This theoretical work has the purpose to thoroughly investigate the problems of shear-horizontal (SH) interfacial acoustic wave propagation guided by the common interface between two dissimilar piezoelectromagnetic hexagonal half-spaces of class 6 mm. At the interface, the mechanical, electrical, and magnetic boundary conditions can support the interfacial SH-wave propagation. The equality of the mechanical displacements and the normal components of the stress tensor (mechanically free interface) were used as the mechanical boundary conditions. The electrical and magnetic boundary conditions can include the electrically closed or electrically open interface, magnetically closed or magnetically open interface, and the others. As a result, twenty two new interfacial SH-waves can propagate in such two-layer structures. Their propagation speeds can be evaluated using the obtained explicit forms and the corresponding existence conditions. Some sample calculations were performed for PZT–Terfenol-D and $BaTiO_3$–$CoFe_2O_4$ composites. The results can be useful for complete understanding of wave processes in two-phase laminated composites in acoustoelectronics and optoelectronics.

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978-3-659-13905-5

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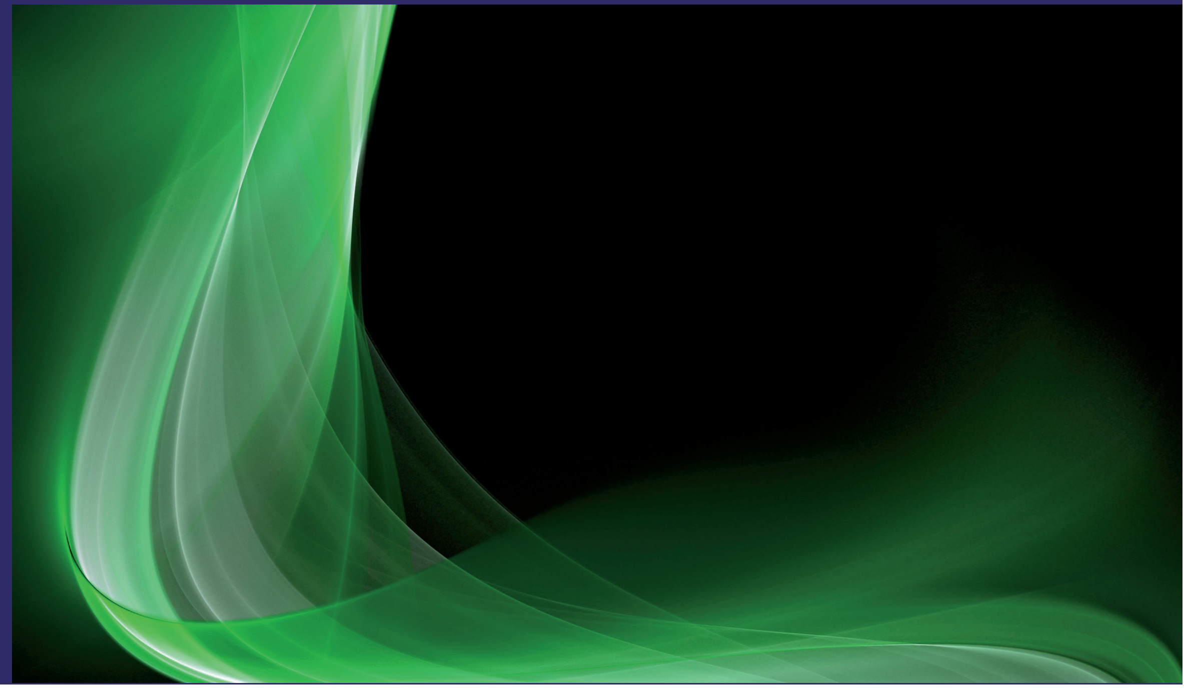

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Impressum/Imprint (nur für Deutschland/only for Germany)
Bibliografische Information der Deutschen Nationalbibliothek: Die Deutsche Nationalbibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über http://dnb.d-nb.de abrufbar.

Coverbild: www.ingimage.com

Verlag: LAP LAMBERT Academic Publishing GmbH & Co. KG
Heinrich-Böcking-Str. 6-8, 66121 Saarbrücken, Deutschland
Telefon +49 681 3720-310, Telefax +49 681 3720-3109
Email: info@lap-publishing.com

Herstellung in Deutschland (siehe letzte Seite)
ISBN: 978-3-659-13905-5

Imprint (only for USA, GB)
Bibliographic information published by the Deutsche Nationalbibliothek: The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at http://dnb.d-nb.de.

Cover image: www.ingimage.com

Publisher: LAP LAMBERT Academic Publishing GmbH & Co. KG
Heinrich-Böcking-Str. 6-8, 66121 Saarbrücken, Germany
Phone +49 681 3720-310, Fax +49 681 3720-3109
Email: info@lap-publishing.com

Printed in the U.S.A.
Printed in the U.K. by (see last page)
ISBN: 978-3-659-13905-5

This work has been fulfilled

within the International Institute of Zakharenko Waves (IIZWs) by

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info@lap-publishing.com http://www.lap-publishing.com

Saarbruecken – Krasnoyarsk, 2012

PREFACE

This theoretical work has the purpose to thoroughly investigate the problems of shear-horizontal (SH) interfacial acoustic wave propagation guided by the common interface between two dissimilar piezoelectromagnetic hexagonal half-spaces of class 6 *mm*. At the interface $x_3 = 0$, the mechanical, electrical, and magnetic boundary conditions can support the interfacial SH-wave propagation. The equality of the mechanical displacements and the normal components of the stress tensor (mechanically free interface) were used as the mechanical boundary conditions. The electrical and magnetic boundary conditions can be the electrically closed or electrically open interface, magnetically closed or magnetically open interface, as well as $\varphi^{I} = \varphi^{II}$, $D_3^{I} = D_3^{II}$, $\psi^{I} = \psi^{II}$, and $B_3^{I} = B_3^{II}$ at $x_3 = 0$ where φ, D_3, ψ, and B_3 stand for the electrical potential, electrical displacement component, magnetic potential, and magnetic flux component, respectively, and the superscripts "I" and "II" signify the first and second piezoelectromagnetic half-spaces.

As a result, it was found that as many as twenty two new interfacial SH-waves can propagate in such two-layer structures. Their propagation speeds can be evaluated using the obtained explicit forms and the existence conditions. The sample calculations were performed for some cases when PZT–Terfenol-D and $BaTiO_3$–$CoFe_2O_4$ composites are used. It is thought that some of the obtained expressions can be also useful for the problems of interfacial SH-wave propagation along the interface between single-phase materials such as piezoelectrics and piezomagnetics. It is obvious that the obtained results can be useful for complete understanding of wave processes in two-phase laminated composite materials with the hexagonal (6 *mm*) symmetry in acoustoelectronics, acoustooptics, and optoelectronics. It is expected that the obtained results can be utilized in fabricating smart materials in the microwave technology. It is thought that the electromagnetic acoustic transducers (EMATs) which are used for investigations of SH-SAW propagation problems can be also exploited for studies of propagation problems of these new interfacial SH-waves.

PACS: 51.40.+p, 62.65.+k, 68.35.Gy, 68.35.Iv, 68.60.Bs, 74.25.Ld, 74.25.Ha, 75.20.En, 75.80.+q, 81.70.Cv

COMMENTS BY THE AUTHOR

This theoretical work was carried out as a research activity existing in the International Institute of Zakharenko Waves (IIZWs). It is expected that this work can cause an interest for researchers and students coping with the acoustic wave propagation in the transversely-isotropic piezoelectromagnetics. It describes a complicated problem such as the wave propagation guided by the interface between two solid half-spaces representing two transversely-isotropic piezoelectromagnetics. It is thought that knowledge of wave properties of such complicated system consisting of two piezoelectromagnetics can be also beneficial to design of smart devices, sensors, actuators, etc. Also, it can represent an interest in constitution of piezoelectromagnetic laminate composites in the microwave technology and non-destructive testing of the composites. This theoretical research can be also useful for the aerospace industry which calls for innovative smart composite materials. Therefore, it is very important to completely understand wave properties of different composites.

This work studies propagation phenomena of the shear-horizontal (SH) interfacial acoustic waves in the layered system consisting of two transversely-isotropic piezoelectromagnetics of class 6 *mm*. This studying subject relates to the disciplines of applied physics and electromagnetic engineering. In physics, ordinary elastic motions in crystals are called acoustic modes. The descriptive term ”acoustic” is used rather than ”elastic”. This is useful because it allows one to distinguish acoustic and optical modes from each other. The optical modes involve internal degrees of freedom within a crystal unit cell. The term ”acoustic” also reflects common terminology among researchers and engineers engaged in developing elastic wave devices for radar and communication systems. This arena of technological development has been strongly influenced by the philosophy, concepts, and techniques of microwave electromagnetics. This is also known as microwave acoustics. Consequently, employment of the term "acoustic" accurately describes the aim and scope of the book.

The International Institute of Zakharenko Waves (IIZWs) was recently created to support researches on different Zakharenko waves, as well as for monitoring the non-dispersive Zakharenko type waves in complex systems such as layered and quantum systems. Also, the IIZWs research is focused on treatments of many complex systems in which dispersive waves can propagate. The well-known examples of dispersive waves are Love and Lamb type waves. The Rayleigh and Bleustein-Gulyaev type waves propagating in layered systems can be also dispersive. The International Institute of Zakharenko Waves also has an interest in different applications of the acoustic waves for signal processing (filters, sensors, etc.) and the structural health monitoring. There are currently more than twenty research papers and books relevant to the IIZWs. These research works also cover some problems of the propagation of the well-known Love, Lamb, Rayleigh, and Bleustein-Gulyaev type waves and discovered new wave phenomena.

It is worth noticing that the IIZWs possessively takes all the planets and smaller natural space bodies in the space outside the Solar System to develop both the IIZWs and the planets concerning economics, ecology, and population. Also, it is thought that this is necessary in order to exclude any sale of the planets and their surfaces by any human or other. This activity of the IIZWs was also created because of some problems to find a spot for the IIZWs on Earth. Note that the single person, namely Mr. Dennis Hope from the United States possesses the planets in the Solar System (but Earth) who sells surfaces of the planets to individuals. It is obvious that the monetary experiment on Earth during thousand years demonstrated a weak power of the financial system to avoid financial problems which cyclically happen. As a result, the following question presents in the air: what is the modern money? It is obvious that monetary systems are coupled only with humans who have given power to each other, but not with any space body such as a planet or star. It is apparent that humans depend on money, but not planets and stars. Indeed, planets and stars are leaving their own lifetimes and their ways of life do not depend on human activities measured in money. Therefore, money can exist only together with the human civilizations. It is not clear that the other civilizations can evaluate their activities in the same way similar to the human civilization does on Earth. Nothing is soundly known about that. It is also noted that only several thousand planets orbiting their own stars can be currently observed in the Star Systems which are situated relatively near the Solar System. This does not mean that only several thousand planets can exist outside the Solar System we can observe. It is expected that in average ten planets can orbit each star of enormous number of Star Systems in our Universe. It is thought that our Universe can accumulate more than 10^{999} stars.

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INTRODUCTION

It is well-known that the simplest example of the shear-horizontal surface acoustic waves (SH-SAWs) is the dispersive Love wave. In 1911, A.E.H. Love [1] has treated a problem of the wave propagation in the two-layer system consisting of a thin film deposited on a substrate assuming that both materials are isotropic. He found that such wave can be guided by the thin film and damp towards the depth of the substrate. The Love wave displacement is perpendicular to both the propagation direction and the surface normal (anti-plane polarization.) There is also the following existence condition for the dispersive Love wave: the speed of the shear-horizontal bulk acoustic wave (SH-BAW) for the thin film should be smaller than that for the substrate. Therefore, it is possible to notice that the Love type wave (LTW) represents a hybridization of these two SH-BAWs for the thin film and the substrate because the LTW phase velocity is localized between these two SH-BAWs. The LTW can only exist in the layered systems and cannot propagate in monomaterials.

It is also well-known that three BAWs can exist in solids such as monocrystals and isotropic monomaterials. It is thought that the reader can reach the famous works cited in Refs. [2-8]. These works are useful to find out more about BAWs and SAWs and their applications. Three BAWs are distinguished as follows: the SH-BAW, shear-vertical BAW, and longitudinal BAW. It is worth noting that the SV-BAW and the LBAW can be also hybridized in order to form a SAW. This problem of the SAW propagation guided by the free surface of an isotropic medium was first studied in 1885 by Lord Rayleigh [9] in connection with the problem of earthquakes. The SAWs called the Rayleigh type waves (RTW) have displacements in the sagittal plane (in-plane polarization) and can exist in isotropic media, monocrystals, and layered systems. Concerning the RTW propagation in isotropic media and crystals, one can find the RTW existence conditions discovered in the recently published paper cited in Ref. [10]. The conditions are written in explicit forms. Dispersive

RTWs propagating in the layered systems were also studied by the author in Refs [11, 12], of which the second studies the piezoelectric case and is available in an on-line open access.

