that no single such BEE can be created for shorter heater pulses. This statement must be experimentally verified. The lifetime can here mean the time duration for system relaxation when the impulse is passed from one domain to a neighbour domain forming a chain reaction of BEE propagation with the

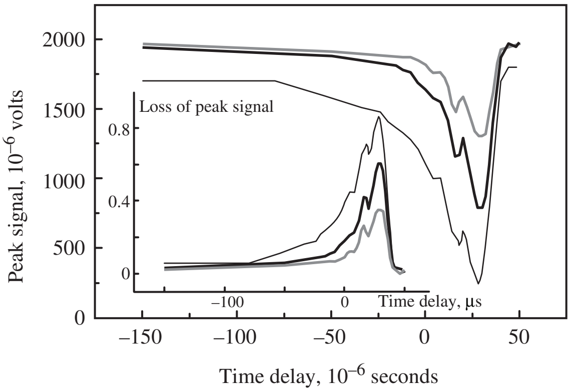

Figure 3. The peak signals from evaporated ^{4}He-atoms liberated by the BEEs being attenuated by scattering in the liquid and detected by the bolometer $\mathbf{B_1}$ in a vacuum above the liquid surface. The main heater power is constant (−13 dB and 0.2 µs). The $\mathbf{H_a}$-power is −20 dB, −23 dB (thick line), and −26 dB (grey line) with the heater pulse length of 4 µs. The insertion shows the loss of peak-signal amplitude.

velocity of ~190 m/s. It is assumed that the real heater pulse to observe no propagating BEEs can be several times shorter owing to the heater relaxation time. Indeed, the pulsed heaters used in such experiments create BEE beams consisting of the single BEEs. With the t_{ps}, it is also possible to evaluate the number of propagating BEEs created by a pulsed heater with the work area of one square millimeter: $\sim 10^{26}$ per second per mm^2. In the population evaluation, the line size of $\sim 1\,\text{Å} = 10^{-10}$ m for a free ^{4}He-atom was taken, because it is thought that the line size of a single propagating BEE is smaller owing to the fact that positive roton wavenumbers are positioned in the BEE energy spectra after the roton minimum towards large wavenumbers.

Indeed, the slower propagating BEEs can participate in the quantum evaporation process. Figure 3 shows the peak signals from evaporated ^{4}He-atoms liberated by the slower propagating BEEs attenuated by scattering in the liquid and detected by the bolometer $\mathbf{B_1}$ already in a vacuum above the liquid surface. In this case, the liquid helium surface was kept at the level when the fastest propagating BEEs travel the same distance of ~22 mm from the **H** to $\mathbf{B_3}$ (after reflection, see Figure 1(b)) comparing with the distance from the **H** to $\mathbf{B_1}$. The loss in signal magnitude for different heater powers can be evaluated with the following formula (see the insertion in Figure 3): $Loss = 1 - (x_j/a)$ where x_j and a are the magnitudes for a current measurement and for the signal without attenuation, respectively. The loss depends on the heater power and achieves 90% for the heater power of −20 dB, ~60% for −23 dB, and ~30% for −26 dB. Also, several extremes in the figure for the quantum evaporation signals give hint to one to conclude existence of reflection of the slower propagating BEEs from metallic surfaces. Note that collimators shown in Figure 1 were positioned on the way between the **H** and $\mathbf{B_1}$. At least two reflection signals can be distinguished in Figure 3. These reflection signals corresponding to the additional extremes are small in magnitude comparing with the main extreme at the delay time of ~28 µs. Also, one can find that they have the same delay of approximately 12 µs from each other. That results in additional distance of (12 µs) × (190 m/s) = 2.28 mm

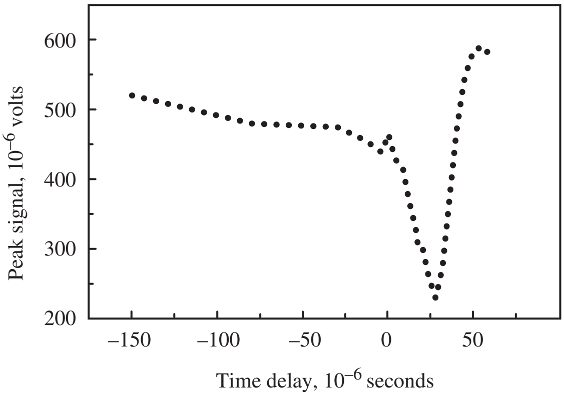

Figure 4. The peak signals from evaporated ^{4}He-atoms liberated by the BEEs being attenuated by scattering in the liquid and detected by the $\mathbf{B_2}$ in a vacuum above the liquid surface. The main heater power is −13 dB and 0.2 µs, and the $\mathbf{H_a}$-power is −20 dB and 4 µs.

