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Aleksey Zakharenko

ALEKSEY ANATOLIEVICH ZAKHARENKO International Institute of Zakharenko Waves (IIZWs, aazaaz@inbox.ru) Education: 1998–Diploma Engineer-Physicist (MSc) Krasnoyarsk State University, Krasnoyarsk, Russia 2008–Certified Interior Designer (AutoCAD, 3ds max) Business School INOPROF, Krasnoyarsk, Russia. Research Experience: Creator of the IIZWs

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Aleksey Zakharenko

PROPAGATION OF SEVEN NEW SH-SAWs IN PIEZOELECTROMAGNETICS OF CLASS 6mm

PROPAGATION OF SEVEN NEW SHEAR - HORIZONTAL SURFACE ACOUSTIC WAVES IN PIEZOELECTROMAGNETICS OF CLASS 6 mm

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ALEKSEY ANATOLIEVICH ZAKHARENKO

(E-mail: aazaaz@inbox.ru)

LAP LAMBERT Academic Publishing GmbH & Co. KG

Dudweiler Landstrasse 99 D - 66123 Saarbruecken

Phone: +49 681 3720-310 Fax: +49 681 3720 -3109

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PREFACE

This book addresses to theoretical investigations of shear-horizontal surface acoustic waves (SH-SAWs) propagating in composites of class 6 mm which can possess piezoelectric and piezomagnetic phases. Applying different electrical and magnetic boundary conditions in the theoretical treatments, wave characteristics of seven new SH-SAWs are obtained in explicit forms. It was found that the new SH-SAWs can have both piezoelectric and piezomagnetic properties. Also, the analytically obtained exact formulae for the new SH-SAW velocities can demonstrate dependencies on the squared speed of light in a vacuum. It was also found that the new SH-SAWs can be coupled with the surface Bleustein-Gulyaev waves and bulk acoustic waves for the piezoelectric phase and the piezomagnetic phase. Calculations of the new SH-SAW characteristics were performed for sample two-phase composites $BaTiO_3$-$(CoO)Fe_2O_3$. It is thought that the obtained results can be useful for complete understanding of wave processes in two-phase and laminated composite materials in acoustoelectronics and acoustooptics. It is also thought that the theoretical results can be utilized in fabricating smart materials in the microwave technology.

PACS: 51.40.+p, 62.65.+k, 68.35.Gy, 68.35.Iv, 68.60.Bs, 74.25.Ld, 74.25.Ha, 75.20.En, 75.80.+q, 81.70.Cv

Keywords: piezoelectromagnetics, magnetoelectric effect, surface Bleustein-Gulyaev waves, surface Melkumyan waves, new surface acoustic waves.

COMMENTS BY THE AUTHOR

This book describes the theoretical work carried out for the International Institute of Zakharenko Waves (IIZWs) as PhD-thesis by the author: Aleksey Anatolievich Zakharenko, corresponding address: 660037, Krasnoyarsk-37, 17701, Krasnoyarsk, Russia (E-mail: aazaaz@inbox.ru). It is thought that this book can be interesting for researchers and students who deal with transversely-isotropic piezoelectromagnetics of class 6 mm. Also, it is thought that this theoretical work can be useful for theoreticians and experimentalist coping with cubic piezoelectromagnetics of class m3m. Indeed, knowledge of wave properties of piezoelectromagnetics is beneficial to design of smart devices, sensors, actuators. It can also represent an interest in some applications in non-destructive testing and evaluation. Also, the obtained results in this book can allow one to choose apt materials to constitute piezoelectromagnetic laminate composites in the microwave technology. It is well-known that innovative smart materials are also created for the aerospace industry.

The International Institute of Zakharenko Waves (IIZWs) was recently created for support of different Zakharenko waves, as well as for monitoring the non-dispersive Zakharenko type waves in complex systems such as layered and quantum systems. Indeed, any complex system in which dispersive waves can propagate is of a great interest for the IIZWs. The well-known examples of dispersive waves are dispersive Rayleigh and Bleustein-Gulyaev type waves as well as Love and Lamb type waves. There are currently more than twenty papers relevant to the IIZWs. The International Institute of Zakharenko Waves also studies different dispersive and non-dispersive waves both theoretically and experimentally, including different applications of the waves for signal processing (filters, sensors, etc.) and the structural health monitoring. Note that one can financially support the research of the International Institute of Zakharenko Waves, using the following bank account: Beneficiary's Bank is the Krasnoyarsk Branch of National Bank TRUST, c/a №

30301840700450000038 with Moscow Branch of National Bank TRUST, Moscow, Russia, SWIFT: MENARU2PMOS, via correspondent account no. 890-0372-664 with the Bank of New York, New York, USA (SWIFT: IRVTUS3N). Beneficiary customer: Aleksey Zakharenko with Beneficiary's account: 42301840300380001087.

It is worth noting that the International Institute of Zakharenko waves possessively takes all the planets and smaller natural space bodies in the space outside the Solar System to develop both the IIZWs and the planets concerning economics, ecology, and population. Also, it is thought that this is necessary in order to exclude any sale of the planets and their surfaces by any human or other. This activity of the IIZWs was also created due to a problem to find a spot for the IIZWs on Earth. Note that the single person, namely Mr. Dennis Hope from the United States possesses the planets in the Solar System (but Earth) who sells surfaces of the planets to individuals. It is also noted that only about 500 planets orbiting stars can be currently observed in Star Systems which are situated relatively near the Solar System. This does not mean that only 500 planets exist outside the Solar System we can observe. It is expected that in average ten planets can orbit each star of enormous number of Star System in our Universe. It is thought that our Universe can accumulate more than 10^{999} stars.

Aleksey A. Zakharenko

Krasnoyarsk, Russia, 2010

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INTRODUCTION

In the late 1960s, shear-horizontal surface acoustic waves (SH-SAWs) called the surface Bleustein-Gulyaev (BG) waves were discovered in transversely-isotropic piezoelectrics [1-2]. It is thought that a transversely-isotropic material of hexagonal class 6 mm is a good example to theoretically demonstrate characteristics of the surface BG-waves. Note that the surface BG-waves can propagate in direction perpendicular to the sixth-order symmetry axis of hexagonal crystals. However, the surface BG-waves can exist not only in the transversely-isotropic piezoelectrics, but also in the transversely-isotropic piezomagnetics. Note that the surface BG-wave velocity is well-known in the arena of physical acoustics of solids and is written in a relatively simple and explicit form as follows: $V_{BG} = V_{te}\{1 - (K_e^2/[(1 + K_e^2)\times(1 + \varepsilon_{11}/\varepsilon_0)])^2\}^{1/2}$ where V_{te} is the shear-horizontal bulk acoustic waves (SH-BAW) and K_e^2 is called the coefficient of the electromechanical coupling (CEMC). The expression for the velocity V_{BG} also couples the material characteristics of a piezoelectrics such as the dielectric permittivity coefficient ε_{11} with the same characteristics ε_0 for a vacuum. Using the electrical boundary condition of the electrically closed surface, the value of $\varepsilon_{11}/\varepsilon_0$ vanishes and the BG-wave velocity is defined by the following simplified form: $V_{BG} = V_{te}\{1 - [K_e^2/(1 + K_e^2)]^2\}^{1/2}$.

Piezoelectric and piezomagnetic properties of anisotropic materials were studied in Ref. [3] by Al'shits and Lyubimov. Also, existence of surface waves in anisotropic elastic half-space with piezoelectric and piezomagnetic properties was theoretically investigated in work [4] by Alshits, Darinskii, and Lothe. According to Ref. [4], the piezomagnetic effect and the piezoelectric effect can be described in the same way. Therefore, the surface BG-wave velocities written above are also true for transversely-isotropic piezomagnetics, using the corresponding material characteristics for the piezomagnetic materials. Also, the recent review paper [5] by Gulyaev, Dikshtein, and Shavrov mentioned that dispersion relations for magnetoelastic SH-waves in ferromagnetics and antiferromagnetics can lead to the surface BG-wave velocity. The more recent review paper [6] written by Gulyaev (the co-discoverer of the surface Bleustein-Gulyaev waves) in collaboration with Hickernell mentioned about impossibility of the existence of the surface BG-waves in cubic piezoelectrics. This means that the surface BG-waves cannot also exist in cubic piezomagnetics. However, shear-horizontal surface acoustic waves called the ultrasonic surface Zakharenko waves (USZWs) can exist in cubic piezoelectrics and cubic piezomagnetics. In 2007, Ref. [7] demonstrated the existence of the new SH-SAWs called the USZWs in cubic piezoelectrics. Also, in Ref. [8] published in 2010, the USZW existence was demonstrated for such well-known cubic piezomagnetics as Galfenol, Terfenol-D, and $CoFe_2O_4$. Indeed, the wave characteristics of the surface BG-waves in the transversely-isotropic crystals and the USZWs in the cubic crystals can dramatically differ from each other.

Also, some composite materials can possess both the piezoelectric and piezomagnetic effects, as well as the magnetoelectric (ME) effect. This is true for both the transversely-isotropic materials and materials with the cubic symmetry of class m3m. Over 300 experimental and theoretical works concerning investigations of the magnetoelectric (ME) effect in composites are reviewed in Ref. [9] by Fiebig. Ref. [10] by Schmid provides the tensor form in the Nye notation [11] of the 58 point groups permitting the linear magnetoelectric effect. Note that there occurs a continuous interest in the study of the magnetoelectric effect in composites for development of smart materials in the microwave technology. It is possible to mention several classical works [12-16] which originally studied composites and the ME-effect. Indeed, piezoelectrics and piezomagnetics can be bonded together to form composites that exhibit the ME-effect. As the result of the ME-effect, an electrical signal can be obtained from the piezoelectrics as a result of the application of a magnetic field to the piezomagnetics. On the other hand, the piezomagnetics can be magnetized because of the application of an electrical field to the piezoelectrics.

Modern researches on the ME-effect in composites are cited in Refs. [17-31]. It is thought that the most popular composites are those in which hexagonal piezoelectrics $BaTiO_3$ and hexagonal piezomagnetics $CoFe_2O_4$ are utilized, see some works in Refs. [32-36]. However, a growing attention is also focused on composite structures consisting of piezomagnetics Terfenol-D and piezoelectrics PZT-5H [33]. Indeed, the concept of electro-magnetic composites has arisen in the last two decades. The electro-magnetic composite materials can exhibit field coupling that is not present in any of the monolithic constituent materials. Such materials can find broaden applications in ultrasonic imaging devices, sensors and actuators for system control, transducers, and many other emerging components. Also, there is a strong interest in theories that provide characteristics of such complex materials and can predict the coupled response of these "smart" composites and structures composed of them. Concerning recent theories, Li in Ref. [37] has studied the electromagneto-acoustic surface Bleustein–Gulyaev wave. Also, Melkumyan in Ref. [38] has discovered new shear-horizontal elastic surface waves in transversely-isotropic magneto-electro-elastic materials, and Ref. [39] by Melkumyan describes new pure SH-SAWs guided by cuts in the materials. The following chapter addresses to constitutive relations for two-phase composite materials.

CHAPTER I. Coupled Governing Equations

It is well-known that for a material with both the piezoelectric and piezomagnetic phases, the mechanical strain tensor **τ**, electrical field vector **E**, and magnetic field vector **H** [34, 40-42] are utilized as independent thermodynamic mechanical, electrical, and magnetic variables, respectively. Therefore, the thermodynamic potential G for a three-dimensional piezoelectromagnetic solid is written as the following function G = G(**τ**, **E**, **H**). As a result, the coupled constitutive relations for linearly-piezoelectromagnetic solids [43, 44] are given by:

$$\sigma_{ij} = C_{ijkl}\tau_{kl} - e_{kij}E_k - h_{kij}H_k \tag{1}$$

$$D_i = e_{ikl}\tau_{kl} + \varepsilon_{ik}E_k + \alpha_{ik}H_k \tag{2}$$

$$B_i = h_{ikl}\tau_{kl} + \alpha_{ik}E_k + \mu_{ik}H_k \tag{3}$$

where the indices *i*, *j*, *k*, and *l* run from 1 to 3. It is clearly seen in equations from (1) to (3) that such a piezomagnetic-piezoelectric composite possesses the elastic stiffness constants C_{ijkl}, piezoelectric constants e_{kij}, piezomagnetic coefficients h_{kij}, dielectric permittivity coefficients ε_{ik}, magnetic permeability coefficients μ_{ik}, and electromagnetic constants α_{ik}.