To study anisotropic media is frequently a very complicated business. It is well-known that the phase velocities (V_{ph}) of the elastic waves propagating in isotropic media do not depend on the propagation direction. In general, natural solids are anisotropic, namely V_{ph} depends on the propagation direction in crystals. The propagation problems of BAWs and SAWs can be significantly complicated in the case when a crystal possesses the piezoelectric or piezomagnetic properties. This is also true for the SH-BAWs and SH-SAWs. However, there are piezoelectrics called the transversely isotropic materials. Some of them possess the hexagonal symmetry of class 6 *mm*. Using the piezoelectrics of this class, Bleustein [13] and Gulyaev [14] in 1960s have discovered the new SH-SAW guided by the free surface of the transversely isotropic piezoelectrics when the propagation direction is perpendicular to the sixfold axis of crystal symmetry. This new SH-SAW called the surface Bleustein-Gulyaev (BG) wave is now well-known and can be treated as instability of the SH-BAW in piezoelectrics. This is also true for piezomagnetics. It is necessary to state that piezoelectrics and piezomagnetics represent the single-phase materials and they are used for transducer applications [15]. It is also indispensable to mention that the BG-wave can be dispersive [16] when the layer-on-substrate structure is treated and the LTW can also exist in the layered systems [17-20] consisting of piezoelectrics. It is also noted that the LTWs are widely used in dispersive SAW filters and sensors, and LTW SAW devices can have the highest sensitivity [21–24]. Some reviews on the subject can be found in Refs. [25-30] and the properties of crystals are perfectly described in excellent and classical books [31, 32].

To complicate the problem of the wave propagation, it is possible to treat two-phase materials. These materials can possess both the piezoelectric and piezomagnetic phases and are therefore called piezoelectromagnetics. In 1990, Al'shits and Lyubimov [33] have performed a crystallographic study of crystal classes. They have the purpose to conveniently describe the properties of crystals and textures as well as co-existence of piezoelectricity and piezomagnetism. As a result, they demonstrated that the two-phase materials such as the transversely isotropic piezoelectromagnetics of class 6 *mm* can exist. Also, piezoelectromagnetics of class 6 *mm* can represent composite materials. The measured material characteristics of the piezoelectromagnetic composites of class 6 *mm* such as $BaTiO_3$-$CoFe_2O_4$ are given in Refs. [34, 35]. The following material parameters can be measured in dependence on the percentage volume fraction (VF) of the piezoelectric phase ($BaTiO_3$) in the $BaTiO_3$-$CoFe_2O_4$ composites: the elastic stiffness constant C, piezoelectric constant e, piezomagnetic coefficient h, dielectric permittivity coefficient ε, magnetic permeability coefficient μ, and electromagnetic constant α. Also, it is natural that a two-phase composite can have an average mass density ρ. Also, it is worth mentioning that the piezoelectromagnetics (PEM) possess the magnetoelectric (ME) effect that can be evaluated by measurements of the magnetoelectric constant α. In PEM composite systems, a linear behavior is usually observed by means of AC magnetic field application. The non-linear ME effect can be also observed in the case of bias magnetic field application. The value of α can be expressed in s/m in SI units. However, it represents a non-dimensional value in Gaussian units. The value of α is very small and can usually reach only several ps/m. For instant, the following value of α is given for crominium oxide: $\alpha(Cr_2O_3)$ = 4.13 ps/m [36]. The ME coefficients for some monocrystals can have significantly larger values: α = 30.6 ps/m for $LiCoPO_4$ [37], and α = 36.7 ps/m for $TbPO_4$ [38]. According to review [36], the value of α can be restricted and this limitation can be written as the following inequality: $\alpha^2 < \varepsilon\mu$. In general, the electromagnetic constants α are written in a tensor form, α_{ij}. With the well-known notations used by Nye [31], Schmid [39] provides the tensor forms of the 58 point groups permitting the linear magnetoelectric effect.

Concerning the wave propagation in the piezoelectromagnetics, theoretical work [40] carried out in 1992 by Al'shits, Darinskii, and Lothe has discussed some problems of SAW existence in the two-phase materials, using different mechanical, electrical, and magnetic boundary conditions. In the case of the wave propagation in the PEMs, the governing equations can couple the mechanical displacements with both the electrical potential φ and the magnetic potential ψ [40-45]. Based on the theoretical treatments developed in the last decades, Arman Melkumyan [45] has discovered twelve new SH-SAWs propagating in the two-phase transversely isotropic materials of class 6 *mm*. He discovered his wide spectrum of the new SH-SAW velocities in 2007. Using different sets of the boundary conditions, he demonstrated the relatively simple explicit forms for each of the twelve new SH-SAW velocities. It is thought that one of the twelve new SH-SAWs discovered by Melkumyan [45] and called the Bleustein-Gulyaev-Melkumyan (BGM) wave [46-48] represents a special interest because the BGM-wave can propagate in both the cubic piezoelectromagnetics [46] and the transversely isotropic piezoelectromagnetic materials [48]. This is true using the same boundary conditions such as the mechanically free, electrically closed, and magnetically open surface. Using the other possible sets of the boundary conditions, recent book [48] revealed the explicit forms for the second spectrum of the seven new SH-SAW velocities existing in the piezoelectromagnetics of class 6 *mm*. To theoretically study the cubic piezoelectromagnetics is significantly more complicated problem and therefore, explicit forms for the new SH-SAW velocities were not demonstrated in recently published book [46]. However, it was possible in work [46] to obtain a convenient common form for the third spectrum of the other seven new SH-SAW velocities to compare them with each other and with the transversely isotropic case.

There are conventional and laminated two-phase composite materials. The space and aircraft technologies have an uninterrupted interest in the composites for various applications. Several books and handbooks on composite materials are cited in Refs. [49-54]. The structure of created composite materials can be complicated and some basic knowledge in crystallography [55, 56] is useful to experimentally determine symmetry classes of obtained new composites. Also, the geometry of a two-phase composite material can be denoted by the following connectivities: 0-0, 0-1, 0-2, 0-3, 1-1, 1-2, 1-3, 2-2, 2-3, and 3-3, where 0, 1, 2, and 3 are the dimensions of piezoelectric-piezomagnetic phases. The laminated composites can be described by the (2-2) connectivity. In the case of thick films it is possible to cope with bulk piezoelectric and piezomagnetic materials and the (3-3) connectivity for the piezoelectric and piezomagnetic phases. However, it is possible that a bulk piezoelectromagnetic monocrystal can have a common interface with another and the mechanical, electrical, and magnetic properties of two contacted piezoelectromagnetic monocrystals (two half-spaces) are different. This can be also true for two different bulk piezoelectromagnetic composites.

This research arena is rapidly developed and therefore, reviews are yearly published to discuss most recent advances in the physics of ME interactions in layered composites and nanostructures. Potential device applications are also reviewed. For instance, the magnetic-field sensors, dual electric-field- and magnetic-field-tunable microwave and millimetre-wave devices can be the potential device applications for the composites. Review works [36, 57-91] on the magnetoelectric effect and composites are recommended for the reader to receive complete information on the subject. Also, it is necessary to mention pioneer works [92-96] on the ME composites. Generally, a continuous interest occurs to study the magnetoelectric effect in composites for development of smart materials in the microwave technology. According to works cited in Refs. [96, 97], modern industry can have an increasing interest in the following possible applications of magnetoelectric materials:

- light computing;
- solid state non-volatile memory;
- magnetic-electric energy converting components;
- multi-state memory which can find application in quantum computing area;
- electrical/optical polarization components which can find applications in communication;
- solid state memories based on spintronics.

Indeed, two-phase materials are multi-promising and therefore, laminated composites of them cause a big interest among different research groups. So, it is necessary to review some achievements in this research field. First of all, it is crucial to mention some important researches carried out with pure piezoelectrics. In 1971, Maerfeld and Tournois [98] have published their collaborative theoretical work on new interfacial SH-wave guided by the common interface of two dissimilar piezoelectrics (semi-infinite media or half-spaces) of class 6 *mm*. This is the single-phase case because both media are pure piezoelectrics. They have also found the existence condition for the new interfacial SH-waves later called the interfacial Maerfeld-Tournois (MT) wave. They stated that the mechanical displacement amplitude of the MT-wave decreases with distance away from the common interface into both media. They also demonstrated the case when the interfacial wave can propagate with the speed of the surface BG-wave [13, 14]. The MT-wave can also propagate along the interface when one of two media is isotropic. It is obvious that one also deals with the BG-wave when a vacuum is used in the theoretical treatments instead of the second piezoelectrics or isotropic half-space. However, the problem of wave propagation is significantly complicated when one medium is piezoelectric and the second medium is piezomagnetic instead of a vacuum or the isotropic medium. It is thought that this case represents the simplest laminated piezoelectromagnetic composite consisting of piezoelectric and piezomagnetic half-spaces with the common interface. The theory developed by Maerfeld and Tournois for the single-phase materials is not suitable for the two-phase composites. Therefore, it is possible to review some papers in which laminated composites were investigated.

To start a review of recent achievements of researchers coping with the laminated composites, it is needed to state that the modern studies of the composites can relate to not only two-layer systems, but also multi-layer structures (sandwich-like systems). Soh and Liu [99] have a purpose to theoretically investigate propagation of interfacial SH-waves along the common interface in a piezoelectric-piezomagnetic bi-material. The piezoelectric half-space is perfectly bonded with the piezomagnetic half-space and both the materials are hexagonal crystals of class 6 *mm* (transversely isotropic materials.) For this case, they have obtained the dispersion relation in an explicit form and discussed two existence conditions for the interfacial SH-waves. Also, they have soundly exhibited that the dispersion relation reduces to that for the surface BG-wave propagation in a pure piezoelectrics as soon as a vacuum is considered instead of the piezomagnetic half-space. However, they did not demonstrated that the dispersion relation obtained by them can reduce to that for the interfacial MT-wave propagation when the second medium is piezoelectric, but not piezomagnetic. Moreover, they even did not cite this excellent and classical work [98]. Probably, they did not know about the existence of the work by Maerfeld and Tournois.

Concerning an imperfect interface, Huang, Li, and Lee [100] have also developed a theory describing interfacial SH-wave propagation in a two-phase piezoelectric/piezomagnetic structure when both the hexagonal materials pertain to class 6 *mm*. They have solidly obtained an exact dispersion relation and the existence condition for the interfacial SH-wave propagation in such bi-material and found that the interfacial imperfection can strongly affect the wave velocity. It is noted that one copes with more complicated case of dispersive waves in the case of the interfacial imperfection. In particular, they stated that for certain combined magnetoelectric composites, interfacial SH-waves cannot exist for perfect interface and exist only for imperfect interface. For the perfectly bonded interface, they stated that their result agrees with that derived in Ref. [99] when the interface is grounded. They have also found that the wave speed always lies between the smaller BG-wave of two constituents, and the smaller SH-BAW (or the interfacial SH-wave for a perfect interface if it exists). Refs. [99, 100] also stated that the findings can be useful for the two-phase composites in the microwave technology. However, Ref. [100] like Ref. [99] did not mention the interfacial MT-wave [98].

In general, the interface of two dissimilar piezoelectrics (piezomagnetics) is assumed to be either bonded perfectly or debonded completely. Ref. [100] also discussed that the interface between any two dissimilar materials cannot be perfectly bonded because of various causes such as microinhomogeneities, microdefects, microdebonding, etc. An interfacial imperfection can weaken the interfacial continuity, and further affect the performance of the composites, in particular the interfacial characteristics. For instance, the effects of the interfacial imperfection on wave propagation in an isotropic elastic bi-material have been analysed in Refs. [101, 102]. Furthermore, the influences of the imperfect interface on the interfacial SH-wave speed have been studied in [103] for two bonded piezoelectrics. Influence of imperfect bonding on interfacial waves propagating along the common interface between bonded piezoelectric and piezomagnetic half-spaces was also studied in work [104] by Melkumyan and Mai. They discussed the cases of absorbent and permeable interfaces and stated that an interfacial imperfection has a significant impact on the interfacial wave existence and on their velocities of propagation. A more complicated case of the interfacial imperfection in an A-B-A heterostructure was considered in recent paper [105] taking into account the geometric symmetry of the system. They have studied surface SH-waves propagating in the multi-layered system with magnetoelectroelastic properties and imperfect (electromagnetically permeable or absorbent, mechanically spring-type) bonding at the interfaces and considered different limit cases. They have numerically obtained the symmetric and asymmetric modes and found that the propagation velocities of the SH waves are limited by the velocities on the homogeneous phases A and B. Indeed, there is an increasing interest in various studies of different multi-layered structures, for instance, see Refs. [106, 107].