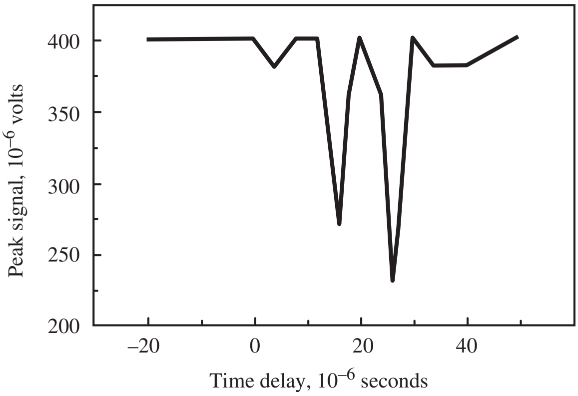

Figure 5. The peak signals from BEEs being attenuated by scattering in the liquid and reflected back into the liquid, and detected by the $\mathbf{B_3}$ in the liquid. The main heater power is −13 dB and 2 µs, and the $\mathbf{H_a}$-power is −20 dB and 4 µs.

gone by the slower propagating BEEs after reflections from metallic surfaces. It is possible that a small part of propagating BEEs created by the attenuating heater $\mathbf{H_a}$ can reflective-wisely come to the crossing point to contribute in the BEE scattering. Note that only single peaks were always observed in many quantum evaporation experiments without the attenuating heater $\mathbf{H_a}$. It is thought that some experiments on the BEE propagation and even the BEE scattering can be feasible when they propagate only in (metallic) tubes with millimeter diameters (or of other sizes) to obtain additional information. That must be completed in the future.

The delay time dependence of recorded magnitudes detected by the bolometer $\mathbf{B_2}$ shown in Figure 4 supports the results concerning signals detected by the bolometer $\mathbf{B_1}$ shown in Figure 3. Figure 5 shows the reflected signals detected by the bolometer $\mathbf{B_3}$ for the fastest propagating BEEs (the first non-dispersive Zakharenko waves). Several minima in Figure 5 allow one to suppose reflection existence of the fastest propagating BEEs from metallic surfaces. However, there are no any experimental records to completely grasp reflection aptitudes of both BEE types from metallic surfaces.

It is thought that the lifetime for (weakly)-dispersive waves in a condensed state matter like liquids and solids is finite and small. These dispersive waves cannot propagate for macroscopic distances without any support on the way. Their energy is dispersing around a source of dispersive waves. It is also thought that the propagation pass for the non-dispersive Zakharenko waves in the condensed matter can be infinitely large, depending on material purity resulting in wave attenuation at scattering centres on the way for long macroscopic distances (even for several metres).

It is possible to discuss the devices, namely heaters, used in the experiments. It is suggested that it is more convenient in the low temperature experiments to use back sides of the Au-heaters to excite BEEs in the liquid as well as ^{4}He atomic beams since a thin gold film on one side of the glass substrate heats the substrate, the substrate can heat the second Au-film on the back side of the substrate, and the second Au-film can heat the liquid helium to excite propagating BEEs. It is thought that in this case it is possible to design the heating devices in the way when the electric wiring of heaters will have no contact with the liquid helium. It is also thought that long heater pulses from 2 μs to 10 μs must be used to heat the glass substrate in order to transfer heat to the Au-film backside. Thus, it is possible to utilise the same heaters made once in many different low temperature experiments to improve comparability of experimental data from different experiments.

4. Conclusions

The experimental results manifested a dramatic loss of peak signals for attenuated BEEs recorded for different experimental configurations. Several extremes can be demonstrated for all cases of the dependence of the peak signal on the attenuating-heater delay time. That can be the clear evidence of scattering of the BEEs of two types between the BEEs in the same and different energy branches (the first and third branches of the BEE energy spectra). Indeed, any study of BEE scattering when both BEE types are present is very complicated concerning perfect understanding of physical processes. Separation of the BEEs (for instance, by a suppression of one type of the propagating BEEs) in order to have only excitations from the same energy branch (for example, the phonon branch) can be a key solution of the problem. However, preparation of experiments for excitation separation can complicate such low temperature experiments in order to have a simplified studying object (for example, only the fastest phonons). It is thought that this simplification of studying object can be the best choice to widely investigate the BEEs in the future.

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