In equations from (1) to (3), σ_{ij} and τ_{kl} are the stress and strain tensor components, respectively. The elastic strain tensor components τ_{kl} are expressed as the following dependence on the partial first derivatives of the mechanical displacement components U_k with respect to the real space components x_l:

$$\tau_{kl} = \frac{1}{2}\left(\frac{\partial U_k}{\partial x_l} + \frac{\partial U_l}{\partial x_k}\right) \tag{4}$$

D_i and B_i are the components of the electrical displacement and the magnetic induction (i.e. magnetic flux) in equations (2) and (3), respectively. Using the electrical potential φ and the magnetic potential ψ in the quasi-static approximation, the components of the electrical field E_k and the magnetic field H_k in equations from (1) to (3) can be defined as follows:

$$E_k = -\frac{\partial \varphi}{x_k} \tag{5}$$

$$H_k = -\frac{\partial \psi}{x_k} \tag{6}$$

Applying the Maxwell equations such as div**D** = 0 and div**B** = 0, the governing mechanical, electrostatic, and magnetostatic equilibriums read

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0 \tag{7}$$

$$\frac{\partial D_i}{\partial x_i} = 0 \tag{8}$$

$$\frac{\partial B_i}{\partial x_i} = 0 \tag{9}$$

It is finally noted that the material constants in equations from (1) to (3) are thermodynamically defined as follows:

$$C_{ijkl} = \left(\frac{\partial \sigma_{ij}}{\partial \tau_{kl}}\right)_{E,H} \tag{10}$$

$$e_{ijk} = -\left(\frac{\partial \sigma_{ij}}{\partial E_k}\right)_{\tau,H} = e_{ikl} = \left(\frac{\partial D_i}{\partial \tau_{kl}}\right)_{E,H} \tag{11}$$

$$h_{ijk} = -\left(\frac{\partial \sigma_{ij}}{\partial H_k}\right)_{\tau,E} = h_{ikl} = \left(\frac{\partial B_i}{\partial \tau_{kl}}\right)_{E,H} \tag{12}$$

$$\varepsilon_{ik} = \left(\frac{\partial D_i}{\partial E_k}\right)_{\tau,H} \tag{13}$$

$$\mu_{ik} = \left(\frac{\partial B_i}{\partial H_k} \right)_{\tau,E} \tag{14}$$

$$\alpha_{ik} = \left(\frac{\partial D_i}{\partial H_k} \right)_{\tau,E} = \left(\frac{\partial B_i}{\partial E_k} \right)_{\tau,H} \tag{15}$$

In equation (10), the elastic stiffness constants C_{ijkl} are defined at constant electrical and magnetic fields. The other material constants in equations from (11) to (15) are also defined at corresponding constant thermodynamic variables. According to the Voigt notation, C_{ijkl}, e_{kij}, h_{kij}, ε_{ik}, μ_{ik}, and α_{ik} can be also written as 6×6, 3×6, 3×6, 3×3, 3×3, and 3×3 matrices [45, 11, 46] which stand for the elastic, piezoelectric, piezomagnetic, dielectric, magnetic, and electromagnetic tensors, respectively. Using the definitions of the thermodynamic variables and material constants, it is possible to write equations of motion for such solids. That is the main purpose of the following chapter.

CHAPTER II. Equations of Motion

Following the definitions given in the previous chapter, the equations of motion of an elastic medium can be written in the following form [47-49]:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 U_i}{\partial t^2} \tag{16}$$

In expression (16), the right term at the factor ρ called the mass density of the medium represents the second partial derivative of the mechanical displacement components U_i with respect to time t. Also, the electrostatics and magnetostatics in the quasi-static approximation read:

$$\frac{\partial D_i}{\partial x_j} = 0 \tag{17}$$

$$\frac{\partial B_i}{\partial x_j} = 0 \tag{18}$$

The coupled equations of motion, namely equations from (16) to (18) for a two-phase solid which possesses both the piezoelectric and piezomagnetic effects, can be than written with the material constants for a piezoelectromagnetic medium in the following form, using equations from (1) to (3):

$$\rho \frac{\partial^2 U_i}{\partial t^2} = C_{ijkl} \frac{\partial^2 U_l}{\partial x_j \partial x_k} + e_{kij} \frac{\partial^2 \varphi}{\partial x_j \partial x_k} + h_{kij} \frac{\partial^2 \psi}{\partial x_j \partial x_k} \tag{19}$$

$$0 = e_{ijk} \frac{\partial^2 U_k}{\partial x_i \partial x_j} - \varepsilon_{ij} \frac{\partial^2 \varphi}{\partial x_i \partial x_j} - \alpha_{ij} \frac{\partial^2 \psi}{\partial x_i \partial x_j} \tag{20}$$

$$0 = h_{ijk} \frac{\partial^2 U_k}{\partial x_i \partial x_j} - \alpha_{ij} \frac{\partial^2 \varphi}{\partial x_i \partial x_j} - \mu_{ij} \frac{\partial^2 \psi}{\partial x_i \partial x_j} \tag{21}$$

Coupled equations from (19) to (21) represent homogeneous partial differential equations of the second order. It is well-known that solutions for such equations can be represented in the following plane wave form [49, 50]:

$$U_i = U_i^0 \exp[\mathrm{j}(k_1 x_1 + k_2 x_2 + k_3 x_3 - \omega t)] \quad (22)$$

$$\varphi = \varphi^0 \exp[\mathrm{j}(k_1 x_1 + k_2 x_2 + k_3 x_3 - \omega t)] \quad (23)$$

$$\psi = \psi^0 \exp[\mathrm{j}(k_1 x_1 + k_2 x_2 + k_3 x_3 - \omega t)] \quad (24)$$

where the index i runs from 1 to 3. In equations from (22) to (24), U_i^0, φ^0, and ψ^0 are the initial amplitudes. The imaginary unity is defined as j = $(-1)^{1/2}$ and ω is the angular frequency defined as $\omega = 2\pi\nu$ where ν is the linear frequency. $\{k_1, k_2, k_3\} = k\{n_1, n_2, n_3\}$ are the components of the wavevector **K** directed towards the wave propagation (see figure 1) and $\{n_1, n_2, n_3\}$ are the directional cosines. Note that the wavenumber k in the direction of wave propagation is defined as $k = 2\pi/\lambda$ where λ is the wavelength.

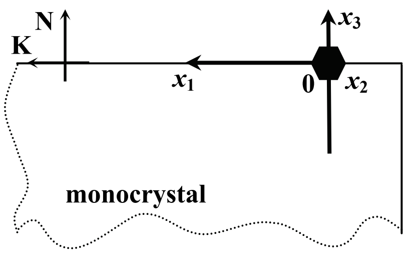

Figure 1. The direction of SH-SAW propagation in a half-space of class 6 mm which possesses the piezoelectric and piezomagnetic effects. The SH-SAW propagation is directed along the wavevector **K** and perpendicular to the vector **N** of surface normal. Also, the vectors **K** and **N** are directed perpendicular to the sixth-order symmetry axis of the crystal directed along the x_2-axis of the rectangular co-ordinate system

Note that equations from (19) to (21) are the coupled equations of motion written in the common form. However, these equations can be readily written in the following simplified form, leaving only equations for waves with the anti-plane polarization and non-zero components of the material tensors for the studied direction of wave propagation (see figure 1):

$$\rho\frac{\partial^2 U_2}{\partial t^2} = C_{66}\frac{\partial^2 U_2}{\partial x_1^2} + C_{44}\frac{\partial^2 U_2}{\partial x_3^2} + e_{16}\frac{\partial^2 \varphi}{\partial x_1^2} + e_{34}\frac{\partial^2 \varphi}{\partial x_3^2} + h_{16}\frac{\partial^2 \psi}{\partial x_1^2} + h_{34}\frac{\partial^2 \psi}{\partial x_3^2} \tag{25}$$

$$0 = e_{16}\frac{\partial^2 U_2}{\partial x_1^2} + e_{34}\frac{\partial^2 U_2}{\partial x_3^2} - \varepsilon_{11}\frac{\partial^2 \varphi}{\partial x_1^2} - \varepsilon_{33}\frac{\partial^2 \varphi}{\partial x_3^2} - \alpha_{11}\frac{\partial^2 \psi}{\partial x_1^2} - \alpha_{33}\frac{\partial^2 \psi}{\partial x_3^2} \tag{26}$$

$$0 = h_{16}\frac{\partial^2 U_2}{\partial x_1^2} + h_{34}\frac{\partial^2 U_2}{\partial x_3^2} - \alpha_{11}\frac{\partial^2 \varphi}{\partial x_1^2} - \alpha_{33}\frac{\partial^2 \varphi}{\partial x_3^2} - \mu_{11}\frac{\partial^2 \psi}{\partial x_1^2} - \mu_{33}\frac{\partial^2 \psi}{\partial x_3^2} \tag{27}$$

In equations from (25) to (27), the mechanical displacement component U_2 is directed along the x_2-axis directed parallel to the sixth-order symmetry axis of a two-phase crystal of class 6 mm, see figure 1. Stating for simplicity that the wavevector **K** is directed along the x_1-axis, the directional cosines are defined as follows: $n_1 = 1$, $n_2 = 0$ and $n_3 \equiv n_3$. As the result, the solutions in the plane wave form are than written for the studied case as follows:

$$U_{2,4,5} = U_{2,4,5}^0 \exp\left[\mathrm{j}k\left(n_1 x_1 + n_3 x_3 - V_{ph} t\right)\right] \tag{28}$$

where the phase velocity is defined as $V_{ph} = \omega/k$. In equation (28), the electrical potential φ and the magnetic potential ψ are written as the fourth and fifth displacement components U_4 and U_5, respectively, which depend on the corresponding initial amplitudes $U_4{}^0 = \varphi^0$ and $U_5{}^0 = \psi^0$. For the studied complicated case of two-phase materials, it is possible to further simplify the theoretical description of the problem because $C_{44} = C_{66} = C$, $e_{16} = e_{34} = e$, $h_{16} = h_{34} = h$, $\varepsilon_{11} = \varepsilon_{33} = \varepsilon$, $\mu_{11} = \mu_{33} = \mu$, and $\alpha_{11} = \alpha_{33} = \alpha$. The material constants are listed in table 1 for the case when the shear-horizontal acoustic waves are coupled with both the electrical and magnetic potentials. The following chapter provides the useful tensor form of the equation of motion and the problem of finding of eigenvalues and eigenvectors.

Table 1. The material characteristics of the piezoelectromagnetic composites consisting of $BaTiO_3$-$CoFe_2O_4$ of class 6 mm. Following Refs. [44, 53], the material constants are given as percentage volume fraction (VF) of $BaTiO_3$ in composites consisting of $BaTiO_3$-$CoFe_2O_4$. Note that the magnetic permeability of a vacuum is μ_0 = 4π×10^{-7} [H × m^{-1}] ~ 12.566371×10^{-7} [(V×s) × (A×m)$^{-1}$]; 10^{-12} Ns/(VC) = ps/m, F = C/V, and T = Tesla = N×(A×m)$^{-1}$. The mass density is assumed the same ρ = 5730 [kg/m^3] and j = (– 1)$^{1/2}$

Composite VF	0%	20%	40%	60%	80%	100%
C, 10^{10} [N/m^2]	4.53	4.50	4.50	4.50	5.00	4.30
e, [C/m^2]	0	0.1	0.2	0.3	0.4	11.6
h, [T]	560	340	220	180	80	0
ε, 10^{-10} [F/m]	0.8	3.3	8.0	9.0	10.0	112.0
μ, 10^{-6} [N/A^2]	– 590	– 390	– 250	– 150	– 80	5.0
α, 10^{-12} [Ns/VC]	0	2.8	4.8	6.0	6.8	0
K_e^2	0	0.00067340067	0.0011111111111	0.00222222222	0.003200000	0.279401993
K_m^2	-0.011733453	-0.00658689459	-0.004302222224	-0.00480000000	-0.001600000	0
K_{em}^2	-0.011733453	-0.00591346104	-0.003191064178	-0.00257767111	0.001600109	0.279401993
V_{EM}, 10^6 [m/s]	4.602873089j	2.78747336669j	2.23606797750j	2.7216552697j	3.535533906j	4.225771274
$V_{EM\alpha}$, 10^6 [m/s]	4.602873089j	2.78747336660j	2.23606797737j	2.7216552694j	3.535533905j	4.225771274
V_α, 10^{12} [m/s]	-	0.357142857143	0.208333333333	0.16666666667	0.147058824	-
$V_{EM0\alpha}$, 10^6 [m/s]	4.607782748j	2.79197507190j	2.24170910463j	2.7331278472j	3.563633399j	3.777639469
$V_{E0M\alpha}$, 10^6 [m/s]	4.367526034j	2.75081502726j	2.22379590861j	2.70836572424j	3.519985271j	4.224101952