It is possible to list some promising two-layer systems which can represent a big experimental and theoretical interest. It is crucial to state that the modern researches on the laminated composites utilise the transversely isotropic materials and materials with the other symmetries, for example, cubic. Since 2000, dramatically enhanced values of the magnetoelectric voltage coefficient (α_{ME}) have been found in laminated composites [108-112] consisting of magnetostrictive and piezoelectric layers epoxied together. The laminated composites can have α_{ME} values of up to 500 larger than any other ME materials. This phenomenon is known as a giant ME effect. It is very popular that the piezomagnetic phase can be represented by $NiFe_2O_4$ [113, 114] or $CoFe_2O_4$ [115, 116] in the laminated piezoelectromagnetic composites. Some cubic piezomagnetics such as the Fe-Ga alloy called Galfenol [109, 119] and the FeBSiC alloy called Metglas [111, 117, 118] are also used. Also, it is possible to meet some works in which the piezoelectric Lithium Niobate ($LiNbO_3$) [44] of trigonal class 3*m* can be utilized. It is thought that the most popular piezomagnetics is the unique alloy called Terfenol-D [108, 110, 120-122]. The most popular piezoelectric phase is $BaTiO_3$, $PbTiO_3$, PZT, and relevant [108, 109, 113-117, 119-122]. There is also an interest in various studies of ring-type piezoelectric-piezomagnetic laminated composites [123-127] and wave propagation in such layered cylinders [123, 124]. This also represents an important problem in addition to those for wave propagation in laminated rectangular plates and half-spaces possessing the common interfaces.

Since 1960 when the soviet physicist Astrov [128] has published his work concerning the magnetoelectric effect in antiferromagnets, much work on the subject can be found. The recent book published in 2011 and cited in Ref. [46] has referred to 255 works including the excellent pioneer papers [92-96, 128-130]. The 2011 review cited in Ref. [57] has mentioned 186 articles, and 2010 and 2009 reviews [58, 59] have 150 and 192 citations, correspondingly. Also, the very famous review paper published in 2005 by Fiebig [36] with 304 references has solidly demonstrated that this research arena is extremely popular. This list of works is far uncompleted. Indeed, the reader can found thousands of works relevant to the magnetoelectric effect and composite materials. In general, these experimental and theoretical works pertain to laborious studies of the magnetoelectric effect and different types of composites consisting of the piezoelectric and piezomagnetic phases. However, there is a lacuna in investigations of the wave propagation problems occurring in the laminated systems of two dissimilar piezoelectromagnetic (composite) materials of class 6 mm which have the common interface perfectly bonded.

Following theoretical work [48] written by the author, it is hoped that the theoretical results, which will be obtained in this work below, can fill up this lacuna and demonstrate that this problem of interfacial wave propagation can be resolved in the case of the transversely isotropic piezoelectromagnetics of class 6 *mm*. It is thought that there are many possibilities for the SH-wave propagation along the common interface of two dissimilar piezoelectromagnetics. These possibilities must be demonstrated. Therefore, Chapter I describes thermodynamics, corresponding constitutive relations, the equations of motion for the treated case, and possible boundary conditions. The following chapters have the purpose to exhibit the dispersion relations for the interfacial waves along the perfectly bonded interface. The final chapter serves for some discussions about the obtained theoretical results.

CHAPTER I

Theory

This first chapter acquaints the reader with the theory utilized for the wave propagation problems when the two-phase materials are studied. For a bulk piezoelectromagnetics, it is necessary to describe the thermodynamics of the medium for the case, constitutive relations, and corresponding equations of motion. Indeed, to review possible mechanical, electrical, and magnetic boundary conditions also represents an important thing for the further theoretical investigations which will be carried out in the following chapters. It is worth noting that this work copes with some suitable problems of the interfacial wave propagation along the common interface of two dissimilar piezoelectromagnetic (composite) materials of class 6 *mm* (transversely isotropic materials.) First of all, a theory will be given which is valid for both piezoelectromagnetics when they can be treated separately. Therefore, no superscript will be used in this case. The superscripts "I" and "II" will be used for the dissimilar piezoelectromagnetics to distinguish them from each other as soon as this will be necessary. It is usual to start with the thermodynamics.

I.1. *Thermodynamics and Constitutive Relations*

Consider a bulk solid possessing the piezoelectric, piezomagnetic, and magnetoelectric effects simultaneously. To thermodynamically describe this complex system, it is possible to use one suitable thermodynamic potential of eight ones used for this purpose. The chosen thermodynamic potential must properly describe thermoelectromagnetoelastic interactions in a piezoelectromagnetic solid. Using one of the thermodynamic potentials called the enthalpy H, general equations for adiabatic rather than isothermal conditions may be obtained. Indeed, the corresponding thermodynamic potentials can derive the equations of piezoelectric, piezomagnetic, and piezoelectromagnetic media [15, 48, 131]. Adiabatic processes can be considered as those with the constant entropy S, where the last represents a level of disorder in the system. It is clear that S = const gives d$S = 0$. For a linear case, it is possible to account only linear terms in a Taylor series for the enthalpy H relative to an equilibrium condition $H(S_0)$.

These linear terms can contain the following thermodynamic variables frequently written in the tensor forms: stress σ_{ij}, strain η_{ij}, electrical field E_i, electrical induction D_i (electrical displacement), magnetic field H_i, magnetic flux B_i (magnetic displacement) where the indexes i and j run from 1 to 3. For a piezoelectromagnetics, energetic terms of such complex system described by a thermodynamic potential can be naturally coupled with the following sub-systems:

- elastic sub-system (thermodynamic variable σ_{ij} or η_{ij});
- electric sub-system (variable D_i or E_i);
- magnetic sub-system (variable B_i or H_i);
- thermal sub-system (temperature T or entropy S).

It is thought that for the problem of wave propagation in a piezoelectromagnetic solid, it is natural to use the thermodynamic functions of which each depends of three independent thermodynamic variables such as the strain η_{ij}, electrical field E_i, and magnetic field H_i. These functions are written as follows:

$$\sigma_{ij} = f_1(\eta_{kl}, E_k, H_k) \tag{I.1}$$

$$D_i = f_2(\eta_{kl}, E_k, H_k) \tag{I.2}$$

$$B_i = f_3(\eta_{kl}, E_k, H_k) \tag{I.3}$$

Using these independent thermodynamic mechanical, electrical, and magnetic variables, it is possible to use the thermodynamic potential G = G(η_{ij}, E_i, H_i) for a three-dimensional piezoelectromagnetic solid [46, 48, 132-135]. As a result, the coupled constitutive relations for a linearly-piezoelectromagnetic solid [46, 48, 136] can be written as follows:

$$\sigma_{ij} = C_{ijkl}\eta_{kl} - e_{kij}E_k - h_{kij}H_k \tag{I.4}$$

$$D_i = e_{ikl}\eta_{kl} + \varepsilon_{ik}E_k + \alpha_{ik}H_k \tag{I.5}$$

$$B_i = h_{ikl}\eta_{kl} + \alpha_{ik}E_k + \mu_{ik}H_k \tag{I.6}$$

where the indices *i*, *j*, *k*, and *l* run from 1 to 3. It is clearly seen in equations from (I.4) to (I.6) that the material parameters of such a piezoelectromagnetic (composite) material are as follows: the elastic stiffness constants C_{ijkl}, piezoelectric constants e_{kij}, piezomagnetic coefficients h_{kij}, dielectric permittivity coefficients ε_{ik}, magnetic permeability coefficients μ_{ik}, and electromagnetic constants α_{ik}.

In equations from (I.4) to (I.6), the first independent thermodynamic variable such as the mechanical strain tensor η_{ij} can be defined by the following well-known strain-displacement relation:

$$\eta_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{I.7}$$

where the indices *i* and *j* run from 1 to 3. Expression (I.7) represents the well-known dependence of the strain tensor components η_{ij} on the corresponding partial first derivatives of the mechanical displacement components U_1, U_2, and U_3 with respect to the real space components x_1, x_2, and x_3.

Also, the second and third independent thermodynamic variables such as the electrical field E_i and the magnetic field H_i , respectively, in equations from (I.4) to (I.6) can be also defined by corresponding partial first derivatives. Using the electrical potential φ and the magnetic potential ψ in the quasi-static (irrotational field) approximation, the components of the electrical field E_i and the magnetic field H_i are determined as the following partial first derivatives with respect to the real space components x_1, x_2, and x_3, respectively:

$$E_i = -\frac{\partial \varphi}{\partial x_i} \tag{I.8}$$

$$H_i = -\frac{\partial \psi}{\partial x_i} \tag{I.9}$$

In the coupled constitutive relations defined by expressions from (I.4) to (I.6), all the material parameters such as C_{ijkl}, e_{kij}, h_{kij}, ε_{ik}, μ_{ik}, and α_{ik} can be also thermodynamically defined. With equations (I.5) and (I.6), the electromagnetic constants α_{ik} can be then written using the following thermodynamic relations:

$$\alpha_{ik} = \left(\frac{\partial D_i}{\partial H_k}\right)_{\eta, E=\text{const}} = \left(\frac{\partial B_i}{\partial E_k}\right)_{\eta, H=\text{const}} \tag{I.10}$$

Using equation (I.6), it is obvious that the magnetic permeability coefficients μ_{ik} can be thermodynamically defined as follows:

$$\mu_{ik} = \left(\frac{\partial B_i}{\partial H_k}\right)_{\eta, E=\text{const}} \tag{I.11}$$

Utilizing equation (I.5), the thermodynamic definition for the dielectric permittivity coefficients ε_{ik} reads:

$$\varepsilon_{ik} = \left(\frac{\partial D_i}{\partial E_k}\right)_{\eta, H=\text{const}} \tag{I.12}$$

With equations (I.4) and (I.6), it is also apparent that the thermodynamic forms of the piezomagnetic coefficients h_{kij} can be obtained as follows:

$$h_{ijk} = -\left(\frac{\partial \sigma_{ij}}{\partial H_k}\right)_{\eta, E=\text{const}} = h_{ikl} = \left(\frac{\partial B_i}{\partial \eta_{kl}}\right)_{E, H=\text{const}} \tag{I.13}$$

Exploiting equations (I.4) and (I.5), the thermodynamic description of the piezoelectric constants e_{kij} can be given by the following definitions:

$$e_{ijk} = -\left(\frac{\partial \sigma_{ij}}{\partial E_k}\right)_{\eta, H=\text{const}} = e_{ikl} = \left(\frac{\partial D_i}{\partial \eta_{kl}}\right)_{E, H=\text{const}} \tag{I.14}$$

Finally, expression (I.4) can be solidly used for the thermodynamic definition of the elastic stiffness constants C_{ijkl}. They can be naturally written as follows:

$$C_{ijkl} = \left(\frac{\partial \sigma_{ij}}{\partial \eta_{kl}}\right)_{E, H=\text{const}} \tag{I.15}$$

Thermodynamic definition (I.15) for the elastic stiffness constants C_{ijkl} states that they can be determined at constant electrical and magnetic fields. Symmetry arguments allow some simplifications of the quantity of the constants C_{ijkl} because the stress and strain tensors are symmetric: $\sigma_{ij} = \sigma_{ji}$ and $\eta_{ij} = \eta_{ji}$. Therefore, the stiffness tensor C_{ijkl} must also have a corresponding degree of symmetry which results in the following simplifications:

$$C_{ijkl} = C_{klij} = C_{jikl} = C_{klji} = C_{ijlk} = C_{lkij} = C_{jilk} = C_{lkji} \tag{I.16}$$

Using the Voigt notation, the (3×3×3×3) tensor form for the elastic stiffness constants C_{ijkl} defined by equalities (I.15) and (I.16) can be compactly written in a form of (6×6) matrix [15, 31, 32, 131, 137, 138]. The transformation procedure of a tensor form into a matrix is well-known. For this purpose, the following rules are used for the indices: 11 → 1, 22 → 2, 33 → 3, 23 → 4, 13 → 5, 12 → 6. So, the indices are changed as $ijkl \rightarrow PQ$ where the new indices P and Q run from 1 to 6. Consequently, one can get the following $C_{ijkl} \rightarrow C_{PQ}$. It is thought that it is unnecessary to give complete theory because the reader can find many excellent and classical books concerning the crystal symmetries and wave propagation in solids such as cited in Refs. [139-144].

The quantities of both the piezomagnetic coefficients h_{kij} and the piezoelectric constants e_{kij} must be also reduced in the similar manner. The symmetry arguments such as $\sigma_{ij} = \sigma_{ji}$ and $\eta_{ij} = \eta_{ji}$ in equations (I.13) and (I.14) can also demonstrate the corresponding degrees of symmetry for the tensors e_{kij} and h_{kij}. The symmetry influences can result in the following equalities:

$$e_{kij} = e_{ijk} = e_{kji} = e_{jik} \quad \text{(I.17)}$$

$$h_{kij} = h_{ijk} = h_{kji} = h_{jik} \quad \text{(I.18)}$$

With the Voigt notation, the (3×3×3) tensor forms for the piezoelectric constants e_{kij} and piezomagnetic coefficients h_{kij} can be rewritten as the asymmetric (6×3) or (3×6) matrices: $e_{kij} \rightarrow e_{kP}$ or $e_{ijk} \rightarrow e_{Pk}$, $h_{kij} \rightarrow h_{kP}$ or $h_{ijk} \rightarrow h_{Pk}$.

In the thermodynamic relations from (I.10) to (I.12), the electromagnetic constants α_{ik}, magnetic permeability coefficients μ_{ik}, and dielectric permittivity coefficients ε_{ik} stand for the following symmetric tensors of the second rank (matrices): $\alpha_{ik} = \alpha_{ki}$, $\mu_{ik} = \mu_{ki}$, $\varepsilon_{ik} = \varepsilon_{ki}$. Indeed, the components of the tensors ε_{ik}, μ_{ik}, and α_{ik} can be also written as (3×3) matrices [31, 32].

It is worth noting that all the tensors defined by thermodynamic relations from (I.10) to (I.15) can be transformed from an original coordinate system (usually, it is a crystallographic coordinate system) into a required one. As soon as coordinate system is changed, the number of independent material constants and their values must be also changed. The values of the new material constants are obtained using the values of the old ones. Exploiting the rules for tensor transformations [5-7], some new values of the material constants with the indexes i, j, k, and l can be obtained by application of the transformation matrices such as a_{im}, a_{jn}, a_{kp}, and a_{lq} to the original values of the material constants with the indexes m, n, p, and q. Therefore, the transformation formulae are written as follows:

$$C_{ijkl} = a_{im} a_{jn} a_{kp} a_{lq} C_{mnpq} \quad (I.19)$$

$$h_{ijk} = a_{im} a_{jn} a_{kp} h_{mnp} \quad (I.20)$$

$$e_{ijk} = a_{im} a_{jn} a_{kp} e_{mnp} \quad (I.21)$$

$$\alpha_{ij} = a_{im} a_{jn} \alpha_{mn} \quad (I.22)$$

$$\mu_{ij} = a_{im} a_{jn} \mu_{mn} \quad (I.23)$$

$$\varepsilon_{ij} = a_{im} a_{jn} \varepsilon_{mn} \quad (I.24)$$

As soon as these complicated transformations are completed, all the tensors of the material parameters in equations from (I.19) to (I.24) can be anew written in their corresponding matrix forms discussed above. It is thought that these matrix forms are convenient for the following theoretical descriptions.

I.2. *Equations of Motion*

It is well-known in physical acoustics that acoustic waves propagating in solids are extremely slow compared with electromagnetic waves propagating in the same materials. The speed of the electromagnetic waves is approximately five orders larger than that of the acoustic waves. However, the acoustic waves can be coupled with both the electrical potential φ and the magnetic potential ψ in the quasi-static (irrotational field) approximation. Therefore, the Maxwell four field equations [145] of the electromagnetic theory must be naturally used. Maxwell has creatively formulated the laws of electrostatics, magnetostatics, and electromagnetism. The Maxwell equations can be also applied to the piezoelectromagnetic solid. The electrostatic and magnetostatic equilibrium equations can be written using the differential forms of the following Maxwell equations:

$$\mathrm{div}\mathbf{D} = 0 \quad (I.25)$$

$$\mathrm{div}\mathbf{B} = 0 \quad (I.26)$$

Equation (I.25) also represents Gauss's law without free charge and currents.

Exploiting the Maxwell equations written above, the governing electrostatic and magnetostatic equilibrium equations respectively read:

$$\frac{\partial D_i}{\partial x_i} = 0 \qquad \text{(I.27)}$$

$$\frac{\partial B_i}{\partial x_i} = 0 \qquad \text{(I.28)}$$

These equations represent the partial first derivatives of the components of the electrical and magnetic displacements, i.e. D_i and B_i, with respect to the real space components x_i, where the index i runs from 1 to 3.

Besides, the governing mechanical equilibrium equation is also written as the following partial first derivative:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0 \qquad \text{(I.29)}$$

where the stress tensor σ_{ij} is expanded in equation (I.4).

Using equation (I.29), wave motions of a piezoelectromagnetic material in dependence on time t can be described by the equation of motion written in the following well-known form [2, 5, 6]:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 U_i}{\partial t^2} \qquad \text{(I.30)}$$

where ρ is the mass density of the piezoelectromagnetics. On the right-hand side in equation (I.30), the partial second derivatives of the mechanical displacement components U_i with respect to time t represent corresponding accelerations.

In addition to equation of motion (I.30), it is necessary to account the electrostatics and magnetostatics in the quasi-static approximation:

$$\frac{\partial D_i}{\partial x_j} = 0 \tag{I.31}$$

$$\frac{\partial B_i}{\partial x_j} = 0 \tag{I.32}$$

where D_i and B_i are defined by equations (I.5) and (I.6), respectively.

Using equations from (I.29) to (I.32), it is possible to write down the coupled equations of motion for a piezoelectromagnetics when the wave propagation can be coupled with both the electrical and magnetic potentials. It is natural to use the electrical (E_i) and magnetic (H_i) fields defined by equations (I.8) and (I.9) for equations from (I.4) to (I.6) in order to write the coupled equations of motion in an expanded form. Employing all these equations mentioned above, the coupled equations of motion, which constitute the wave propagation in a piezoelectromagnetics possessing the piezoelectric, piezomagnetic, and piezoelectromagnetic effects, are then composed as follows:

$$\rho \frac{\partial^2 U_i}{\partial t^2} = C_{ijkl} \frac{\partial^2 U_l}{\partial x_j \partial x_k} + e_{kij} \frac{\partial^2 \varphi}{\partial x_j \partial x_k} + h_{kij} \frac{\partial^2 \psi}{\partial x_j \partial x_k} \tag{I.33}$$

$$0 = e_{ijk} \frac{\partial^2 U_k}{\partial x_i \partial x_j} - \varepsilon_{ij} \frac{\partial^2 \varphi}{\partial x_i \partial x_j} - \alpha_{ij} \frac{\partial^2 \psi}{\partial x_i \partial x_j} \tag{I.34}$$

$$0 = h_{ijk} \frac{\partial^2 U_k}{\partial x_i \partial x_j} - \alpha_{ij} \frac{\partial^2 \varphi}{\partial x_i \partial x_j} - \mu_{ij} \frac{\partial^2 \psi}{\partial x_i \partial x_j} \tag{I.35}$$

where the indexes i, j, k, and l run from 1 to 3.

It is well-known that these homogeneous partial differential equations of the second order written above must have solutions in the plane wave forms [2, 5, 6]. Therefore, these solutions read:

$$U_I = U_I^0 \exp[\mathrm{j}(k_1 x_1 + k_2 x_2 + k_3 x_3 - \omega t)] \tag{I.36}$$

where the index I runs from 1 to 5. In equation (I.36) there is the following: $U_I = U_i$ for $I = i$, $U_4 = \varphi$, and $U_5 = \psi$.

Also, U_I^0, j = $(-1)^{1/2}$, and ω stand for the initial amplitudes, imaginary unity [146], and angular frequency, respectively. The initial amplitudes such as U_1^0, U_2^0, U_3^0, $U_4^0 = \varphi^0$, and $U_5^0 = \psi^0$ should be determined further, and the angular frequency is defined by $\omega = 2\pi\nu$ where ν is the linear frequency. In equation (I.36), the parameters such as k_1, k_2, and k_3 are the components of the wavevector **K** directed towards the wave propagation: $(k_1, k_2, k_3) = k(n_1, n_2, n_3)$ where the directional cosines denoted by n_1, n_2, and n_3 are introduced. For convenience, they can be defined as follows: $n_1 = 1$, $n_2 = 0$, and $n_3 \equiv n_3$. It is also noted that the wavenumber k in the direction of wave propagation can be naturally normalized by the wavelength λ as follows: $k\lambda = 2\pi$.

It is blatant that the utilization of solutions (I.36) and the directional cosines for corresponding substitutions into coupled equations from (I.33) to (I.35) can lead to the five homogeneous equations. These five homogeneous equations can be naturally combined in the following compact form [46, 48]:

$$\left(GL_{IJ} - \delta_{IJ}\rho V_{ph}\right)U_I^0 = 0 \tag{I.37}$$

where ρ is the mass density and the indices I and J run from 1 to 5. In the parentheses on the left-hand side in equation (I.37), GL_{IJ} stands for the components of the modified tensor in the well-known Green-Christoffel equation [46, 48], δ_{IJ} represents the well-known Kronecker delta-function such as $\delta_{IJ} = 1$ for $I = J$, $\delta_{IJ} = 0$ for $I \neq J$, and $\delta_{44} = \delta_{55} = 0$.

Also, the phase velocity denoted by V_{ph} in equation (I.37) symbolizes the relationship between the angular frequency ω and the wavenumber k in the direction of wave propagation:

$$V_{ph} = \omega / k \tag{I.38}$$

In equation (I.37), the suitable phase velocity can be revealed as soon as several procedures can be completed. First of all, it is necessary to find all the eigenvalues and the corresponding eigenvector for each eigenvalue. In this case, the eigenvalues represent all the suitable values of n_3 and the eigenvector can be expressed in the following common form:

$$\left(U_1^0, U_2^0, U_3^0, U_4^0, U_5^0\right) \tag{I.39}$$

It is worth noting that compact tensor form (I.37) of the coupled equations of motion is well-known and can be found in many research publications concerning the wave propagation problems in solids. It is also central to state that the modified Green-Christoffel tensor GL_{IJ} is symmetric, i.e. $GL_{IJ} = GL_{JI}$. For that reason, it has only 15 independent tensor components. It is thought that exploiting coupled equations from (I.33) to (I.35), it is practical for the reader to obtain the explicit forms for all the tensor components. However, this is not the purpose of this work because it studies the problems of the SH-wave propagation in the piezoelectromagnetic materials. Therefore, the following subsection demonstrates the simplifications for the case. Indeed, some GL-tensor components can become equal to zero when acoustic waves propagate in certain directions on certain cuts.

I.3. *SH-Wave Propagation in PEMs of Class 6 mm*

There are certain cuts and certain propagation directions in the transversely isotropic piezoelectromagnetic (PEM) materials [45, 48, 99, 100, 147-149] in which the pure SH-waves coupled with both the electrical and magnetic potentials can propagate. This is true for the SH-waves guided by the interface between a vacuum and the piezoelectromagnetics [45, 48, 147-149] and when the second piezoelectromagnetics is treated instead of a vacuum [99, 100]. However, the second case is significantly more complicated and therefore, still incompletely studied. This study corresponds to the SH-wave propagation along the perfectly bonded interface between two dissimilar PEMs. The problems of the different interfacial imperfections are not included in this research because they represent complicated theoretical investigations which must be separately studied.

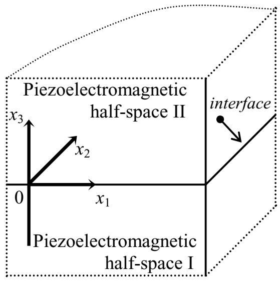

Figure I.1. The rectangular coordinate system for the layered system consisting of two dissimilar piezoelectromagnetic half-spaces solidly coupled at the common interface. For both the transversely isotropic materials of class 6 *mm*, the propagation direction is along the x_1-axis and perpendicular to the sixfold symmetry axis directed along the x_2-axis. The anti-plane polarized interfacial SH-waves can damp towards the positive values of the x_3-axis in half-space II and towards the negative values of the x_3-axis in half-space I.