CHAPTER III. The Tensor Form of Equations of Motion

Using material constants, it is thought that a tensor form of the equation of motion is more convenient for further theoretical analysis. Substituting the mechanical displacement components U_i, the electrical potential $\varphi = U_4$, and the magnetic potential $\psi = U_5$ in equations from (22), (23), and (24) into equations from (19) to (21), the equations of motion in the common form can be written in the well-known tensor form [51]:

$$(\mathrm{GL}_{sw} - \delta_{sw}\rho V_{ph})U_s^0 = 0 \tag{29}$$

in which GL_{sw} are the tensor components in the modified Green-Christoffel equation. In the Green-Christoffel equation, s and w run from 1 to 5, δ_{sw} is the Kronecker delta for $s < 4$ and $w < 4$, $\delta_{44} = 0$ and $\delta_{55} = 0$. The eigenvector components are defined as follows $U_s^0 = \left\langle U_1^0, U_2^0 = U^0, U_3^0, U_4^0 = \varphi^0, U_5^0 = \psi^0 \right\rangle$ which correspond to each eigenvalue k_3. Note that the Green-Christoffel equation can split into two independent sets of equations in many highly-symmetric directions of wave propagation. The main purpose of this book is to theoretically investigate shear-horizontal surface acoustic waves (SH-SAWs) which correspond to the one of the sets of independent equations. In the case of SH-SAWs, the eigenvector $\left\langle U^0, \varphi^0, \psi^0 \right\rangle$ corresponds to an eigenvalue k_3 which represents an imaginary number (or a complex number in the common case). Also, it is assumed that $k_3 < 0$ to satisfy wave damping towards the depth of a monocrystal. This is the surface wave condition for the co-ordinate system shown in figure 1.

Using the material constants C, e, h, ε, μ, and α defined in the previous chapter, the GL-components which correspond to equations from (25) to (27) are as follows:

$$\mathrm{GL}_{22} = C(1 + n_3^2)$$

$$\mathrm{GL}_{24} = \mathrm{GL}_{42} = e(1 + n_3^2)$$

$$\mathrm{GL}_{25} = \mathrm{GL}_{52} = h(1 + n_3^2)$$

$$\mathrm{GL}_{44} = -\varepsilon(1 + n_3^2)$$

$$\mathrm{GL}_{45} = \mathrm{GL}_{54} = -\alpha(1 + n_3^2)$$

$$\mathrm{GL}_{55} = -\mu(1 + n_3^2)$$

in which $n_3 = k_3/k$.

Therefore, the following system of three homogeneous equations for the pure SH-wave reads:

$$\begin{pmatrix} Cm - \rho V_{ph}^2 & em & hm \\ em & -\varepsilon m & -\alpha m \\ hm & -\alpha m & -\mu m \end{pmatrix} \begin{pmatrix} U^0 \\ \varphi^0 \\ \psi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{30}$$

where $m = 1 + n_3^2$.

It is obvious that the determinant of the coefficient matrix in equations (30) can be written as

$$m \times m \times \begin{vmatrix} Cm - \rho V_{ph}^2 & em & hm \\ e & -\varepsilon & -\alpha \\ h & -\alpha & -\mu \end{vmatrix} = 0 \tag{31}$$

Equation (31) is served for determination of six normalized eigenvalues n_3. It is clearly seen in equation (31) that the first and second factors readily give the following four solutions:

$$n_3^{(1,2)} = n_3^{(3,4)} = \pm \mathrm{j} \tag{32}$$

Expending the determinant of the coefficient matrix in equation (31), the rest two eigenvalues are determined from the following equation:

$$\left(1+K_{em}^2\right)m-\left(V_{ph}/V_{t4}\right)^2=0 \tag{33}$$

in which

$$K_{em}^2=\frac{\mu e^2+\varepsilon h^2-2\alpha eh}{C\left(\varepsilon\mu-\alpha^2\right)} \tag{34}$$

is called the coefficient of the magnetoelectromechanical coupling (CMEMC). Also, the velocity V_{t4} is determined as follows:

$$V_{t4}=\sqrt{C/\rho} \tag{35}$$

which corresponds to the speed of the shear-horizontal bulk acoustic wave (SH-BAW) in the case of zero value of the CMEMC. Therefore, the rest two eigenvalues can be expressed as follows:

$$n_3^{(5,6)}=\pm \mathrm{j}\sqrt{1-\left(V_{ph}/V_{tem}\right)^2} \tag{36}$$

where the velocity V_{tem} is the speed of the SH-BAW coupled with both the electrical potential and the magnetic potential:

$$V_{tem}=V_{t4}\left(1+K_{em}^2\right)^{1/2} \tag{37}$$

Setting the material constants e, h, ε, μ, and α equal to zero in expression (34), the CMEMC reduces to the well-known coefficient of the electromechanical coupling (CEMC) for a purely piezoelectric material defined as follows:

$$K_e^2=\frac{e^2}{\varepsilon C} \tag{38}$$

Using $e = 0$ and $\alpha = 0$ in expression (34), the CMEMC also reduces to the coefficient of the magnetomechanical coupling (CMMC) for a purely piezomagnetic crystal defined as follows:

$$K_m^2 = \frac{h^2}{\mu C} \tag{39}$$

Note that for the problem of SH-SAW, the suitable three eigenvalues in equations (32) and (36) should have a negative sign. The corresponding eigenvectors $\langle U^0, \varphi^0, \psi^0 \rangle$ can be determined by solving the system of three equations (30). Indeed, it is thought that it is natural to define the eigenvector component U^0 from the first equation in equations (30), namely

$$U^0 = -\frac{em}{a}\varphi^0 - \frac{hm}{a}\psi^0 \tag{40}$$

In equation (40), the parameter a is expressed as follows:

$$a = mC - \rho V_{ph}^2 \tag{41}$$

Excluding the component U^0 from the second and third equations in equations (30), one can obtain the following equations to determine a coupling between the components φ^0 and ψ^0:

$$\left(\frac{me^2}{a} + \varepsilon\right)\varphi^0 + \left(\frac{meh}{a} + \alpha\right)\psi^0 = 0 \tag{42}$$

$$\left(\frac{meh}{a} + \alpha\right)\varphi^0 + \left(\frac{mh^2}{a} + \mu\right)\psi^0 = 0 \tag{43}$$

It is apparent that the components φ^0 and ψ^0 can be determined from equation (42). In addition, the components φ^0 and ψ^0 can be also determined from equation (43). Using equation (42), one can check that the components φ^0 and ψ^0 are defined as follows:

$$\varphi^0 = \frac{meh}{a} + \alpha \tag{44}$$

$$\psi^0 = -\frac{me^2}{a} - \varepsilon \tag{45}$$

For this case, it is possible to write the following useful expressions which can significantly simplify further theoretical investigations:

$$m^{(1)} = m^{(3)} = 0 \tag{46}$$

$$n_3^{(1)} = n_3^{(3)} = -\mathrm{j} \tag{47}$$

$$U^{0(1)} = U^{0(3)} = 0 \tag{48}$$

$$\varphi^{0(1)} = \varphi^{0(3)} = \alpha \tag{49}$$

$$\psi^{0(1)} = \psi^{0(3)} = -\varepsilon \tag{50}$$

$$m^{(5)} = \left(V_{ph}/V_{tem}\right)^2 \tag{51}$$

$$n_3^{(5)} = -\mathrm{j}\sqrt{1-m^{(5)}} = -\mathrm{j}b \tag{52}$$

$$a^{(5)} = -m^{(5)}CK_{em}^2 \tag{53}$$

$$U^{0(5)} = \frac{e\alpha - h\varepsilon}{CK_{em}^2} \tag{54}$$

$$\varphi^{0(5)} = -\frac{eh}{CK_{em}^2} + \alpha \tag{55}$$

$$\psi^{0(5)} = \frac{e^2}{CK_{em}^2} - \varepsilon \tag{56}$$

Also, the following equality is useful for calculations:

$$e\varphi^{0(3)} + h\psi^{0(3)} = e\varphi^{0(5)} + h\psi^{0(5)} = e\alpha - h\varepsilon \tag{57}$$

Notwithstanding, in the second case it is possible to state that the components φ^0 and ψ^0 can be also defined from equation (43) as follows:

$$\varphi^0 = \frac{mh^2}{a} + \mu \tag{58}$$

$$\psi^0 = -\frac{meh}{a} - \alpha \tag{59}$$

For the second set of the eigenvector components in equations (58) and (59), it is also possible to write the following useful expressions for the eigenvector components $\langle U^0, \varphi^0, \psi^0 \rangle$ which correspond to the eigenvalues in equations (47) and (52):

$$U^{0(1)} = U^{0(3)} = 0 \tag{60}$$

$$\varphi^{0(1)} = \varphi^{0(3)} = \mu \tag{61}$$

$$\psi^{0(1)} = \psi^{0(3)} = -\alpha \tag{62}$$

$$U^{0(5)} = \frac{e\mu - h\alpha}{CK_{em}^2} \tag{63}$$

$$\varphi^{0(5)} = -\frac{h^2}{CK_{em}^2} + \mu \tag{64}$$

$$\psi^{0(5)} = \frac{eh}{CK_{em}^2} - \alpha \tag{65}$$

For this case, the equality in expression (57) is already written in the following useful form:

$$e\varphi^{0(3)} + h\psi^{0(3)} = e\varphi^{0(5)} + h\psi^{0(5)} = e\mu - h\alpha \tag{66}$$

Notice that the first set of the eigenvector components couples ε and α, but the second set couples μ and α. It is also noticed that the second set of the eigenvector components can lead to interesting results when it is used in calculations instead of the first set. This will be analytically demonstrated below.

For the free space, the elastic constant of a vacuum is C_0 = 0.001 Pa [52]. This value of C_0 is thirteen orders smaller than that for a solid. Therefore, it is too negligible to account it in calculations. Also, the free space dielectric permittivity constant is $\varepsilon_0 = 10^{-7}/(4\pi C_L^2) = 8.854187817\times10^{-12}$ [F/m] where $C_L = 2.99782458\times10^8$ [m/s] is the speed of light in a vacuum. The Laplace equation of type $\Delta\varphi_f = 0$ and the electrical potential can be written as follows, respectively:

$$\left(k_1^2 + k_3^2\right)\varphi_{f0} = 0 \tag{67}$$

$$\varphi_{f0} = F^{(E0)} \exp(-k_1 x_3) \exp[j(k_1 x_1 - \omega t)] \tag{68}$$

Also, the free space magnetic permeability constant $\mu_0 = 4\pi\times10^{-7}$ [H/m] = $12.5663706144\times10^{-7}$ [H/m] must be used and Laplace's equation of type $\Delta\psi_f = 0$ is written in the following form:

$$\left(k_1^2 + k_3^2\right)\psi_{f0} = 0 \tag{69}$$

The magnetic potential in a vacuum can be written as follows:

$$\psi_{f0} = F^{(M0)} \exp(-k_1 x_3) \exp[j(k_1 x_1 - \omega t)] \tag{70}$$

Note that the electrical and magnetic potentials in expressions (68) and (70) must decrease for $k_1 > 0$ with increase in the coordinate $x_3 > 0$ (see figure 1).

For a solid piezoelectromagnetics, the complete mechanical displacement U^Σ, complete electrical potential φ^Σ, and complete magnetic potential ψ^Σ can be written in the plane wave form as follows:

$$U^\Sigma = \sum_{p=1,3,5} F^{(p)} U^{0(p)} \exp\left[jk\left(n_1 x_1 + n_3^{(p)} x_3 - V_{ph} t\right)\right] \tag{71}$$

$$\varphi^{\Sigma} = \sum_{p=1,3,5} F^{(p)} \varphi^{0(p)} \exp\left[\mathrm{j}k\left(n_1 x_1 + n_3^{(p)} x_3 - V_{ph} t\right)\right] \tag{72}$$

$$\psi^{\Sigma} = \sum_{p=1,3,5} F^{(p)} \psi^{0(p)} \exp\left[\mathrm{j}k\left(n_1 x_1 + n_3^{(p)} x_3 - V_{ph} t\right)\right] \tag{73}$$

The weight factors $F^{(1)}$, $F^{(3)}$, and $F^{(5)}$ can be determined from equations in which suitable boundary conditions are accounted. The following chapter addresses to the presentation of realization of the mechanical boundary condition as well as possible electrical and magnetic boundary conditions.