Figure I.1 shows the configuration for the two-layer structure. The propagating SH-wave can be guided by the common interface, is directed along the x_1-axis, and must damp towards the depth of either solid. The anti-plane polarization of the interfacial SH-wave represents the mechanical displacements directed along the sixfold symmetry axis of either piezoelectromagnetics of class 6 *mm*. The studied propagation direction leads to the fact that the coupled equations of motion written in compact tensor form (I.37) can be decomposed. This decomposition allows one to separately treat the equations of motion for the in-plane polarized waves and those for the anti-plane polarized waves. Using equation (I.37), the SH-wave propagation can be then expressed by the following three homogeneous equations:

$$\begin{pmatrix} GL_{22}-\rho V_{ph}^2 & GL_{24} & GL_{25} \\ GL_{42} & GL_{44} & GL_{45} \\ GL_{52} & GL_{54} & GL_{55} \end{pmatrix} \begin{pmatrix} U^0 \\ \varphi^0 \\ \psi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{I.40}$$

where $U^0 = U_2^0$. In equations (I.40), the eigenvector has the following components:

$$\left(U^0, \varphi^0, \psi^0\right) \tag{I.41}$$

The eigenvalues can be found when the determinant of the coefficient matrix in equations (I.40) equals to zero. Therefore, it is possible to inscribe the following:

$$\begin{vmatrix} GL_{22}-\rho V_{ph}^2 & GL_{24} & GL_{25} \\ GL_{42} & GL_{44} & GL_{45} \\ GL_{52} & GL_{54} & GL_{55} \end{vmatrix} = 0 \tag{I.42}$$

where the GL-components are expressed as follows:

$$GL_{22} = C(1+n_3^2) \tag{I.43}$$

$$GL_{44} = -\varepsilon(1+n_3^2) \tag{I.44}$$

$$GL_{55} = -\mu(1+n_3^2) \tag{I.45}$$

$$GL_{24} = GL_{42} = e(1+n_3^2) \tag{I.46}$$

$$GL_{25} = GL_{52} = h(1+n_3^2) \tag{I.47}$$

$$GL_{45} = GL_{54} = -\alpha(1+n_3^2) \tag{I.48}$$

In expressions from (I.43) to (I.48), the directional cosine is defined by $n_3 = k_3/k$ and the independent material constants for the case are C, e, h, ε, μ, and α where $C = C_{44} = C_{66}$, $e = e_{16} = e_{34}$, $h = h_{16} = h_{34}$, $\varepsilon = \varepsilon_{11} = \varepsilon_{33}$, $\mu = \mu_{11} = \mu_{33}$, and $\alpha = \alpha_{11} = \alpha_{33}$ [48].

Expanding the GL-tensor components defined by equations from (I.43) to (I.48), it is obvious that determinant (I.42) of the coefficient matrix can be written as follows:

$$\begin{vmatrix} Cm-\rho V_{ph}^2 & em & hm \\ e & -\varepsilon & -\alpha \\ h & -\alpha & -\mu \end{vmatrix} \times m \times m = 0 \quad \text{(I.49)}$$

where $m = 1 + n_3^2$.

It is blatant that m = 0 in equation (I.49) can soundly satisfy the equality. Therefore, two the same factors such as m can determine four of six normalized eigenvalues n_3. They read:

$$n_3^{(1,2)} = n_3^{(3,4)} = \pm \mathrm{j} \quad \text{(I.50)}$$

Also, the determinant in equation (I.49) can reveal the rest two eigenvalues n_3. Expanding the determinant, the following secular equation can be obtained:

$$\left(1 + K_{em}^2\right)m - \left(V_{ph}/V_{t4}\right)^2 = 0 \quad \text{(I.51)}$$

In equation (I.51), the phase velocity V_{ph} is defined by expression (I.38). Also, V_{ph} and K_{em}^2 stand for the speed of the shear-horizontal bulk acoustic wave (SH-BAW) uncoupled with both the electrical and magnetic potentials and the coefficient of the magnetoelectromechanical coupling (CMEMC), respectively. They read:

$$V_{t4} = \sqrt{C/\rho} \quad \text{(I.52)}$$

$$K_{em}^2 = \frac{\mu e^2 + \varepsilon h^2 - 2\alpha e h}{C\left(\varepsilon\mu - \alpha^2\right)} \quad \text{(I.53)}$$

As a result, equation (I.51) can provide the rest two eigenvalues obtained in the following form:

$$n_3^{(5,6)} = \pm \mathrm{j}\sqrt{1-\left(V_{ph}/V_{tem}\right)^2} \qquad \text{(I.54)}$$

where the velocity V_{tem} is the speed of the SH-BAW coupled with both the electrical and magnetic potential. It is defined by the following formula:

$$V_{tem} = V_{t4}\left(1+K_{em}^2\right)^{1/2} \qquad \text{(I.55)}$$

So, the first problem such as the determination of the eigenvalues is resolved. Employing the obtained eigenvalues for equations (I.40), it is possible to obtain all the explicit forms of the corresponding eigenvectors. Equation (I.40) can be rewritten in the following form:

$$\begin{pmatrix} Cm-\rho V_{ph}^2 & em & hm \\ em & -\varepsilon m & -\alpha m \\ hm & -\alpha m & -\mu m \end{pmatrix}\begin{pmatrix} U^0 \\ \varphi^0 \\ \psi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \text{(I.56)}$$

where the eigenvector $\left(U^0, \varphi^0, \psi^0\right)$ must be found.

It is thought that it is natural to define the eigenvector component U^0 from the first equation in equations (I.56). As a result, U^0 can be expressed as the following dependence on both φ^0 and ψ^0:

$$U^0 = -\frac{em}{A}\varphi^0 - \frac{hm}{A}\psi^0 \qquad \text{(I.57)}$$

where

$$A = C\left[m-\left(V_{ph}/V_{t4}\right)^2\right] \qquad \text{(I.58)}$$

Utilizing definition (I.57) for the second and third equations in equations (I.56), one can obtain the following equations which demonstrate the coupling between the components φ^0 and ψ^0:

$$\left(\frac{me^2}{A}+\varepsilon\right)\varphi^0+\left(\frac{meh}{A}+\alpha\right)\psi^0=0 \tag{I.59}$$

$$\left(\frac{meh}{A}+\alpha\right)\varphi^0+\left(\frac{mh^2}{A}+\mu\right)\psi^0=0 \tag{I.60}$$

It is necessary to state that equations (I.57), (I.59), and (I.60) can reveal all the eigenvector components. For the two equal eigenvalues obtained from the following equation $m = 0$, equations (I.56) can be written in the following simplified form:

$$\begin{pmatrix} 0-\rho V_{ph}^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} U^0 \\ \varphi^0 \\ \psi^0 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{I.61}$$

It is obvious in equation (I.61) that the single possibility to have a wavevector of non-zero length for the non-zero eigenvalue is the situation when $U^0 = 0$ and there are uncertain non-zero values for both φ^0 and ψ^0. Indeed, $U^0 = 0$ for $m = 0$ agrees with expression (I.57). The certain values of both φ^0 and ψ^0 can be determined from equations (I.59) and (I.60). Also, the value of the phase velocity V_{ph} for $m = 0$ and $U^0 = 0$ in equation (I.61) is uncertain and can therefore have any non-zero value. It is thought that it is natural to couple these two uncertain eigenvectors for $m = 0$ with the third eigenvector corresponding to the eigenvalues defined by expression (I.54). The following useful expressions can be written for eigenvalues (I.54) coupled with the phase velocity V_{ph}:

$$m^{(5,6)}=\left(V_{ph}/V_{tem}\right)^2 \tag{I.62}$$

$$n_3^{(5,6)}=\pm \mathrm{j}\sqrt{1-m^{(5,6)}}=\pm \mathrm{j}b \tag{I.63}$$

$$A^{(5,6)} = -m^{(5,6)} C K_{em}^2 \tag{I.64}$$

Also, one can check that for $m \neq 0$, equalities (I.59) and (I.60) are satisfied when the eigenvector components φ^0 and ψ^0 are expressed as follows, using equation (I.59):

$$\varphi^0 = \frac{meh}{A} + \alpha \tag{I.65}$$

$$\psi^0 = -\frac{me^2}{A} - \varepsilon \tag{I.66}$$

Using the eigenvector components defined by expressions (I.57), (I.65), and (I.66), the first eigenvector for eigenvalues (I.63) can be formed.

There is however the second case to satisfy equalities (I.59) and (I.60). Using equation (I.60), it is blatant that the eigenvector components φ^0 and ψ^0 can be also defined as follows:

$$\varphi^0 = \frac{mh^2}{A} + \mu \tag{I.67}$$

$$\psi^0 = -\frac{meh}{A} - \alpha \tag{I.68}$$

Thus, the eigenvector components defined by expressions (I.57), (I.67), and (I.68) can form the second eigenvector for the same eigenvalues defined by expression (I.63).

It is necessary to state that to know these two sets of the eigenvector components is very important because they can lead to two different solutions for the phase velocity V_{ph}. This fact was first revealed in book [48] for the problems of the propagation of the shear-horizontal surface acoustic wave guide by the free surface of the transversely isotropic piezoelectromagnetic material. This fact can also complicate the investigations of the interfacial SH-wave propagation along the common interface of two hexagonal PEMs. Indeed, it is indispensable to distinguish two dissimilar PEMs in the theory. The superscripts "I" will be used below to distinguish the eigenvalues and eigenvectors for the first PEM half-space from those for the second PEM half-space marked by the superscripts "II".

For the first PEM half-space, three eigenvalues are purely imaginary and must have negative signs because the SH-wave must damp towards the depth of the PEM1 with $x_3 < 0$, see figure I.1. This will be demonstrated in the formulae for the complete displacements in this subsection below. Therefore, the PEM1 eigenvalues can be written as follows:

$$n_3^{\mathrm{I}(1)} = n_3^{\mathrm{I}(3)} = -\mathrm{j} \tag{I.69}$$

$$n_3^{\mathrm{I}(5)} = -\mathrm{j}\sqrt{1-\left(V_{ph}/V_{tem}^{\mathrm{I}}\right)^2} = -\mathrm{j}b^{\mathrm{I}} \tag{I.70}$$

Using the φ^0 and ψ^0 defined by equations (I.65) and (I.66), the corresponding eigenvectors for the eigenvalues (I.69) and (I.70) are respectively composed as follows:

$$\left(U^{0\mathrm{I}(1)},\varphi^{0\mathrm{I}(1)},\psi^{0\mathrm{I}(1)}\right)=\left(U^{0\mathrm{I}(3)},\varphi^{0\mathrm{I}(3)},\psi^{0\mathrm{I}(3)}\right)=\left(0,\alpha^{\mathrm{I}},-\varepsilon^{\mathrm{I}}\right) \tag{I.71}$$

$$\left(U^{0\mathrm{I}(5)},\varphi^{0\mathrm{I}(5)},\psi^{0\mathrm{I}(5)}\right)=\left(\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2},-\frac{e^{\mathrm{I}}h^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+\alpha^{\mathrm{I}},\frac{\left(e^{\mathrm{I}}\right)^2}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-\varepsilon^{\mathrm{I}}\right) \tag{I.72}$$

where the non-zero eigenvector components in expression (I.71) were also obtained by using equations (I.65) and (I.66) because the same equations were used to obtain eigenvector components (I.72). This results in the following equalities:

$$e^{\mathrm{I}}\varphi^{0\mathrm{I}(3)}+h^{\mathrm{I}}\psi^{0\mathrm{I}(3)}=e^{\mathrm{I}}\varphi^{0\mathrm{I}(5)}+h^{\mathrm{I}}\psi^{0\mathrm{I}(5)}=e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}} \tag{I.73}$$

It is worth noting that expressions (I.71) and (I.72) define the first set of the eigenvector components for the first PEM half-space.

Using the φ^0 and ψ^0 defined by equations (I.67) and (I.68), it is possible to form the second set of the eigenvector components for the same eigenvalues defined by expressions (I.69) and (I.70). The eigenvectors respectively read:

$$\left(U^{0\mathrm{I}(1)},\varphi^{0\mathrm{I}(1)},\psi^{0\mathrm{I}(1)}\right)=\left(U^{0\mathrm{I}(3)},\varphi^{0\mathrm{I}(3)},\psi^{0\mathrm{I}(3)}\right)=\left(0,\mu^{\mathrm{I}},-\alpha^{\mathrm{I}}\right) \tag{I.74}$$

$$\left(U^{0\mathrm{I}(5)},\varphi^{0\mathrm{I}(5)},\psi^{0\mathrm{I}(5)}\right)=\left(\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2},-\frac{\left(h^{\mathrm{I}}\right)^2}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+\mu^{\mathrm{I}},\frac{e^{\mathrm{I}}h^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-\alpha^{\mathrm{I}}\right) \tag{I.75}$$

One can check that the following useful equalities, which can significantly simplify the further analytics, also occur for these eigenvector components:

$$e^{\mathrm{I}}\varphi^{0\mathrm{I}(3)}+h^{\mathrm{I}}\psi^{0\mathrm{I}(3)}=e^{\mathrm{I}}\varphi^{0\mathrm{I}(5)}+h^{\mathrm{I}}\psi^{0\mathrm{I}(5)}=e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}} \tag{I.76}$$

It is necessary to state that all the eigenvector components for the first PEM half-space defined by expressions (I.71), (I.72), (I.74), and (I.75) do not depend on the phase velocity V_{ph}. This is also true for the second PEM half-space.

For the second PEM half-space shown in figure I.1, it is crucial to exploit the superscript “II” in order to distinguish it from the first PEM half-space. It is also central to state that for this PEM half-space occupying the space with $x_3 > 0$ in the figure, it is necessary to choose only the eigenvalues with positive signs. Such choice of the positive signs for the eigenvalues is caused by the fact that the interfacial SH-wave must also damp towards the depth of this PEM. Consequently, three eigenvalues read:

$$n_3^{\mathrm{II}(2)}=n_3^{\mathrm{II}(4)}=+\mathrm{j} \tag{I.77}$$

$$n_3^{\mathrm{II}(6)}=+\mathrm{j}\sqrt{1-\left(V_{ph}/V_{tem}^{\mathrm{II}}\right)^2}=+\mathrm{j}b^{\mathrm{II}} \tag{I.78}$$

Utilizing equations (I.57), (I.65), and (I.66) for this case, the corresponding eigenvectors and the useful equalities are respectively written as follows:

$$\left(U^{0\text{II}(2)},\varphi^{0\text{II}(2)},\psi^{0\text{II}(2)}\right)=\left(U^{0\text{II}(4)},\varphi^{0\text{II}(4)},\psi^{0\text{II}(4)}\right)=\left(0,\alpha^{\text{II}},-\varepsilon^{\text{II}}\right) \quad (\text{I}.79)$$

$$\left(U^{0\text{II}(6)},\varphi^{0\text{II}(6)},\psi^{0\text{II}(6)}\right)=\left(\frac{e^{\text{II}}\alpha^{\text{II}}-h^{\text{II}}\varepsilon^{\text{II}}}{C^{\text{II}}\left(K_{em}^{\text{II}}\right)^2},-\frac{e^{\text{II}}h^{\text{II}}}{C^{\text{II}}\left(K_{em}^{\text{II}}\right)^2}+\alpha^{\text{II}},\frac{\left(e^{\text{II}}\right)^2}{C^{\text{II}}\left(K_{em}^{\text{II}}\right)^2}-\varepsilon^{\text{II}}\right) \quad (\text{I}.80)$$

$$e^{\text{II}}\varphi^{0\text{II}(4)}+h^{\text{II}}\psi^{0\text{II}(4)}=e^{\text{II}}\varphi^{0\text{II}(6)}+h^{\text{II}}\psi^{0\text{II}(6)}=e^{\text{II}}\alpha^{\text{II}}-h^{\text{II}}\varepsilon^{\text{II}} \quad (\text{I}.81)$$

With equations (I.57), (I.67), and (I.68), they can be expressed as follows:

$$\left(U^{0\text{II}(2)},\varphi^{0\text{II}(2)},\psi^{0\text{II}(2)}\right)=\left(U^{0\text{II}(4)},\varphi^{0\text{II}(4)},\psi^{0\text{II}(4)}\right)=\left(0,\mu^{\text{II}},-\alpha^{\text{II}}\right) \quad (\text{I}.82)$$

$$\left(U^{0\text{II}(6)},\varphi^{0\text{II}(6)},\psi^{0\text{II}(6)}\right)=\left(\frac{e^{\text{II}}\mu^{\text{II}}-h^{\text{II}}\alpha^{\text{II}}}{C^{\text{II}}\left(K_{em}^{\text{II}}\right)^2},-\frac{\left(h^{\text{II}}\right)^2}{C^{\text{II}}\left(K_{em}^{\text{II}}\right)^2}+\mu^{\text{II}},\frac{e^{\text{II}}h^{\text{II}}}{C^{\text{II}}\left(K_{em}^{\text{II}}\right)^2}-\alpha^{\text{II}}\right) \quad (\text{I}.83)$$

$$e^{\text{II}}\varphi^{0\text{II}(4)}+h^{\text{II}}\psi^{0\text{II}(4)}=e^{\text{II}}\varphi^{0\text{II}(6)}+h^{\text{II}}\psi^{0\text{II}(6)}=e^{\text{II}}\mu^{\text{II}}-h^{\text{II}}\alpha^{\text{II}} \quad (\text{I}.84)$$

Finally, it is possible to state that the obtained eigenvalues and eigenvectors for both piezoelectromagnetics are used to determine the suitable phase velocities for all the interfacial SH-waves guided by the common interface. Various electrical and magnetic boundary conditions applied to the common interface can reveal the suitable SH-wave velocities. This is the problem of the following chapters. The following subsection of this chapter will review the possible electrical and magnetic boundary conditions. It is also needed to write down the complete mechanical displacement, complete electrical potential, and complete magnetic potential denoted by U^{Σ}, φ^{Σ}, and ψ^{Σ}, respectively, for both half-spaces shown in figure I.1.

Using the superscript "I" for the first PEM half-space, these complete parameters can be written in the following plane wave forms:

$$U^{\text{I}\Sigma}=F^{\text{I}(5)}U^{0\text{I}(5)}\exp\left[\text{j}k\left(x_1-\text{j}b^{\text{I}}x_3-V_{ph}t\right)\right] \quad (\text{I}.85)$$

$$\varphi^{\text{I}\Sigma}=\left(F^{\text{I}(1)}+F^{\text{I}(3)}\right)\varphi^{0\text{I}(3)}\exp\left[\text{j}k\left(x_1-\text{j}x_3-V_{ph}t\right)\right]+F^{\text{I}(5)}\varphi^{0\text{I}(5)}\exp\left[\text{j}k\left(x_1-\text{j}b^{\text{I}}x_3-V_{ph}t\right)\right] \quad (\text{I}.86)$$

$$\psi^{\text{I}\Sigma}=\left(F^{\text{I}(1)}+F^{\text{I}(3)}\right)\psi^{0\text{I}(3)}\exp\left[\text{j}k\left(x_1-\text{j}x_3-V_{ph}t\right)\right]+F^{\text{I}(5)}\psi^{0\text{I}(5)}\exp\left[\text{j}k\left(x_1-\text{j}b^{\text{I}}x_3-V_{ph}t\right)\right] \quad (\text{I}.87)$$

where $x_3 < 0$, $b^{\mathrm{I}} > 0$, and $n_1 = 1$ are accounted. In equation (I.85), it was also accounted that $U^{0\mathrm{I}(1)} = U^{0\mathrm{I}(3)} = 0$. Also, the $\varphi^{0\mathrm{I}(1)} = \varphi^{0\mathrm{I}(3)}$ and $\psi^{0\mathrm{I}(1)} = \psi^{0\mathrm{I}(3)}$ are used in equations (I.86) and (I.87), respectively. This significantly simplifies the complete parameters. The weight factors such as $F^{\mathrm{I}} = F^{\mathrm{I}(1)} + F^{\mathrm{I}(3)}$ and $F^{\mathrm{I}2} = F^{\mathrm{I}(5)}$ can be determined from equations in which suitable boundary conditions are exploited. It is also noted that equations from (I.85) to (I.87) are true for both the sets of the eigenvector components.

Employing the superscript "II" for the second PEM half-space, the complete parameters written below can be expressed in the same manner:

$$U^{\mathrm{II}\Sigma} = F^{\mathrm{II}(6)} U^{0\mathrm{II}(6)} \exp\left[\mathrm{j}k\left(x_1 + \mathrm{j}b^{\mathrm{II}} x_3 - V_{ph} t\right)\right] \quad (\mathrm{I}.88)$$

$$\varphi^{\mathrm{II}\Sigma} = \left(F^{\mathrm{II}(2)} + F^{\mathrm{II}(4)}\right)\varphi^{0\mathrm{II}(4)} \exp\left[\mathrm{j}k\left(x_1 + \mathrm{j}x_3 - V_{ph} t\right)\right] + F^{\mathrm{II}(6)} \varphi^{0\mathrm{II}(6)} \exp\left[\mathrm{j}k\left(x_1 + \mathrm{j}b^{\mathrm{II}} x_3 - V_{ph} t\right)\right] \quad (\mathrm{I}.89)$$

$$\psi^{\mathrm{II}\Sigma} = \left(F^{\mathrm{II}(2)} + F^{\mathrm{II}(4)}\right)\psi^{0\mathrm{II}(4)} \exp\left[\mathrm{j}k\left(x_1 + \mathrm{j}x_3 - V_{ph} t\right)\right] + F^{\mathrm{II}(6)} \psi^{0\mathrm{II}(6)} \exp\left[\mathrm{j}k\left(x_1 + \mathrm{j}b^{\mathrm{II}} x_3 - V_{ph} t\right)\right] \quad (\mathrm{I}.90)$$

where $x_3 > 0$, $b^{\mathrm{II}} > 0$, $n_1 = 1$, $U^{0\mathrm{II}(2)} = U^{0\mathrm{II}(4)} = 0$, $\varphi^{0\mathrm{II}(2)} = \varphi^{0\mathrm{II}(4)}$, and $\psi^{0\mathrm{II}(2)} = \psi^{0\mathrm{II}(4)}$ are also utilized. Using various electrical and magnetic boundary conditions, the explicit forms of the following weight factors $F^{\mathrm{II}} = F^{\mathrm{II}(2)} + F^{\mathrm{II}(4)}$ and $F^{\mathrm{II}2} = F^{\mathrm{II}(6)}$ must be also found for this case. Also, equations from (I.88) to (I.90) are true for both the sets of the eigenvector components.

I.4. *Mechanical, Electrical, and Magnetic Boundary Conditions at the Common Interface*

It is expected that the common interface between two dissimilar hexagonal (6 *mm*) piezoelectromagnetics can allow the propagation of the interfacial SH-waves. However, this is still unclear. In this theoretical work, some mechanical boundary conditions will be applied. Besides, the applied electrical and magnetic boundary conditions at the interface $x_3 = 0$ (see figure I.1) can be different. The following electrical boundary conditions can occur: the electrically closed interface ($\varphi = 0$), electrically open interface (D_3 = 0), and the continuity of both φ and D_3 at the common interface between two dissimilar PEM half-spaces, i.e. $\varphi^{\mathrm{I}} = \varphi^{\mathrm{II}}$ and $(D_3)^{\mathrm{I}} = (D_3)^{\mathrm{II}}$, where D_3 is the normal component of the electrical displacements. Also, the magnetic boundary conditions are as follows: the magnetically closed interface (B_3 = 0), magnetically open interface (ψ = 0), and the continuity of both ψ and B_3 at $x_3 = 0$, i.e. $\psi^{\mathrm{I}} = \psi^{\mathrm{II}}$ and $(B_3)^{\mathrm{I}} = (B_3)^{\mathrm{II}}$, where B_3 is the normal component of the magnetic flux. The realization of the mechanical, electrical, and magnetic boundary conditions is perfectly described in Ref. [40] by Al'shits, Darinskii, and Lothe.

First of all, it is basic to write down the mechanical boundary conditions. It is obvious that it is natural to require the equality of the mechanical displacements at the common interface $x_3 = 0$, see figure I.1. This condition can be written as the following equality:

$$F^{\mathrm{I}(1)}U^{0\mathrm{I}(1)} + F^{\mathrm{I}(3)}U^{0\mathrm{I}(3)} + F^{\mathrm{I}(5)}U^{0\mathrm{I}(5)} = F^{\mathrm{II}(2)}U^{0\mathrm{II}(2)} + F^{\mathrm{II}(4)}U^{0\mathrm{II}(4)} + F^{\mathrm{II}(6)}U^{0\mathrm{II}(6)} \quad \text{(I.91)}$$

Using the fact such as $U^{0\mathrm{I}(1)} = U^{0\mathrm{I}(3)} = 0$ and $U^{0\mathrm{II}(2)} = U^{0\mathrm{II}(4)} = 0$, the mechanical boundary condition in equation (I.91) reduces to the following:

$$F^{\mathrm{I}2}U^{0\mathrm{I}(5)} = F^{\mathrm{II}2}U^{0\mathrm{II}(6)} \quad \text{(I.92)}$$

where $F^{\mathrm{I}2} = F^{\mathrm{I}(5)}$ and $F^{\mathrm{II}2} = F^{\mathrm{II}(6)}$ were used.

The second mechanical boundary condition involves the normal component of the stress tensor σ_{32} at the interface $x_3 = 0$. It is also possible to require the continuity the stress tensor component σ_{32} at $x_3 = 0$. This condition can be demonstrated by the following equality:

$$\sigma_{32}^{\mathrm{I}} = \sigma_{32}^{\mathrm{II}} \quad \text{(I.93)}$$

where

$$\sigma_{32}^{\mathrm{I}} = F^{\mathrm{I}} k n_3^{\mathrm{I}(3)} \left[C^{\mathrm{I}} U^{0\mathrm{I}(3)} + e^{\mathrm{I}} \varphi^{0\mathrm{I}(3)} + h^{\mathrm{I}} \psi^{0\mathrm{I}(3)} \right] + F^{\mathrm{I}2} k n_3^{\mathrm{I}(5)} \left[C^{\mathrm{I}} U^{0\mathrm{I}(5)} + e^{\mathrm{I}} \varphi^{0\mathrm{I}(5)} + h^{\mathrm{I}} \psi^{0\mathrm{I}(5)} \right] \quad (\mathrm{I}.94)$$

$$\sigma_{32}^{\mathrm{II}} = F^{\mathrm{II}} k n_3^{\mathrm{II}(4)} \left[C^{\mathrm{II}} U^{0\mathrm{II}(4)} + e^{\mathrm{II}} \varphi^{0\mathrm{II}(4)} + h^{\mathrm{II}} \psi^{0\mathrm{II}(4)} \right] + F^{\mathrm{II}2} k n_3^{\mathrm{II}(6)} \left[C^{\mathrm{II}} U^{0\mathrm{II}(6)} + e^{\mathrm{II}} \varphi^{0\mathrm{II}(6)} + h^{\mathrm{II}} \psi^{0\mathrm{II}(6)} \right] \quad (\mathrm{I}.95)$$

In equation (I.94), $F^{\mathrm{I}} = F^{\mathrm{I}(1)} + F^{\mathrm{I}(3)}$ and $F^{\mathrm{I}2} = F^{\mathrm{I}(5)}$ were used. Also, $F^{\mathrm{II}} = F^{\mathrm{II}(2)} + F^{\mathrm{II}(4)}$ and $F^{\mathrm{II}2} = F^{\mathrm{II}(6)}$ were utilized in equation (I.95).