CHAPTER IV. Mechanical, Electrical, and Magnetic Boundary Conditions

Indeed, it is expected that the mechanical, electrical, and magnetic boundary conditions of a studied piezoelectromagnetics of class 6 mm which occupies the half-space $x_3 < 0$ (see figure 1) must be satisfied. It is well-known that the electrical boundary conditions must satisfy the cases of the electrically closed surface ($\varphi = 0$) and electrically open surface ($D_3 = 0$). The electrically closed surface can be realized by surface metallization. Also, the magnetic boundary conditions of the magnetically closed surface ($B_3 = 0$) and magnetically open surface ($\psi = 0$) can occur. The realization of the mechanical, electrical, and magnetic boundary conditions is described in Ref. [4]. According to Ref. [4], the case of magnetically open surface can be realized when a crystal surface contacts with a ferromagnetic covering characterized by a relative magnetic susceptibility $\mu_r >> 1$.

For the mechanically free surface, the mechanical boundary condition for the normal component of the stress tensor $\sigma_{32}(x_3 = 0) = 0$ at the interface between the crystal surface and a vacuum reads:

$$\begin{aligned}\sigma_{32} &= F_1\left[Ck_3^{(1)}U^{0(1)} + ek_3^{(1)}\varphi^{0(1)} + hk_3^{(1)}\psi^{0(1)}\right]\\ &+ F_2\left[Ck_3^{(3)}U^{0(3)} + ek_3^{(3)}\varphi^{0(3)} + hk_3^{(3)}\psi^{0(3)}\right]\\ &+ F_3\left[Ck_3^{(5)}U^{0(5)} + ek_3^{(5)}\varphi^{0(5)} + hk_3^{(5)}\psi^{0(5)}\right]\end{aligned} \tag{74}$$

where $F_1 = F^{(1)}$, $F_2 = F^{(3)}$, and $F_3 = F^{(5)}$.

The electrical boundary conditions at the interface are as follows:

1e) continuity of the electrical displacement normal component D_3 at the interface $x_3 = 0$, namely $D_3 = D_3^f$ where

$$\begin{aligned}D_3 &= F_1\left[ek_3^{(1)}U^{0(1)} - \varepsilon k_3^{(1)}\varphi^{0(1)} - \alpha k_3^{(1)}\psi^{0(1)}\right]\\ &+ F_2\left[ek_3^{(3)}U^{0(3)} - \varepsilon k_3^{(3)}\varphi^{0(3)} - \alpha k_3^{(3)}\psi^{0(3)}\right]\\ &+ F_3\left[ek_3^{(5)}U^{0(5)} - \varepsilon k_3^{(5)}\varphi^{0(5)} - \alpha k_3^{(5)}\psi^{0(5)}\right]\end{aligned} \tag{75}$$

and the vacuum characteristics D_3^f is

$$D_3^f = -F_E \varphi_0^f \, \mathrm{j} k_1 \varepsilon_0 \tag{76}$$

2e) continuity of the electrical potential φ at the interface, i.e. $\varphi = \varphi^f$ where

$$\varphi = F_1 \varphi^{0(1)} + F_2 \varphi^{0(3)} + F_3 \varphi^{0(5)} \tag{77}$$

and the electrical potential φ^f in a vacuum is

$$\varphi^f = F_E \varphi_0^f \tag{78}$$

Also, the magnetic boundary conditions can be written as follows:

1m) continuity of the magnetic flux normal component B_3 at $x_3 = 0$, namely $B_3 = B_3^f$ where

$$\begin{aligned} B_3 = {} & F_1\left[hk_3^{(1)} U^{0(1)} - \alpha k_3^{(1)} \varphi^{0(1)} - \mu k_3^{(1)} \psi^{0(1)}\right] \\ & + F_2\left[hk_3^{(3)} U^{0(3)} - \alpha k_3^{(3)} \varphi^{0(3)} - \mu k_3^{(3)} \psi^{0(3)}\right] \\ & + F_3\left[hk_3^{(5)} U^{0(5)} - \alpha k_3^{(5)} \varphi^{0(5)} - \mu k_3^{(5)} \psi^{0(5)}\right] \end{aligned} \tag{79}$$

and the value of B_3^f for a vacuum is

$$B_3^f = -F_M \psi_0^f \, \mathrm{j} k_1 \mu_0 \tag{80}$$

2m) continuity of the magnetic potential ψ at $x_3 = 0$, i.e. $\psi = \psi^f$ where

$$\psi = F_1 \psi^{0(1)} + F_2 \psi^{0(3)} + F_3 \psi^{0(5)} \tag{81}$$

and the magnetic potential ψ^f in a vacuum is

$$\psi^f = F_M \psi_0^f \tag{82}$$

The following chapters study the influence of different electrical and magnetic boundary conditions. It is thought that the case of the electrically closed surface ($\varphi = 0$) and the magnetically open surface ($\psi = 0$) is a common realization of the boundary conditions to commence the analysis.

CHAPTER V. The Case of $\sigma_{32} = 0$, $\varphi = 0$, and $\psi = 0$

Using the mechanical boundary condition at the interface $x_3 = 0$ (see figure 1) in the form of $\sigma_{32} = 0$ and the other conditions of $\varphi = 0$ and $\psi = 0$, three equations can be than written following equations (74), (77), and (81):

$$F\left[Cn_3^{(3)}U^{0(3)} + en_3^{(3)}\varphi^{0(3)} + hn_3^{(3)}\psi^{0(3)}\right] + F_3\left[Cn_3^{(5)}U^{0(5)} + en_3^{(5)}\varphi^{0(5)} + hn_3^{(5)}\psi^{0(5)}\right] = 0 \qquad (83)$$

$$F\varphi^{0(3)} + F_3\varphi^{0(5)} = 0 \qquad (84)$$

$$F\psi^{0(3)} + F_3\psi^{0(5)} = 0 \qquad (85)$$

where $n_3 = k_3/k$ and $F = F_1 + F_2$ because two equal eigenvalues $n_3^{(1)} = n_3^{(3)} = -\mathrm{j}$ in equation (47) give equal eigenvectors. First of all, it is possible to use the first set of the eigenvector components and useful relations given in equations from (48) to (57). For this case, equations from (83) to (85) after some transformations can be written as follows:

$$(e\alpha - h\varepsilon)CK_{em}^2 F + bC(e\alpha - h\varepsilon)\left(1 + K_{em}^2\right)F_3 = 0 \qquad (86)$$

$$e\alpha CK_{em}^2 F - \left(e^2 h - e\alpha CK_{em}^2\right)F_3 = 0 \qquad (87)$$

$$-h\varepsilon CK_{em}^2 F + \left(e^2 h - h\varepsilon CK_{em}^2\right)F_3 = 0 \qquad (88)$$

In equation (86), the factor b is defined by formula (52). It is clearly seen in equations from (86) to (88) that equations (87) and (88) are coupled with equation (86) through the factors at the weight function F. Therefore, by excluding F one can obtain the following simplified relation:

$$b\left(1 + K_{em}^2\right) - K_{em}^2 = 0 \qquad (89)$$

Using formula (52) for b, the SH-SAW velocity V_{BGM} in this case of the first set of the eigenvector components for the boundary conditions of the electrically closed surface ($\varphi = 0$) and the magnetically open surface ($\psi = 0$) is written as follows:

$$V_{BGM} = V_{tem}\left[1-\left(\frac{K_{em}^2}{1+K_{em}^2}\right)^2\right]^{1/2} \quad (90)$$

where the coefficient of the magnetoelectromechanical coupling (CMEMC) K_{em}^2 and the shear-horizontal bulk acoustic wave (SH-BAW) velocity V_{tem} are defined by formulae (34) and (37), respectively. Indeed, formula (90) looks like the well-known formula for the velocity V_{BGEC} of surface Bleustein-Gulyaev (BG) wave [1, 2] propagating along the electrically closed surface ($\varphi = 0$) in a pure piezoelectrics. This formula for the surface BG-wave can be obtained from equation (90) by setting $h = 0$ and $\alpha = 0$ in V_{tem} and K_{em}^2 and reads:

$$V_{BGEC} = V_{te}\left[1-\left(\frac{K_e^2}{1+K_e^2}\right)^2\right]^{1/2} \quad (91)$$

where the coefficient of the electromechanical coupling (CEMC) K_e^2 is defined by formula (38) and the SH-BAW velocity V_{te} in a pure piezoelectrics is as follows:

$$V_{te} = V_{t4}\left(1+K_e^2\right)^{1/2} \quad (92)$$

where the velocity V_{t4} is defined in equation (35).

Also, formula (90) looks like the well-known formula for the velocity V_{BGMO} of surface BG-wave propagating along the magnetically open surface ($\psi = 0$) in a pure piezomagnetics. This formula can be obtained from equation (90) by setting $e = 0$ and $\alpha = 0$ in V_{tem} and K_{em}^2. This results in the following formula:

$$V_{BGMO} = V_{tm}\left[1 - \left(\frac{K_m^2}{1 + K_m^2}\right)^2\right]^{1/2} \qquad (93)$$

where the coefficient of the magnetomechanical coupling (CMMC) K_m^2 is defined by formula (39) and the SH-BAW velocity V_{tm} in a pure piezomagnetics is written as follows:

$$V_{tm} = V_{t4}\left(1 + K_m^2\right)^{1/2} \qquad (94)$$

The SH-SAW velocity V_{BGM} in equation (90) was first obtained by Arman Melkumyan in his theoretical work [38, 29] published several years ago. Therefore, the SH-SAW propagating in a piezoelectromagnetics with the velocity V_{BGM} is called the surface Bleustein-Gulyaev-Melkumyan (BGM) wave because formulae (91) and (93) for the surface BG-waves in a pure piezoelectrics or piezomagnetics can be obtained from formula (90), but not vice versa. Also, it is obvious that formula (91) can be readily transformed into the form of formula (93) by setting $e \to h$ and $\varepsilon \to \mu$, and vice versa.

For the second set of the eigenvector components and useful relations given in equations from (60) to (66), one can also obtain the following simplified equations after several transformations applied to equations from (83) to (85):

$$(e\mu - h\alpha)CK_{em}^2 F + bC(e\mu - h\alpha)(1 + K_{em}^2)F_3 = 0 \qquad (95)$$

$$e\mu CK_{em}^2 F - (eh^2 - e\mu CK_{em}^2)F_3 = 0 \qquad (96)$$

$$-h\alpha CK_{em}^2 F + (eh^2 - h\alpha CK_{em}^2)F_3 = 0 \qquad (97)$$

It is apparent that equations from (95) to (97) are also coupled like those in the case of equations from (86) to (88). Indeed, after a subtraction of equations (96) and (97) from equation (95) the resulting equation will be equation (89), from which the velocity of the surface Bleustein-Gulyaev-Melkumyan wave [38, 29] is also written in the form of equation (90). The weight functions F and F_3 can be also determined from equation (95) or (86) as follows:

$$F = b\left(1 + K_{em}^2\right) \tag{98}$$

$$F_3 = -K_{em}^2 \tag{99}$$

where the parameter b in equation (98) for this case is as follows:

$$b = \frac{K_{em}^2}{1 + K_{em}^2} \tag{100}$$

Using a sum of equations (96) and (97) or (87) and (88), the functions F and F_3 can be alternatively related by $F = -F_3$ where F = 1. This procedure to determine the functions F and F_3 is also applicable to the other cases described below.

It is worth noting that the utilization of the first and second sets of the eigenvector components does not always lead to the same resulting velocity like that treated in this chapter. The following chapter treats the other electrical and magnetic boundary conditions described in the previous chapter and demonstrates that the two different sets of the eigenvector components can lead to two possible solutions for new SH-SAWs.