One of the electrical boundary conditions can represent the following requirement for the electrical potential at the common interface $x_3 = 0$:

$$\varphi^{\mathrm{I}} = \varphi^{\mathrm{II}} \quad (\mathrm{I}.96)$$

where

$$\varphi^{\mathrm{I}} = F^{\mathrm{I}} \varphi^{0\mathrm{I}(3)} + F^{\mathrm{I}2} \varphi^{0\mathrm{I}(5)} \quad (\mathrm{I}.97)$$

$$\varphi^{\mathrm{II}} = F^{\mathrm{II}} \varphi^{0\mathrm{II}(4)} + F^{\mathrm{II}2} \varphi^{0\mathrm{II}(6)} \quad (\mathrm{I}.98)$$

In equations (I.97) and (I.9), it was respectively accounted that $\varphi^{0\mathrm{I}(1)} = \varphi^{0\mathrm{I}(3)}$ and $\varphi^{0\mathrm{II}(2)} = \varphi^{0\mathrm{II}(4)}$. Besides, $\varphi^{\mathrm{I}} = 0$ and $\varphi^{\mathrm{II}} = 0$ can be used instead of condition (I.96).

The other electrical boundary condition at the common interface $x_3 = 0$ couples the normal components $(D_3)^{\mathrm{I}}$ and $(D_3)^{\mathrm{II}}$ of the electrical displacements for the first and second PEM half-spaces. The continuity requirement at the interface $x_3 = 0$ can be expressed as follows:

$$D_3^{\mathrm{I}} = D_3^{\mathrm{II}} \quad (\mathrm{I}.99)$$

where

$$D_3^{\mathrm{I}} = F^{\mathrm{I}} k n_3^{\mathrm{I}(3)} \left[e^{\mathrm{I}} U^{0\mathrm{I}(3)} - \varepsilon^{\mathrm{I}} \varphi^{0\mathrm{I}(3)} - \alpha^{\mathrm{I}} \psi^{0\mathrm{I}(3)} \right] + F^{\mathrm{I}2} k n_3^{\mathrm{I}(5)} \left[e^{\mathrm{I}} U^{0\mathrm{I}(5)} - \varepsilon^{\mathrm{I}} \varphi^{0\mathrm{I}(5)} - \alpha^{\mathrm{I}} \psi^{0\mathrm{I}(5)} \right] \quad (\mathrm{I}.100)$$

$$D_3^{\mathrm{II}} = F^{\mathrm{II}} k n_3^{\mathrm{II}(4)} \left[e^{\mathrm{II}} U^{0\mathrm{II}(4)} - \varepsilon^{\mathrm{II}} \varphi^{0\mathrm{II}(4)} - \alpha^{\mathrm{II}} \psi^{0\mathrm{II}(4)} \right] + F^{\mathrm{II}2} k n_3^{\mathrm{II}(6)} \left[e^{\mathrm{II}} U^{0\mathrm{II}(6)} - \varepsilon^{\mathrm{II}} \varphi^{0\mathrm{II}(6)} - \alpha^{\mathrm{II}} \psi^{0\mathrm{II}(6)} \right] \quad (\mathrm{I}.101)$$

It is also essential to state that $(D_3)^{\mathrm{I}} = 0$ and $(D_3)^{\mathrm{II}} = 0$ can be used as the possible electrical boundary conditions at $x_3 = 0$. This is the case of the electrically open interface.

Besides, it is fundamental to treat the magnetic boundary conditions at the common interface $x_3 = 0$. It is natural that the continuity requirement for the magnetic potential can be also used. As a result, it is possible to demonstrate this requirement as the following equality:

$$\psi^{\mathrm{I}} = \psi^{\mathrm{II}} \tag{I.102}$$

where

$$\psi^{\mathrm{I}} = F^{\mathrm{I}}\psi^{0\mathrm{I}(3)} + F^{\mathrm{I}2}\psi^{0\mathrm{I}(5)} \tag{I.103}$$

$$\psi^{\mathrm{II}} = F^{\mathrm{II}}\psi^{0\mathrm{II}(4)} + F^{\mathrm{II}2}\psi^{0\mathrm{II}(6)} \tag{I.104}$$

The following equalities $\psi^{0\mathrm{I}(1)} = \psi^{0\mathrm{I}(3)}$ and $\psi^{0\mathrm{II}(2)} = \psi^{0\mathrm{II}(4)}$ were taken into account in equations (I.103) and (I.104), respectively.

Finally, the following magnetic boundary condition is written for the normal component of the magnetic flux denoted by B_3. At $x_3 = 0$, the continuity condition for the magnetic flux component B_3 must be fulfilled, namely

$$B_3^{\mathrm{I}} = B_3^{\mathrm{II}} \tag{I.105}$$

where

$$B_3^{\mathrm{I}} = F^{\mathrm{I}} k n_3^{\mathrm{I}(3)}\left[h^{\mathrm{I}} U^{0\mathrm{I}(3)} - \alpha^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} - \mu^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] + F^{\mathrm{I}2} k n_3^{\mathrm{I}(5)}\left[h^{\mathrm{I}} U^{0\mathrm{I}(5)} - \alpha^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} - \mu^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] \tag{I.106}$$

$$B_3^{\mathrm{II}} = F^{\mathrm{II}} k n_3^{\mathrm{II}(4)}\left[h^{\mathrm{II}} U^{0\mathrm{II}(4)} - \alpha^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} - \mu^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] + F^{\mathrm{II}2} k n_3^{\mathrm{II}(6)}\left[h^{\mathrm{II}} U^{0\mathrm{II}(6)} - \alpha^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} - \mu^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] \tag{I.107}$$

Employing $(B_3)^{\mathrm{I}} = 0$ and $(B_3)^{\mathrm{II}} = 0$ in equations (I.106) and (I.107), these equations can also represent a set of the magnetic boundary conditions. This is the well-known case of the magnetically closed interface.

It is thought that it is important to briefly discuss the hexagonal piezoelectromagnetic (composite) materials which can be used to perfectly bond together. The following final subsection of this chapter provides the discussion. It is stated that in this work, the numerical calculations of the velocities of the interfacial SH-waves will be not carried out for a large number of such structures because this is not the main purpose. The purpose of this study is to obtain the explicit forms of the interfacial SH-wave velocities and the possible existence conditions. This is the basics which can be used in the further researches.

I.5. *Piezoelectromagnetic Composite Materials*

It is thought that the most popular hexagonal (6 *mm*) piezoelectromagnetic composite materials are $BaTiO_3$–$CoFe_2O_4$ and PZT–Terfenol-D. They are well-know already for the last two decades [150]. These two-phase composites possess both the piezoelectric and piezomagnetic phases. The piezoelectric phase of these composites consists of the well-known hexagonal piezoelectrics such as $BaTiO_3$ and PZT, respectively. The piezomagnetic phase of these composites is formed by the hexagonal piezomagnetics such as $CoFe_2O_4$ and Terfenol-D, respectively. Such composite materials can find broaden applications in ultrasonic imaging devices, sensors and actuators for system control, transducers, and many other emerging components. Also, various theories providing characteristics of such complex materials, as well as "smart" composites and structures composed of them can represent a large interest.

The average material properties of these two popular composites [150-154] are listed in table I.1. It can be assumed that there is approximately equal volume fraction of one phase (inclusions) into the other phase called matrix. This is the 3-0 or 0-3 connectivity for the two-phase materials. Besides, the 2-2 connectivity of the laminated composites is also very popular because such composites can exhibit very large values of the electromagnetic constant α. It is worth noting that the value of α is restricted by the following inequality [36, 59]: $\alpha^2 < \varepsilon\mu$. Using the table data, one can check that the value of $\varepsilon\mu$ for PZT–Terfenol-D is approximately one order smaller than that for the $BaTiO_3$–$CoFe_2O_4$ composite. Thus, it is obvious that it is practical to compare both values of α^2 and $\varepsilon\mu$ among different composites. The corresponding values of the electromagnetic constant α for the composites are not given in the table. This constant can have either positive or negative sign. This can depend on the preparation method and connectivity. It is also noted that the sign of α can depend on the direction of the magnetic field. It is also well-known that the values of α for composites can be several orders larger than those for some native magnetoelectric monocrystals. The piezoelectromagnetic monocrystals such as $LiCoPO_4$, $TbPO_4$, $TbMnO_3$, $TbMn_2O_5$, $BiFeO_3$, Cr_2O_3, and $BiMnO_3$, which possess simultaneously both the ferroelectric and ferromagnetic properties, are known already for the last decades. They demonstrate very small magnetoelectric coupling to be practical. However, very small values of α for the piezoelectromagnetic monocrystals are not critical in the case of investigation of wave propagation guided by the common interface of two dissimilar piezoelectromagnetic half-spaces. It is expected that to know wave parameters is very important because existence conditions for interfacial SH-waves can frequently require some similarity for the wave characteristics of two dissimilar PEM half-spaces. This statement can be verified in the following chapters.

Table I.1. The material constants for the hexagonal (6 *mm*) piezoelectromagnetic composite materials such as $BaTiO_3$–$CoFe_2O_4$ and PZT-5H–Terfenol-D.

Composite material	ρ [kg/m^3]	C, 10^{10} [N/m^2]	e [C/m^2]	h [T]	ε, 10^{-10} [F/m]	μ, 10^{-6} [N/A^2]
$BaTiO_3$–$CoFe_2O_4$	5730	4.40	5.80	275	56.4	81.0
PZT-5H–Terfenol-D	8500	1.45	8.50	83.8	75.0	2.61

The purpose of this work is to obtain the explicit forms for velocities of all the new interfacial SH-waves propagating along the common interface between two dissimilar piezoelectromagnetics and to discuss obtained existence conditions. Also, it is important to compare obtained results with the previous achievements. So, it is necessary to start theoretical investigations of the influence of different electrical and magnetic boundary conditions on the existence of the interfacial SH-waves. It is thought that it is natural to commence the analysis with the case of the electrically closed ($\varphi = 0$) and magnetically open ($\psi = 0$) interface. This is the main purpose of the following chapter.

CHAPTER II

The Case of $\varphi^{\mathrm{I}} = \varphi^{\mathrm{II}} = 0$ and $\psi^{\mathrm{I}} = \psi^{\mathrm{II}} = 0$ at the Interface

This chapter provides the theoretical investigations of the interfacial SH-waves guided by the common interface between two dissimilar PEM half-spaces shown in figure I.1. The interfacial waves propagate along the electrically closed and magnetically open interface. The mechanical boundary conditions include the continuity of both the mechanical displacement and the normal component of the stress tensor at the interface $x_3 = 0$: $U^{\mathrm{I}} = U^{\mathrm{II}}$ and $(\sigma_{32})^{\mathrm{I}} = (\sigma_{32})^{\mathrm{II}}$. They are defined by conditions (I.92) and (I.93) from the previous chapter, respectively. The electrically closed interface results in the following conditions: $\varphi^{\mathrm{I}} = 0$ and $\varphi^{\mathrm{II}} = 0$, where φ^{I} and φ^{II} are correspondingly defined by expressions (I.97) and (I.98). Also, equations (I.103) and (I.104) can provide the following two magnetic boundary conditions for the magnetically open interface: $\psi^{\mathrm{I}} = 0$ and $\psi^{\mathrm{II}} = 0$,

As a result, six homogeneous equations based on the mechanical, electrical, and magnetic boundary conditions are composed as follows:

$$F^{\mathrm{I}2}U^{0\mathrm{I}(5)} - F^{\mathrm{II}2}U^{0\mathrm{II}(6)} = 0 \tag{II.1}$$

$$\begin{aligned}&F^{\mathrm{I}}\left[e^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] + F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}U^{0\mathrm{I}(5)} + e^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right]\\&+F^{\mathrm{II}}\left[e^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] + F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}U^{0\mathrm{II}(6)} + e^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0\end{aligned} \tag{II.2}$$

$$F^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + F^{\mathrm{I}2}\varphi^{0\mathrm{I}(5)} = 0 \tag{II.3}$$

$$F^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + F^{\mathrm{II}2}\varphi^{0\mathrm{II}(6)} = 0 \tag{II.4}$$

$$F^{\mathrm{I}}\psi^{0\mathrm{I}(3)} + F^{\mathrm{I}2}\psi^{0\mathrm{I}(5)} = 0 \tag{II.5}$$

$$F^{\mathrm{II}}\psi^{0\mathrm{II}(4)} + F^{\mathrm{II}2}\psi^{0\mathrm{II}(6)} = 0 \tag{II.6}$$

Equations from (II.1) to (II.6) are served for the determination of the phase velocity V_{ph} of the interfacial SH-wave. Also, these equations are responsible for the determination of the explicit forms of six weight factors such as $F^{\mathrm{I}(1)}$, $F^{\mathrm{I}(3)}$, $F^{\mathrm{I}(5)}$, $F^{\mathrm{II}(2)}$, $F^{\mathrm{II}(4)}$, and $F^{\mathrm{II}(6)}$ because $F^{\mathrm{I}} = F^{\mathrm{I}(1)} + F^{\mathrm{I}(3)}$, $F^{\mathrm{I}2} = F^{\mathrm{I}(5)}$, $F^{\mathrm{II}} = F^{\mathrm{II}(2)} + F^{\mathrm{II}(4)}$, and $F^{\mathrm{II}2} = F^{\mathrm{II}(6)}$. So, it is necessary to find the explicit forms of these four weight factors used in equations from (II.1) to (II.6) instead of six ones.