CHAPTER VI. The Case of Continuity of D_3 and B_3 at $x_3 = 0$

In this case of continuity of both the D_3 and B_3 at the crystal surface toward the free space, modified equations (74), (75), and (79) are written as follows:

$$F\left[Cn_3^{(3)}U^{0(3)} + en_3^{(3)}\varphi^{0(3)} + hn_3^{(3)}\psi^{0(3)}\right] + F_3\left[Cn_3^{(5)}U^{0(5)} + en_3^{(5)}\varphi^{0(5)} + hn_3^{(5)}\psi^{0(5)}\right] = 0 \quad (101)$$

$$F\left[ek_3^{(3)}U^{0(3)} - (\varepsilon k_3^{(3)} - \mathrm{j}\varepsilon_0)\varphi^{0(3)} - \alpha k_3^{(3)}\psi^{0(3)}\right] + F_3\left[ek_3^{(5)}U^{0(5)} - (\varepsilon k_3^{(5)} - \mathrm{j}\varepsilon_0)\varphi^{0(5)} - \alpha k_3^{(5)}\psi^{0(5)}\right] = 0 \quad (102)$$

$$F\left[hk_3^{(3)}U^{0(3)} - \alpha k_3^{(3)}\varphi^{0(3)} - (\mu k_3^{(3)} - \mathrm{j}\mu_0)\psi^{0(3)}\right] + F_3\left[hk_3^{(5)}U^{0(5)} - \alpha k_3^{(5)}\varphi^{0(5)} - (\mu k_3^{(5)} - \mathrm{j}\mu_0)\psi^{0(5)}\right] = 0 \quad (103)$$

where ε_0 and μ_0 in equations (102) and (103) are the corresponding characteristics of the free space.

These three equations written above can be significantly simplified by application of the first set of the eigenvector components and the useful relations in equations from (48) to (57). After several transformations, these three equations are than written as follows:

$$\varepsilon(\mu + \mu_0)CK_{em}^2 F + b\varepsilon C(\mu + \mu_0)(1 + K_{em}^2)F_3 = 0 \quad (104)$$

$$-\alpha^2 CK_{em}^2 F + (\alpha eh - \alpha^2 CK_{em}^2)F_3 = 0 \quad (105)$$

$$\left[\varepsilon(\mu + \mu_0) - \alpha^2\right]CK_{em}^2 F - \mu_0(e^2 - \varepsilon CK_{em}^2)F_3 = 0 \quad (106)$$

It is overt that equations (104) and (105) are not independent from equation (106). A subtraction of these two equations from the third equation leads to the following secular equation for determination of the phase velocity of surface SH-waves:

$$\mu_0(e^2 - \varepsilon CK_{em}^2) + \alpha eh - \alpha^2 CK_{em}^2 + b\varepsilon C(\mu + \mu_0)(1 + K_{em}^2) = 0 \quad (107)$$

Therefore, the velocity of the first new SH-SAW can be written in the following explicit form:

$$V_{new1} = V_{tem}\left[1-\left(\frac{K_{em}^2 - K_e^2 + \alpha^2 C_L^2 \dfrac{\varepsilon_0}{\varepsilon}\left(K_{em}^2 - \dfrac{eh}{\alpha C}\right)}{\left(1+K_{em}^2\right)\left(1+\dfrac{\mu}{\mu_0}\right)}\right)^2\right]^{1/2} \tag{108}$$

where the following constant

$$C_L^2 = \frac{1}{\varepsilon_0 \mu_0} \tag{109}$$

represents the squared speed of light in a vacuum. In equation (108), the CMEMC K_{em}^2, the CEMC K_e^2, and the SH-BAW velocity V_{tem} are defined by formulae (34), (38), and (37), respectively. It is thought that equation (108) represents the velocity of the new SH-SAWs for the coupled piezomagnetic phase. This is actually true because the term $K_{em}^2 - K_e^2$ represents a subtraction of the purely piezoelectric phase from the coupled piezoelectromagnetic phase. The second complicated term at the following factor $\alpha^2 C_L^2(\varepsilon_0/\varepsilon)$ represents a subtraction of the piezoelectromagnetic exchange phase from the coupled piezoelectromagnetic phase. Also, the value of $\varepsilon_0/\varepsilon$ in $\alpha^2 C_L^2(\varepsilon_0/\varepsilon)$ can represent a weight parameter coming from the purely piezoelectric phase and can be very small in the case of a large value of the dielectric permittivity coefficient ε for a piezoelectromagnetics.

In general, a value of the electromagnetic constant α is very small. Notwithstanding, this is not always true. Nevertheless, it is possible to write the velocity of the first new SH-SAW for the case of the material constant $\alpha = 0$ in the following simplified form:

$$V_{n1} = V_{tem0}\left[1-\left(\frac{K_m^2}{(1+K_e^2+K_m^2)(1+\mu/\mu_0)}\right)^2\right]^{1/2} \quad (110)$$

where the velocity V_{tem0} of the shear-horizontal bulk acoustic wave (SH-BAW) is defined by the following expression:

$$V_{tem0} = V_{t4}\left(1+K_e^2+K_m^2\right)^{1/2} \quad (111)$$

In equation (111), the CMMC K_m^2 and the SH-BAW velocity V_{t4} are defined by formulas (39) and (35), respectively. It is clearly seen in equations (110) and (111) that when the piezoelectric constant $e = 0$, than equation (110) reduces to the well-known velocity V_{BGpm} of the surface Bleustein-Gulyaev waves [1, 2] in a pure piezomagnetics:

$$V_{BGpm} = V_{tm}\left[1-\left(\frac{K_m^2}{(1+K_m^2)(1+\mu/\mu_0)}\right)^2\right]^{1/2} \quad (112)$$

In equation (112), the SH-BAW velocity V_{tm} is defined by equation (94). Note that it is well-known that the velocity V_{BGpm} is situated slightly below the velocity V_{tm}. However, in the case of zero piezomagnetic coefficient, $h = 0$, equation (110) reduces to the SH-BAW velocity V_{te} defined by equation (92) which is a wave characteristics for a pure piezoelectrics. Therefore, it is expected that the velocities of the new SH-SAWs in equations (108) and (110) can probably lie within the phase velocity interval between the SH-BAW velocity V_{te} for a pure piezoelectrics and the SH-SAW velocity V_{BGpm} (or the SH-BAW velocity V_{tm} in the case of a weak piezomagnetics) for a pure piezomagnetics. Moreover, it doesn't matter that $V_{te} > V_{BGpm}$ ($V_{te} > V_{tm}$) or $V_{te} < V_{BGpm}$ ($V_{te} < V_{tm}$). This can mean that a piezoelectromagnetics can possess incorporative properties of the piezoelectric phase and the piezomagnetic phase.

For this case of the first set of the eigenvector components, the weight functions F and F_3 can be also determined. Indeed, a sum of equations (104) and (105) gives a new equation with the same factor at the function F like that in equation (106). Therefore, the functions F and F_3 are determined from this new equation as follows:

$$F = \alpha eh - \alpha^2 CK_{em}^2 + b\varepsilon C(\mu + \mu_0)(1 + K_{em}^2) \quad (113)$$

$$F_3 = -[\varepsilon(\mu + \mu_0) - \alpha^2]CK_{em}^2 \quad (114)$$

where the parameter b is written in equation (108) as follows:

$$b = \frac{K_{em}^2 - K_e^2 + \alpha^2 C_L^2 \frac{\varepsilon_0}{\varepsilon}\left(K_{em}^2 - \frac{eh}{\alpha C}\right)}{(1 + K_{em}^2)(1 + \mu/\mu_0)} \quad (115)$$

It is worth here noting that the functions F and F_3 defined by equations (113) and (114) are those for substitutions in equations from (71) to (73).

For comparison with the first set of the eigenvector components in equations from (48) to (57), the second set of the eigenvector components and the useful relations given in equations from (60) to (66) can be used. The utilization of them for the substitutions into equations from (101) to (103) and further transformations of the equations result in the following coupled equations:

$$(\varepsilon + \varepsilon_0)\mu CK_{em}^2 F + b(\varepsilon + \varepsilon_0)\mu C(1 + K_{em}^2)F_3 = 0 \quad (116)$$

$$[(\varepsilon + \varepsilon_0)\mu - \alpha^2]CK_{em}^2 F - \varepsilon_0(h^2 - \mu CK_{em}^2)F_3 = 0 \quad (117)$$

$$-\alpha^2 CK_{em}^2 F + (\alpha eh - \alpha^2 CK_{em}^2)F_3 = 0 \quad (118)$$

It is also blatant that equations (116) and (118) are not independent from equation (117). A subtraction of these two equations from equation (117) results in the following equation for determination of the phase velocity of surface SH-waves:

$$\varepsilon_0\left(h^2 - \mu C K_{em}^2\right) + \alpha e h - \alpha^2 C K_{em}^2 + b(\varepsilon + \varepsilon_0)\mu C\left(1 + K_{em}^2\right) = 0 \tag{119}$$

Equation (119) reveals the velocity of the second new SH-SAW written in the following form:

$$V_{new2} = V_{tem}\left[1 - \left(\frac{K_{em}^2 - K_m^2 + \alpha^2 C_L^2 \dfrac{\mu_0}{\mu}\left(K_{em}^2 - \dfrac{eh}{\alpha C}\right)}{\left(1 + K_{em}^2\right)\left(1 + \dfrac{\varepsilon}{\varepsilon_0}\right)}\right)^2\right]^{1/2} \tag{120}$$

where the speed of light C_L in a vacuum is also defined by equation (109). It is also thought that equation (116) represents the velocity of the new SH-SAW for the coupled piezoelectric phase. This is also true because the term $K_{em}^2 - K_m^2$ represents a subtraction of the purely piezomagnetic phase from the coupled piezoelectromagnetic phase. In equation (120), the second term at the following factor $\alpha^2 C_L^2(\mu_0/\mu)$ has the same exchange function like that in equation (108) with the weight parameter μ_0/μ instead of $\varepsilon_0/\varepsilon$. Indeed, the value of μ_0/μ in $\alpha^2 C_L^2(\mu_0/\mu)$ can also represent a weight parameter of the purely piezomagnetic phase. This value can be very small in the case of a large value of the magnetic permeability coefficient μ for a piezoelectromagnetics. Also, in the case of a very small value of the electromagnetic constant α, it is possible to write the velocity of the second new SH-SAW in the following simplified form:

$$V_{n2} = V_{tem0}\left[1 - \left(\frac{K_e^2}{\left(1 + K_e^2 + K_m^2\right)\left(1 + \varepsilon/\varepsilon_0\right)}\right)^2\right]^{1/2} \tag{121}$$

where the SH-BAW velocity V_{tem0} is defined by equation (111). For the piezomagnetic coefficient $h = 0$ or $h \to 0$, equation (121) also reduces to the well-known velocity V_{BGpe} of the surface Bleustein-Gulyaev waves [1, 2] in a purely piezoelectric crystal:

$$V_{BGpe} = V_{te}\left[1-\left(\frac{K_e^2}{\left(1+K_e^2\right)\left(1+\varepsilon/\varepsilon_0\right)}\right)^2\right]^{1/2} \tag{122}$$

It is also noticed that it is well-known that the velocity V_{BGpe} is situated slightly below the velocity V_{te}. However, in the case of piezoelectric constant $e = 0$, equation (121) reduces to the SH-BAW velocity V_{tm} defined by equation (94) which is a wave characteristics for a pure piezomagnetics. Therefore, it is expected that the velocities of the second new SH-SAWs in equations (120) and (121) can probably lie within the phase velocity interval between the SH-BAW velocity V_{tm} for a pure piezomagnetics and the SH-SAW velocity V_{BGpe} (or the SH-BAW velocity V_{te} in the case of a weak piezoelectrics) for a pure piezoelectrics. In this case, a piezoelectromagnetics can also possess incorporative properties of the piezoelectric phase and the piezomagnetic phase.

For the second set of the eigenvector components, the functions F and F_3 can be also determined. Using a sum of equations (116) and (118), the resulting new equation gives the same factor at F like that in equation (117). Therefore, the functions F and F_3 can be defined as follows:

$$F = \alpha eh - \alpha^2 C K_{em}^2 + b\left(\varepsilon+\varepsilon_0\right)\mu C\left(1+K_{em}^2\right) \tag{123}$$

$$F_3 = -\left[\left(\varepsilon+\varepsilon_0\right)\mu - \alpha^2\right] C K_{em}^2 \tag{124}$$

where the parameter b is written in equation (120) as follows:

$$b = \frac{K_{em}^2 - K_m^2 + \alpha^2 C_L^2 \frac{\mu_0}{\mu}\left(K_{em}^2 - \frac{eh}{\alpha C}\right)}{\left(1+K_{em}^2\right)\left(1+\varepsilon/\varepsilon_0\right)} \tag{125}$$

Indeed, the functions F and F_3 defined by equations (123) and (124) also serve for substitutions in equations from (71) to (73). The following chapter studies the wave characteristics for a combination of the electrical and magnetic boundary conditions written in this and previous chapters.

CHAPTER VII. The Case of φ = 0 and Continuity of B_3

In the case of the electrically closed surface (φ = 0) and continuity of the B_3 above the crystal surface toward the free space, the three equations are written as follows:

$$F\left[Cn_3^{(3)}U^{0(3)} + en_3^{(3)}\varphi^{0(3)} + hn_3^{(3)}\psi^{0(3)}\right] + F_3\left[Cn_3^{(5)}U^{0(5)} + en_3^{(5)}\varphi^{0(5)} + hn_3^{(5)}\psi^{0(5)}\right] = 0 \quad (126)$$

$$F\varphi^{0(3)} + F_3\varphi^{0(5)} = 0 \quad (127)$$

$$\begin{aligned} &F\left[hk_3^{(3)}U^{0(3)} - \alpha k_3^{(3)}\varphi^{0(3)} - (\mu k_3^{(3)} - \mathrm{j}\mu_0)\psi^{0(3)}\right] \\ &+ F_3\left[hk_3^{(5)}U^{0(5)} - \alpha k_3^{(5)}\varphi^{0(5)} - (\mu k_3^{(5)} - \mathrm{j}\mu_0)\psi^{0(5)}\right] = 0 \end{aligned} \quad (128)$$

One can check here that equations from (126) to (128) can be also reduced to equations from (104) to (106) in the case of the first set of the eigenvector components and the useful relations in equations from (48) to (57). This is true because equation (87) transforms into equation (105) by multiplication with a factor of α/e. Therefore, the results in this case are the same to the velocities of the first new SH-SAWs defined by equations (108) and (110) obtained in the previous chapter.

However, utilizing the second set of the eigenvector components and the useful relations given in equations from (60) to (66), one can obtain three equations in the following forms:

$$\left(\varepsilon\mu - \alpha^2\right)CK_{em}^2F + bC\left(\varepsilon\mu - \alpha^2\right)\left(1 + K_{em}^2\right)F_3 = 0 \quad (129)$$

$$\varepsilon\mu CK_{em}^2F - \left(\varepsilon h^2 - \varepsilon\mu CK_{em}^2\right)F_3 = 0 \quad (130)$$

$$-\alpha^2CK_{em}^2F + \left(\alpha eh - \alpha^2CK_{em}^2\right)F_3 = 0 \quad (131)$$

It is clearly seen in equations from (129) to (131) that equation (131) is identical to equation (115). Therefore, equation (130) can transform into equation (96) by a factor of ε/e, and equation (129) can be also transformed into equation (113) by a corresponding factor. It is flagrant that these three equations are also not independent. A subtraction of equations (130) and (131) from equation (129) results in the following:

$$bC\left(\varepsilon\mu-\alpha^2\right)\left(1+K_{em}^2\right)+\varepsilon h^2-\varepsilon\mu CK_{em}^2-\alpha eh+\alpha^2CK_{em}^2=0 \qquad (132)$$

It is interesting that equation (132) does not contain the characteristics of the free space such as the dielectric permittivity constant ε_0 and the magnetic permeability constant μ_0. As the result, equation (132) reveals the following explicit form for the velocity of the third new SH-SAW:

$$V_{new3}=V_{tem}\left[1-\left(\frac{\frac{V_{EM\alpha}^2}{V_{EM}^2}\left(K_{em}^2-K_m^2\right)-\frac{V_{EM\alpha}^2}{V_\alpha^2}\left(K_{em}^2-\frac{eh}{\alpha C}\right)}{1+K_{em}^2}\right)^2\right]^{1/2} \qquad (133)$$

where the velocities $V_{EM\alpha}$, V_{EM}, and V_α represent the material properties of a piezoelectromagnetics and are defined as follows:

$$V_{EM\alpha}^2=\frac{1}{\varepsilon\mu-\alpha^2} \qquad (134)$$

$$V_{EM}^2=\frac{1}{\varepsilon\mu} \qquad (135)$$

$$V_\alpha^2=\frac{1}{\alpha^2} \qquad (136)$$

Note that the velocities $V_{EM\alpha}$ and V_{EM} can represent the speeds of the electromagnetic waves in a piezoelectromagnetics with and without accounting of the constant α for the electromagnetic effect, respectively. With $\alpha = 0$, equation (133) can also reduce to the following simplified form for the velocity of the third new SH-SAW:

$$V_{n3} = V_{tem0}\left[1-\left(\frac{K_e^2}{1+K_e^2+K_m^2}\right)^2\right]^{1/2} \tag{137}$$

where the SH-BAW velocity V_{tem0} is defined by equation (111). With $h = 0$, equation (137) also recovers the classical BG-wave velocity V_{BGEC} given in equation (91) for a pure piezoelectrics with the metallized surface ($\varphi = 0$). However, equation (137) for the material constant $e = 0$ gives the SH-BAW velocity V_{tm} defined by equation (94).

The functions F and F_3 can be also determined for the second set of the eigenvector components. Indeed, a sum of equations (130) and (131) results in the new equation with the same factor at F similarly to that in equation (129). Therefore, the functions F and F_3 can be determined in this case from equation (129) as follows:

$$F = bC\left(\varepsilon\mu-\alpha^2\right)\left(1+K_{em}^2\right) \tag{138}$$

$$F_3 = -\left(\varepsilon\mu-\alpha^2\right)CK_{em}^2 \tag{139}$$

where the parameter b can be written following equation (133) as follows:

$$b = \frac{\varepsilon\mu\left(K_{em}^2-K_m^2\right)-\alpha^2\left(K_{em}^2-\dfrac{eh}{\alpha C}\right)}{\left(\varepsilon\mu-\alpha^2\right)\left(1+K_{em}^2\right)} \tag{140}$$

The F and F_3 expressed in equations (138) and (139) also serve for substitutions in equations from (71) to (73). The following chapter studies the wave characteristics for the case of the magnetically open surface ($\psi = 0$) and continuity of D_3.

CHAPTER VIII. The Case of $\psi = 0$ and Continuity of D_3

Using equations (101), (102), and (85) for the case of the magnetically open surface ($\psi = 0$) and continuity of the D_3, three equations are written as follows:

$$F\left[Cn_3^{(3)}U^{0(3)} + en_3^{(3)}\varphi^{0(3)} + hn_3^{(3)}\psi^{0(3)}\right] + F_3\left[Cn_3^{(5)}U^{0(5)} + en_3^{(5)}\varphi^{0(5)} + hn_3^{(5)}\psi^{0(5)}\right] = 0 \quad (141)$$

$$\begin{aligned} &F\left[ek_3^{(3)}U^{0(3)} - (\varepsilon k_3^{(3)} - \mathrm{j}\varepsilon_0)\varphi^{0(3)} - \alpha k_3^{(3)}\psi^{0(3)}\right] \\ &+ F_3\left[ek_3^{(5)}U^{0(5)} - (\varepsilon k_3^{(5)} - \mathrm{j}\varepsilon_0)\varphi^{0(5)} - \alpha k_3^{(5)}\psi^{0(5)}\right] = 0 \end{aligned} \quad (142)$$

$$F\psi^{0(3)} + F_3\psi^{0(5)} = 0 \quad (143)$$

Therefore, these three equations with the first set of the eigenvector components can be transformed for this case into the following forms:

$$(\varepsilon\mu - \alpha^2)CK_{em}^2 F + bC(\varepsilon\mu - \alpha^2)(1 + K_{em}^2)F_3 = 0 \quad (144)$$

$$-\alpha^2 CK_{em}^2 F + (\alpha eh - \alpha^2 CK_{em}^2)F_3 = 0 \quad (145)$$

$$\varepsilon\mu CK_{em}^2 F - (\mu e^2 - \varepsilon\mu CK_{em}^2)F_3 = 0 \quad (146)$$

The resulting equation for the coupled equations from (144) to (146) is as follows:

$$bC(\varepsilon\mu - \alpha^2)(1 + K_{em}^2) - \alpha eh + \alpha^2 CK_{em}^2 + \mu e^2 - \varepsilon\mu CK_{em}^2 = 0 \quad (147)$$

As the result, equation (147) leads to the following velocity of the fourth new SH-SAW:

$$V_{new4} = V_{tem}\left[1 - \left(\frac{\frac{V_{EM\alpha}^2}{V_{EM}^2}(K_{em}^2 - K_e^2) - \frac{V_{EM\alpha}^2}{V_\alpha^2}\left(K_{em}^2 - \frac{eh}{\alpha C}\right)}{1 + K_{em}^2}\right)^2\right]^{1/2} \quad (148)$$

where the velocities $V_{EM\alpha}$, V_{EM}, and V_α are defined by equations from (134) to (136), respectively. It is clearly seen that equations (148) and (133) are very similar. The single difference is that the CEMC K_e^2 is used in equation (148) instead of the CMMC K_m^2 used in equation (133). One can also obtain that equation (148) can be readily simplified to the following form:

$$V_{n4} = V_{tem0}\left[1 - \left(\frac{K_m^2}{1 + K_e^2 + K_m^2}\right)^2\right]^{1/2} \tag{149}$$

With $e = 0$, the simplified form in equation (149) for the velocity of the fourth new SH-SAW also recovers the classical BG-wave velocity V_{BGMO} given by equation (93) for a pure piezomagnetics with the magnetically open surface ($\psi = 0$). However, equation (149) for $h = 0$ gives the SH-BAW velocity V_{te} defined by equation (92).

The functions F and F_3 can be also determined from equations (144) to (146). Therefore, the functions F and F_3 can be determined in this case from equation (144) as follows:

$$F = bC\left(\varepsilon\mu - \alpha^2\right)\left(1 + K_{em}^2\right) \tag{150}$$

$$F_3 = -\left(\varepsilon\mu - \alpha^2\right)CK_{em}^2 \tag{151}$$

where the parameter b can be written following equation (148) as follows:

$$b = \frac{\varepsilon\mu\left(K_{em}^2 - K_e^2\right) - \alpha^2\left(K_{em}^2 - \dfrac{eh}{\alpha C}\right)}{\left(\varepsilon\mu - \alpha^2\right)\left(1 + K_{em}^2\right)} \tag{152}$$

The F and F_3 expressed in equations (150) and (151) are also substituted into equations from (71) to (73).

Using the second set of the eigenvector components, one can also write the coupled three equations. These coupled equations can be also transformed into equations from (116) to (118) in the case of the second set of the eigenvector components and the useful relations in equations from (60) to (66). Therefore, the results in this case are the same to the velocities of the second new SH-SAWs defined by equations (120) and (121) obtained in the previous chapter. The following chapters describe the wave characteristics for the other possible combinations of the electrical and magnetic boundary conditions.

CHAPTER IX. The Case of D_3 = 0 and ψ = 0

For the case of the electrically open surface (D_3 = 0) and the magnetically open surface (ψ = 0), three coupled equations are written following those written in equations from (141) to (143) with the following single difference: ε_0 should be excluded in equation (142). Therefore, utilization of the first set of the eigenvector components gives the following coupled equations:

$$\varepsilon\mu CK_{em}^2 F + b\varepsilon\mu C\left(1 + K_{em}^2\right)F_3 = 0 \tag{153}$$

$$0F + 0F_3 = 0 \tag{154}$$

$$\varepsilon\mu CK_{em}^2 F - \left(\mu e^2 - \varepsilon\mu CK_{em}^2\right)F_3 = 0 \tag{155}$$

It is clearly seen in the above written equations that equation (154) can be omitted from the consideration. Therefore, equations (153) and (155) reveal the following SH-SAW velocity:

$$V_{M1} = V_{tem}\left[1 - \left(\frac{K_{em}^2 - K_e^2}{1 + K_{em}^2}\right)^2\right]^{1/2} \tag{156}$$

This SH-SAW velocity V_{M1} defined by equation (156) was first obtain by Arman Melkumyan in his recent works [38, 29] for the case of the mechanically free, electrically open (D_3 = 0) and magnetically open (ψ = 0) surface. Therefore, the SH-SAW velocity V_{M1} in equation (156) can be called the first surface Melkumyan wave.

The functions F and F_3 for the first surface Melkumyan wave can be determined from equation (153). Therefore, the functions are expressed as follows:

$$F = b\varepsilon\mu C\left(1 + K_{em}^2\right) \tag{157}$$

$$F_3 = -\varepsilon\mu CK_{em}^2 \tag{158}$$

where the parameter b is defined following equation (156) as follows:

$$b = \frac{K_{em}^2 - K_e^2}{1 + K_{em}^2} \tag{159}$$

Using the second set of the eigenvector components, the coupled three equations are then written as follows:

$$\varepsilon\mu C K_{em}^2 F + b\varepsilon\mu C\left(1 + K_{em}^2\right)F_3 = 0 \tag{160}$$

$$\left[\varepsilon\mu - \alpha^2\right]C K_{em}^2 F + 0F_3 = 0 \tag{161}$$

$$-\alpha^2 C K_{em}^2 F + \left(\alpha eh - \alpha^2 C K_{em}^2\right)F_3 = 0 \tag{162}$$

Therefore, equations (160) and (162) give the following velocity of the fifth new shear-horizontal surface acoustic wave:

$$V_{new5} = V_{tem}\left[1 - \left(\frac{V_{EM}^2}{V_\alpha^2}\frac{K_{em}^2 - \dfrac{eh}{\alpha C}}{1 + K_{em}^2}\right)^2\right]^{1/2} \tag{163}$$

where the velocities V_{EM} and V_α are defined by equations (126) and (127), respectively. The functions F and F_3 for the case of the fifth new SH-SAW can be determined from a sum of equations (160) and (162) as follows:

$$F = \alpha eh - \alpha^2 C K_{em}^2 + b\varepsilon\mu C\left(1 + K_{em}^2\right) \tag{164}$$

$$F_3 = -\left(\varepsilon\mu - \alpha^2\right)C K_{em}^2 \tag{165}$$

where the parameter b is defined following equation (163) as follows:

$$b = \frac{\alpha^2}{\varepsilon\mu} \frac{K_{em}^2 - \dfrac{eh}{\alpha C}}{1 + K_{em}^2} \tag{166}$$

However, equation (161) gives the following simple values for the functions F and F_3 such as F = 0 and F_3 = 1. It is noticed that the functions are also substituted into equations from (71) to (73).

CHAPTER X. The Case of $B_3 = 0$ and $\varphi = 0$

It is also possible to treat the case of the mechanically free, electrically closed ($\varphi = 0$) and magnetically closed ($B_3 = 0$) surface. Here, equations from (126) to (128) are used with exclusion of μ_0 in equation (128). Using the first set of the eigenvector components, the coupled three equations can be then written after some simplifications in the following forms:

$$\varepsilon\mu CK_{em}^2 F + b\varepsilon\mu C\left(1 + K_{em}^2\right)F_3 = 0 \tag{167}$$

$$-\alpha^2 CK_{em}^2 F + \left(\alpha eh - \alpha^2 CK_{em}^2\right)F_3 = 0 \tag{168}$$

$$\left(\varepsilon\mu - \alpha^2\right)CK_{em}^2 F + 0F_3 = 0 \tag{169}$$

These coupled equations lead to the velocity of the fifth new SH-SAW defined by equation (163).

Using the second set of the eigenvector components, the coupled three equations are written as follows:

$$\varepsilon\mu CK_{em}^2 F + b\varepsilon\mu C\left(1 + K_{em}^2\right)F_3 = 0 \tag{170}$$

$$\varepsilon\mu CK_{em}^2 F - \left(\varepsilon h^2 - \varepsilon\mu CK_{em}^2\right)F_3 = 0 \tag{171}$$

$$0F + 0F_3 = 0 \tag{172}$$

Coupled equations (170) and (171) give the following SH-SAW velocity:

$$V_{M2} = V_{tem}\left[1 - \left(\frac{K_{em}^2 - K_m^2}{1 + K_{em}^2}\right)^2\right]^{1/2} \tag{173}$$

This SH-SAW velocity V_{M2} defined by equation (173) was also obtain by Arman Melkumyan in his theoretical works [38, 29] for the case of the mechanically free, electrically and magnetically closed surface. Therefore, the SH-SAW velocity V_{M2} in equation (173) can be called the second surface Melkumyan wave. The functions F and F_3 for the second surface Melkumyan wave are determined from equation (170) as follows:

$$F = b\varepsilon\mu C\left(1 + K_{em}^2\right) \tag{174}$$

$$F_3 = -\varepsilon\mu C K_{em}^2 \tag{175}$$

where the parameter b is defined following equation (173) as follows:

$$b = \frac{K_{em}^2 - K_m^2}{1 + K_{em}^2} \tag{176}$$

CHAPTER XI. The Case of $D_3 = 0$ and $B_3 = 0$

For this case of the mechanically free, electrically open ($D_3 = 0$) and magnetically closed ($B_3 = 0$) surface, any SH-SAW solution cannot be found for both the sets of the eigenvector components. This is true because the single solution for this case represents the SH-BAW velocity V_{tem}. Note that Arman Melkumyan in his works [38, 29] reported the same result, namely no SH-SAW solutions for the case of $D_3 = 0$ and $B_3 = 0$.

CHAPTER XII. The Case of $D_3 = 0$ and Continuity of B_3

Using the first set of the eigenvector components, the coupled three equations for this case are written as follows:

$$\left[\varepsilon(\mu + \mu_0) - \alpha^2\right] C K_{em}^2 F + bC\left[\varepsilon(\mu + \mu_0) - \alpha^2\right]\left(1 + K_{em}^2\right) F_3 = 0 \quad (177)$$

$$0F + 0F_3 = 0 \quad (178)$$

$$\left[\varepsilon(\mu + \mu_0) - \alpha^2\right] C K_{em}^2 F - \mu_0\left(e^2 - \varepsilon C K_{em}^2\right) F_3 = 0 \quad (179)$$

They result in the following velocity of the sixth new SH-SAW:

$$V_{new6} = V_{tem}\left[1 - \left(\frac{\varepsilon}{\varepsilon_0}\frac{V_{EM0\alpha}^2}{C_L^2}\frac{K_{em}^2 - K_e^2}{1 + K_{em}^2}\right)^2\right]^{1/2} \quad (180)$$

where the speed C_L of light in a vacuum is defined by equation (109). The formula for the velocity $V_{EM0\alpha}$ reads

$$V_{EM0\alpha}^2 = \frac{1}{\varepsilon(\mu + \mu_0) - \alpha^2} \quad (181)$$

The functions F and F_3 for the case of the sixth new SH-SAW can be determined from equation (177) as follows:

$$F = bC\left[\varepsilon(\mu + \mu_0) - \alpha^2\right]\left(1 + K_{em}^2\right) \quad (182)$$

$$F_3 = -\left[\varepsilon(\mu + \mu_0) - \alpha^2\right] C K_{em}^2 \quad (183)$$

where the parameter b is defined following equation (180) as follows:

$$b = \frac{\varepsilon\mu_0}{\varepsilon(\mu+\mu_0)-\alpha^2}\frac{K_{em}^2 - K_e^2}{1+K_{em}^2} \tag{184}$$

It is worth noting here that the functions F and F_3 are also substituted into equations from (71) to (73).

Using the second set of the eigenvector components, the coupled three equations for this case can be written in the following forms:

$$\varepsilon\mu CK_{em}^2 F - b\varepsilon\mu C\left(1+K_{em}^2\right)F_3 = 0 \tag{185}$$

$$\left[\varepsilon\mu - \alpha^2\right]CK_{em}^2 F + 0F_3 = 0 \tag{186}$$

$$-\alpha^2 CK_{em}^2 F - \left(\alpha eh - \alpha^2 CK_{em}^2\right)F_3 = 0 \tag{187}$$

Therefore, the resulting velocity represents the velocity of the fifth new SH-SAW and is defined by equation (163).

CHAPTER XIII. The Case of $B_3 = 0$ and Continuity of D_3

Using the first set of the eigenvector components, the coupled three equations for this case are then written as follows:

$$\varepsilon\mu C K_{em}^2 F + b\varepsilon\mu C\left(1 + K_{em}^2\right)F_3 = 0 \tag{188}$$

$$-\alpha^2 C K_{em}^2 F + \left(\alpha e h - \alpha^2 C K_{em}^2\right)F_3 = 0 \tag{189}$$

$$\left(\varepsilon\mu - \alpha^2\right)C K_{em}^2 F + 0F_3 = 0 \tag{190}$$

Therefore, the resulting velocity is also defined by equation (163) which demonstrates the velocity of the fifth new SH-SAW.

For the second set of the eigenvector components, the coupled three equations for this case read:

$$\left[(\varepsilon + \varepsilon_0)\mu - \alpha^2\right]C K_{em}^2 F + bC\left[(\varepsilon + \varepsilon_0)\mu - \alpha^2\right]\left(1 + K_{em}^2\right)F_3 = 0 \tag{191}$$

$$\left[(\varepsilon + \varepsilon_0)\mu - \alpha^2\right]C K_{em}^2 F - \varepsilon_0\left(h^2 - \mu C K_{em}^2\right)F_3 = 0 \tag{192}$$

$$0F + 0F_3 = 0 \tag{193}$$

They also result in the following velocity of the seventh new SH-SAW:

$$V_{new7} = V_{tem}\left[1 - \left(\frac{\mu}{\mu_0}\frac{V_{E0M\alpha}^2}{C_L^2}\frac{K_{em}^2 - K_m^2}{1 + K_{em}^2}\right)^2\right]^{1/2} \tag{194}$$

where the velocity $V_{E0M\alpha}$ is defined as follows:

$$V_{E0M\alpha}^2 = \frac{1}{(\varepsilon + \varepsilon_0)\mu - \alpha^2} \tag{195}$$

The functions F and F_3 for the case of the sixth new SH-SAW can be determined from equation (191) as follows:

$$F = bC\left[(\varepsilon + \varepsilon_0)\mu - \alpha^2\right]\left(1 + K_{em}^2\right) \quad (196)$$

$$F_3 = -\left[(\varepsilon + \varepsilon_0)\mu - \alpha^2\right]CK_{em}^2 \quad (197)$$

where the parameter b is defined following equation (194) as follows:

$$b = \frac{\varepsilon_0\mu}{(\varepsilon + \varepsilon_0)\mu - \alpha^2}\frac{K_{em}^2 - K_m^2}{1 + K_{em}^2} \quad (198)$$

Note that the functions F and F_3 are also substituted into equations from (71) to (73).

CHAPTER XIV. Calculations and Discussion

First of all, it is possible to discuss the material properties listed in table 1. The material constants in the table are given as percentage volume fraction of $BaTiO_3$ in composites consisting of $BaTiO_3$-$CoFe_2O_4$ [44, 53]. Note that Refs. [44, 53] give zero piezoelectric coefficient e for all the composites listed in table 1, but purely piezoelectric material $BaTiO_3$ in the last column of the table. On the other hand, Ref. [54] by Wang and Mai provides the value of $e = e_p/2$ for 50% $BaTiO_3$ in $BaTiO_3$-$CoFe_2O_4$, where e_p is that for the pure $BaTiO_3$. Therefore, it is thought that it is a good idea to use small (see table 1) but non-zero values of e for the composites in the calculations. It is also noted that Refs. [44, 53] give the magnetic permeability coefficient $\mu < 0$ for $CoFe_2O_4$ and the composites. Conversely, Ref. [54] provides the magnetic permeability coefficient $\mu > 0$ for both $CoFe_2O_4$ and the composite. It is thought that $\mu < 0$ is preferred because it is given in almost all papers concerning studies of the composites. However, the negative sign for the magnetic permeability coefficient μ results in the negative sign for the coefficient of the magnetomechanical coupling (CMMC) K_m^2 and can also result in the negative sign for the coefficient of the magnetoelectromechanical coupling (CMEMC) K_{em}^2 in the case of Abs(K_m^2) > Abs(K_e^2), see table 1. Therefore, this is actually an interesting case. Indeed, the negative sign for the magnetic permeability coefficient μ also results in the imaginary values for such material characteristics as the velocities $V_{EM\alpha}$, V_{EM}, $V_{EM0\alpha}$, and $V_{E0M\alpha}$ also listed table 1 and defined by equations (134), (135), (181), and (195), respectively. Also, the electromagnetic constant α for the composites in the table is very small. This results in the large values of the velocity V_α defined by equation (136). It is worth noting that the value of the electromagnetic constant α can be miles larger. It is thought that a large value of the constant α can give a significant rise to the CMEMC K_{em}^2, hence, to the velocities of the surface Bleustein-Gulyaev-Melkumyan wave defined by equation (90), to the first and second surface Melkumyan waves defined by equations (156) and (173), and to the seven new SH-

SAWs correspondingly defined by equations (108), (120), (133), (148), (163), (180), and (194).

Table 2. The wave characteristics for the piezoelectromagnetic composite materials of class 6 mm listed in table 1. The characteristics are given as percentage volume fraction (VF) of $BaTiO_3$ in the $BaTiO_3$-$CoFe_2O_4$ composites

Composite VF	**0%**	**20%**	**40%**	**60%**	**80%**	**100%**
V_{t4}, **[m/s]**	**2811.71818686**	**2802.39239604**	**2802.39239604**	**2802.39239604**	**2953.98095634**	**2739.40924320**
V_{te}, **[m/s]**	**2811.71818686**	**2803.33580370**	**2803.94884848**	**2805.50443741**	**2958.70355080**	**3098.56369689**
V_{tm}, **[m/s]**	**2795.17393167**	**2793.14761564**	**2796.35764091**	**2795.65856397**	**2951.61682554**	**2739.40924320**
V_{tem}, **[m/s]**	**2795.17393167**	**2794.09419095**	**2797.91751629**	**2798.77824255**	**2956.34335715**	**3098.56369689**
V_{tem0}, **[m/s]**	**2795.17393167**	**2794.09414474**	**2797.91745041**	**2798.77809289**	**2956.34319660**	**3098.56369689**
V_{BGM}, **[m/s]**	**2794.97691742**	**2794.04475419**	**2797.90317946**	**2798.76889633**	**2956.33958460**	**3023.77282194**
V_{BGEC}, **[m/s]**	**2811.71818686**	**2803.33516894**	**2803.94712149**	**2805.49754091**	**2958.68849869**	**3023.77282194**
V_{BGpe}, **[m/s]**	**2811.71818686**	**2803.33580327**	**2803.94884827**	**2805.50443675**	**2958.70354964**	**3098.56365079**
V_{BGMO}, **[m/s]**	**2794.97691742**	**2793.08621536**	**2796.33153761**	**2795.62604638**	**2951.61303535**	**2739.40924320**
V_{BGpm}, **[m/s]**	**2795.17393077**	**2793.14761500**	**2796.35764024**	**2795.65856165**	**2951.61682457**	**2739.40924320**
V_{new1}, **[m/s]**	**2795.17393077**	**2794.09419031**	**2797.91751562**	**2798.77824023**	**2956.34335618**	**3098.56369689**
V_{new2}, **[m/s]**	**2795.17393167**	**2794.09419051**	**2797.91751608**	**2798.77824189**	**2956.34335598**	**3098.56365079**
V_{new3}, **[m/s]**	**2795.17393167**	**2794.09354984**	**2797.91577803**	**2798.77129588**	**2956.32826844**	**3023.77282190**
V_{new4}, **[m/s]**	**2794.97691742**	**2794.03285336**	**2797.89145691**	**2798.74583429**	**2956.33958536**	**3098.56369689**
V_{M1}, **[m/s]**	**2794.97691742**	**2794.03285367**	**2797.89145720**	**2798.74583501**	**2956.33958562**	**3098.56369689**
V_{new5}, **[m/s]**	**2795.17393167**	**2794.09419095**	**2797.91751629**	**2798.77824255**	**2956.34335715**	**3098.56369689**
V_{M2}, **[m/s]**	**2795.17393167**	**2794.09354981**	**2797.91577796**	**2798.77129555**	**2956.32826793**	**3023.77282194**
V_{new6}, **[m/s]**	**2795.17393077**	**2794.09419031**	**2797.91751562**	**2798.77824024**	**2956.34335619**	**3098.56369689**
V_{new7}, **[m/s]**	**2795.17393167**	**2794.09419051**	**2797.91751608**	**2798.77824189**	**2956.34335599**	**3098.56365079**
$(K_{new1})^2$	**0.00014096726**	**0.00003538607**	**0.00001024834**	**0.00000667720**	**0.00000255112**	**0.04827454416**
$(K_{new2})^2$	**0.00014096797**	**0.00003538678**	**0.00001024834**	**0.00000667863**	**0.00000255112**	**0.04827451518**

Table 2 lists the wave characteristics of the composites. The characteristic velocities are calculated with the corresponding formulae written in the previous chapters. First of all, it is indispensable to discuss about the dependence of the first, second, sixth, and seventh new SH-SAWs on the speed of light in a vacuum. This can mean that the existence of the new waves is caused by the contact of the free surface of such a solid with a vacuum. It is also clearly seen in the corresponding formulas for these new SH-SAWs that the character of the dependence on the speed C_L for the first and second new SH-SAWs differs from that for the sixth and seventh new SH-SAWs. For the first and second new SH-SAWs, the speed C_L appears in the second term of the corresponding value of the parameter b, because the first term is $K_{em}^2 - K_e^2$ or $K_{em}^2 - K_m^2$. It is thought that the second term can give a significant rise for a large value of such material parameter as the electromagnetic constant α. For the sixth and seventh new SH-SAWs, the corresponding parameter b depends only on the corresponding single term. It is certainly seen in equations (180) and (194) that the parameter b can be larger in these cases for small values of the material constants ε and μ and a large value of the CMEMC K_{em}^2. Note that a large value of the parameter b results in the position of the new SH-SAWs just below the SH-BAW velocity V_{tem}. It is well-known for such surface waves that the larger is the value of b^2, the lower is situated the velocity of a surface wave from the SH-BAW velocity V_{tem}. This actually results in a smaller penetration depth, and more energy of such surface waves can be localized at the surface of a solid.

On the other hand, the constants ε_0 and μ_0 of a vacuum do not participate in the propagation of the third, fourth, and fifth new SH-SAWs. This is also true for the velocities V_{BGM}, V_{M1}, and V_{M2}. Indeed, these velocities and the velocities of these three new SH-SAWs depend only on the composite material constants, particularly on the dielectric permittivity coefficient ε, magnetic permeability coefficient μ, and the electromagnetic constant α. However, it is thought that the dependence on the material constants for these three new SH-SAWs is more complicated, because the corresponding parameters b for the velocities V_{BGM}, V_{M1}, and V_{M2} depend only on the coefficients K_e^2, K_m^2, and K_{em}^2. Indeed, for the third and fourth new SH-SAWs defined by equations (133) and (148) there are two terms which demonstrate sophisticated dependencies on the $\varepsilon\mu$ and α. This is also somewhat true for the fifth new SH-SAWs defined by equation (163), for which the parameters b depends only the single term.

It is also thought that the fifth new SH-SAWs can lie significantly below the SH-BAW velocity V_{tem} for a large value of the electromagnetic constant α and an enough small value of $\varepsilon\mu$, because the V_{new5} position just below the velocity V_{tem} strongly depends on the following relation $\alpha^2/\varepsilon\mu$. Notwithstanding, this is still not obvious. The maximum value of the constant α for the composites listed in table 1 is as high as 6.8 ps/m. According to Ref. [9], the largest magnetoelectrical coefficients $\alpha \sim$ 30-40 ps/m have been observed for $LiCoPO_4$, $Y_3Fe_5O_{12}$ (YIG has the cubic symmetry of class m3m) and $TbPO_4$. However, Refs. [44, 53] also give the largest value of $\alpha_{33} \sim$ 2750 ps/m for 40% $BaTiO_3$ in $BaTiO_3$-$CoFe_2O_4$. It is worth noting that the largest value of α_{33} does not participate in calculations for the treated cases of the transversely-isotropic materials, because the surface waves can propagate in the propagation directions in which $\alpha_{33} \to \alpha_{22}$ and $\alpha_{22} \to \alpha_{33}$ to cope with the case of $\alpha_{11} = \alpha_{33} = \alpha$. Note that the electromagnetic response is limited by the following relation:

$$\alpha^2 < \varepsilon\mu \tag{199}$$

Therefore, the magnetoelectric (ME) effect is exhaustively studied in many composites consisting of hexagonal materials of class 6 mm and cubic materials of class m3m. It is natural that the particular purpose of many experimental investigations of the ME-effect is to observe a maximally-possible value of the electromagnetic constant α for a composite in dependence on the percentage volume fraction of a piezoelectrics or a piezomagnetics in two-phase composites.

It is possible to discuss about the calculated data listed in table 2. First of all, it is stressed that it is trying to deal with all the cases. It is thought that it is preferable to compare various materials with large values of the electromagnetic constant α for a concrete case of the electrical and magnetic boundary conditions. This is true because the boundary conditions can be realized in different ways following Ref. [4]. It is clearly seen in table 2 that all the velocities of the new SH-SAWs for the cases of $\alpha \neq 0$ are situated just below the SH-BAW velocity V_{tem}. Note that the case of $\alpha = 0$ occurs only for pure $BaTiO_3$ and pure $CoFe_2O_4$. It is also noted that for $\mu < 0$ (see 20%, 40%, and 60% $BaTiO_3$ in the composites in table 1) the SH-BAW velocity V_{tem} is smaller than the SH-BAW velocity V_{t4} which serves for the case of $e = 0$, $h = 0$, and $\alpha = 0$. For 80% $BaTiO_3$ in the composite with $\mu < 0$ there occurs $V_{tem} > V_{t4}$. Also, there occurs $V_{tem} < V_{te}$ for all the composites in table 2. For the composites with small α, the difference among the velocities of the new SH-SAWs can be seen only in the last digits after the decimal point. Also, for some cases, even the last digits cannot demonstrate the fact that the new SH-SAW velocities lie just below the SH-BAW velocity V_{tem}. This occasion is discussed below. It is worth noting that the new SH-SAW velocities are defined by exact formulae obtained in this work. Therefore, any even very small difference between a new SH-SAW velocity and the SH-BAW velocity V_{tem} due to a very small value of $\alpha \neq 0$ is trustable. This is also true for comparison of one new SH-SAW velocity with the other or with the velocities V_{M1}, V_{M2}, and V_{tem}.

It is emphasized that the calculated values of the velocity V_{new5} of the fifth new SH-SAW do not demonstrate the position of the velocity V_{new5} which should be just below the SH-BAW velocity V_{tem}. This is clearly seen in table 2 due to very small values of the electromagnetic constant α for the composites. However, it is possible to calculate the values of the velocities V_{tem} and V_{new5} with more digits after the decimal point. For example, for the percentage volume fraction of 80% $BaTiO_3$ in $BaTiO_3$-$CoFe_2O_4$, the velocities can be calculated with the following accuracy: V_{tem} = 2956.3433571533(10389124) m/s and V_{new5} = 2956.3433571533(06028796) m/s, where the digits in the parentheses demonstrate that the velocity V_{new5} is actually situated just below the SH-BAW velocity V_{tem}. This can present a big difficulty to numerically find this type of the new surface waves in non-hexagonal piezoelectromagnetic composites, for instance, in cubic piezoelectromagnetic composites. Following the results of this work, it is thought that at least nine new SH-SAWs can be numerically distinguished in cubic piezoelectromagnetic composites. It is also thought that all cubic piezoelectromagnetic composites can be also divided into two groups: the first group is for those with $K_{em}^2 < 1/3$ and the second for $K_{em}^2 >$ 1/3 following cubic piezoelectrics [7] and cubic piezomagnetics [8]. It is also offered that the difference $\Delta_{V5} = (V_{tem} - V_{new5})$ can serve as an indicator for the magnetoelectric effect in addition to evaluation of the electromagnetic constant α for a piezoelectromagnetic composite.

Also, table 2 gives sample evaluations of the coefficients K_{new1} and K_{new2} for the case of piezoelectromagnetoelastic composites, in order to have an analogy to the cases of purely piezoelectric phase and purely piezomagnetic phase. It is thought that the coefficients K_{new1} and K_{new2} can be evaluated with the following formulae:

$$K_{new1}^2 = 2\frac{V_{new1} - V_{BGM}}{V_{new1}} \quad (200)$$

$$K_{new2}^2 = 2\frac{V_{new2} - V_{BGM}}{V_{new2}} \quad (201)$$

where the velocities V_{BGM}, V_{new1}, and V_{new2} are defined by equations (90), (108), and (120), respectively. Indeed, these coefficients in equations (200) and (201) are significantly smaller for the composites than those for pure $BaTiO_3$ in the last column of table 2 and pure $CoFe_2O_4$ in the second column of the table. For the other five new SH-SAWs, it is thought that it is still uncertain which suitable coefficients like those calculated from equations (200) and (201) can be used for the evaluation. Therefore, they were not evaluated.

CONCLUSION

This theoretical work demonstrated existence of seven new piezoelectromagnetic SH-SAWs which can propagate in a transversely-isotropic monocrystal of hexagonal class 6 mm. Utilizing different electrical and magnetic boundary conditions in the theory, a coupling of the new SH-SAWs with the surface Bleustein-Gulyaev waves and bulk acoustic waves was analytically shown. Also, it was analytically shown that the new SH-SAWs can reveal the dependence on the squared speed of light in a vacuum. Sample calculations were carried out for the well-known mixture of piezoelectrics $BaTiO_3$ and piezomagnetics $(CoO)Fe_2O_3$. It is thought that the obtained results can be useful for theoreticians working with two-phase materials and developers of smart devices in the microwave technology.

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