Also, it is indispensable to state that the corresponding eigenvalues and eigenvector components must be substituted into these equations. The extra difficulty in these theoretical treatments is the fact that there are two different sets of the eigenvector components for either of two PEM half-spaces. As a result, it is apparent that the following three possible configurations must be theoretically treated: (i) the corresponding first sets of the eigenvector components are used for two dissimilar PEM half-spaces; (ii) the corresponding second sets of the eigenvector components are used for them; and (iii) the first set is used for the first PEM half-space and the second set is used for the second PEM half-space. The third case represents the combination of the sets of the eigenvector components. This is actually possible because one copes here with two dissimilar PEM half-spaces. Also, it is possible to mention the fourth case when the second set can be used for the first PEM half-space and the first set can be used for the second PEM half-space. However, it is flagrant that the third and fourth cases are the same because the first PEM half-space can be readily replaced by the second, or vice versa. Therefore, three possibilities are recorded in this chapter below as well as in the following chapters for the other boundary conditions.

II.1. ***The first sets of the eigenvector components***

Using the corresponding eigenvalues and the first sets of the eigenvector components for the first and second PEM half-spaces, namely equations from (I.69) to (I.73) for the first half-space and equations from (I.77) to (I.81) for the second half-space, equations from (II.1) to (II.6) composed in the common forms can be rewritten for this case as follows:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \qquad \text{(II.7)}$$

$$\begin{aligned}&F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]\\&+F^{\mathrm{II}}\left[e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]=0\end{aligned} \qquad \text{(II.8)}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\alpha^{\mathrm{I}}-F^{\mathrm{I}2}\left(\frac{\left(e^{\mathrm{I}}\right)^2h^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-e^{\mathrm{I}}\alpha^{\mathrm{I}}\right)=0 \qquad \text{(II.9)}$$

$$F^{\mathrm{II}}e^{\mathrm{II}}\alpha^{\mathrm{II}}-F^{\mathrm{II}2}\left(\frac{\left(e^{\mathrm{II}}\right)^2h^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}-e^{\mathrm{II}}\alpha^{\mathrm{II}}\right)=0 \qquad \text{(II.10)}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\varepsilon^{\mathrm{I}}+F^{\mathrm{I}2}\left(\frac{\left(e^{\mathrm{I}}\right)^2h^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)=0 \qquad \text{(II.11)}$$

$$-F^{\mathrm{II}}h^{\mathrm{II}}\varepsilon^{\mathrm{II}}+F^{\mathrm{II}2}\left(\frac{\left(e^{\mathrm{II}}\right)^2h^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right)=0 \qquad \text{(II.12)}$$

It is clearly seen in equations from (II.9) to (II.12) that the electrical and magnetic boundary conditions for the first and second PEM half-spaces such as $\varphi^{\mathrm{I}} = 0$, $\varphi^{\mathrm{II}} = 0$, $\psi^{\mathrm{I}} = 0$, and $\psi^{\mathrm{II}} = 0$ are written in corresponding modified forms. It is apparent that expression (II.7) can determine the coupling between the weight factors $F^{\mathrm{I}2}$ and $F^{\mathrm{II}2}$. Using the known factors $F^{\mathrm{I}2}$ and $F^{\mathrm{II}2}$ for equation (II.8), it is possible to reveal the relationship between the other two weight factor, F^{I} and F^{II}. The explicit forms of the weight factors will be demonstrated below. To determine the phase velocity of the interfacial SH-wave, it is obvious that it is necessary to successively subtract all the equations from (II.9) to (II.12) from equation (II.8). Consequently, in order to determine the phase velocity, it is necessary and enough to deal only with the following forms of the obtained two equations:

$$F^{\mathrm{I}2} = F^{\mathrm{II}2} \frac{e^{\mathrm{II}}\alpha^{\mathrm{II}} - h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}} \frac{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2} \tag{II.13}$$

$$\begin{aligned} &F^{\mathrm{I}2}\left[b^{\mathrm{I}}C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2} + \left(b^{\mathrm{I}} - 1\right)\left(e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)\right] \\ &+ F^{\mathrm{II}2}\left[b^{\mathrm{II}}C^{\mathrm{II}}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}} - h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2} + \left(b^{\mathrm{II}} - 1\right)\left(e^{\mathrm{II}}\alpha^{\mathrm{II}} - h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right)\right] = 0 \end{aligned} \tag{II.14}$$

Equation (II.13) or (II.14) can represent the relationship between the weight factors $F^{\mathrm{I}2}$ and $F^{\mathrm{II}2}$. Using equations (II.8), (II.11) and (II.12), the rest two weight factors such as F^{I} and F^{II} can be also written as some dependencies on $F^{\mathrm{II}2}$. Therefore, all the values of the weight factors can be determined because it is natural to choose $F^{\mathrm{II}2} = 1$. On the other hand, all the weight factors can be normalized by the factor of $\left(\left(F^{\mathrm{I}}\right)^2 + \left(F^{\mathrm{I}2}\right)^2 + \left(F^{\mathrm{II}}\right)^2 + \left(F^{\mathrm{II}2}\right)^2\right)^{-1/2}$ to get the following vector (F^{I}, $F^{\mathrm{I}2}$, F^{II}, $F^{\mathrm{II}2}$) of the unit length. Thus, F^{I} and F^{II} can have the following definite values:

$$F^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}} - h^{\mathrm{II}}\varepsilon^{\mathrm{II}}} = -F^{\mathrm{II}2}\left(\frac{\left(K_e^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2} - 1\right) - F^{\mathrm{II}2}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}b^{\mathrm{I}}\left[\frac{1}{\left(K_{em}^{\mathrm{II}}\right)^2} + \frac{\left(K_{em}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}\right] - F^{\mathrm{II}2}b^{\mathrm{II}}\left[\frac{1}{\left(K_{em}^{\mathrm{II}}\right)^2} + 1\right] \tag{II.15}$$

$$F^{\mathrm{II}} = -F^{\mathrm{II}2}\frac{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}\left(\frac{\left(K_e^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2} - 1\right) - F^{\mathrm{II}2}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}b^{\mathrm{I}}\left[\frac{1}{\left(K_{em}^{\mathrm{II}}\right)^2} + \frac{\left(K_{em}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}\right] - F^{\mathrm{II}2}b^{\mathrm{II}}\left[\frac{1}{\left(K_{em}^{\mathrm{II}}\right)^2} + 1\right] \tag{II.16}$$

Also, it is possible to account the relationship between $F^{\mathrm{I}2}$ and $F^{\mathrm{II}2}$ defined by expression (II.13). Accounting this relationship, equation (II.14) can readily reduce to the following simplified form:

$$C^{\mathrm{I}}\left[b^{\mathrm{I}} + \left(K_{em}^{\mathrm{I}}\right)^2\left(b^{\mathrm{I}} - 1\right)\right] + C^{\mathrm{II}}\left[b^{\mathrm{II}} + \left(K_{em}^{\mathrm{II}}\right)^2\left(b^{\mathrm{II}} - 1\right)\right] = 0 \tag{II.17}$$

In equation (II.17), the velocity V_{new1} of the first new interfacial SH-wave is defined by $X = \left(V_{new1}/V_{tem}^{\mathrm{I}}\right)^2$ and the functions b^{I} and b^{II} depend on the velocity V_{new1} as follows:

$$b^{\mathrm{I}} = \sqrt{1 - X} \tag{II.18}$$

$$b^{\mathrm{II}} = \sqrt{1 - X\left(V_{tem}^{\mathrm{I}} / V_{tem}^{\mathrm{II}}\right)^2} \tag{II.19}$$

It is understandable that $X < 1$ must occur because the eigenvalues defined by expressions (I.70) and (I.78) from the previous chapter should be always imaginary. It was discussed that the imaginary eigenvalues are required because the interfacial SH-wave must damp toward the depth of either piezoelectromagnetics (PEM). It is apparently seen in expressions (I.70) and (I.78) that these eigenvalues can be always imaginary when both conditions such as $V_{new1} < V_{tem}^{\mathrm{I}}$ and $V_{new1} < V_{tem}^{\mathrm{II}}$ are satisfied, where V_{tem}^{I} and V_{tem}^{II} stand for the SH-BAW velocities for the first and second piezoelectromagnetics denoted by PEM1 and PEM2, respectively. For simplicity, it is possible to choose $V_{tem}^{\mathrm{I}} < V_{tem}^{\mathrm{II}}$. It is blatant that the case of $V_{tem}^{\mathrm{I}} > V_{tem}^{\mathrm{II}}$ can be rearranged, i.e. PEM1 → PEM2 and PEM2 → PEM1, to have $V_{tem}^{\mathrm{I}} < V_{tem}^{\mathrm{II}}$ again. Also, the value of the velocity V_{new1} must unequal to zero because $X = 0$ due to $V_{new1} = 0$ is undesirable. Therefore, the following existence conditions for the velocity V_{new1} can exist:

$$0 < X < 1 \tag{II.20}$$

$$1 - \left(\frac{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2 + C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}{C^{\mathrm{II}}\left(1 + \left(K_{em}^{\mathrm{II}}\right)^2\right)}\right)^2 < \left(V_{tem}^{\mathrm{I}} / V_{tem}^{\mathrm{II}}\right)^2 < 1 \tag{II.21}$$

Besides, equation (II.17) for determination of the velocity V_{new1} of the first new interfacial SH-wave can be rewritten as follows:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}} b^{\mathrm{I}}\left[1 + \left(K_{em}^{\mathrm{I}}\right)^2\right] + b^{\mathrm{II}}\left[1 + \left(K_{em}^{\mathrm{II}}\right)^2\right] = \frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\left(K_{em}^{\mathrm{I}}\right)^2 + \left(K_{em}^{\mathrm{II}}\right)^2 \tag{II.22}$$

To have no square roots in equation (II.22), it is necessary to square both the left-hand and right-hand sides, to combine all the terms without the square roots on the right-hand side, and to square anew. As a result, the following quadratic equation for the determination of the explicit form for the velocity V_{new1} can be obtained:

$$PX^2 + QX + W = 0 \tag{II.23}$$

where

$$P = \left(p_1^2 + p_2^2\left(V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}\right)^2\right)^2 - 4p_1^2 p_2^2\left(V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}\right)^2 \tag{II.24}$$

$$Q = 2\left(p_3^2 - p_1^2 - p_2^2\right)\left[p_1^2 + p_2^2\left(V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}\right)^2\right] + 4p_1^2 p_2^2\left[1 + \left(V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}\right)^2\right] \tag{II.25}$$

$$W = \left(p_3^2 - p_1^2 - p_2^2\right)^2 - 4p_1^2 p_2^2 \tag{II.26}$$

$$p_1 = \frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\left[1 + \left(K_{em}^{\mathrm{I}}\right)^2\right],\ p_2 = 1 + \left(K_{em}^{\mathrm{II}}\right)^2,\ p_3 = \frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\left(K_{em}^{\mathrm{I}}\right)^2 + \left(K_{em}^{\mathrm{II}}\right)^2 \tag{II.27}$$

It is well-known that a quadratic equation can have two solutions. For this case, they can be introduced as follows:

$$X_{1,2} = \left(\frac{V_{new1}}{V_{tem}^{\mathrm{I}}}\right)^2 = \frac{-Q \pm \sqrt{Q^2 - 4PW}}{2P} \tag{II.28}$$

This does not mean that two solutions can exist because the parameters P, Q, and W are very complicated. They can result in complex values of X. However, the value of X should be real. Moreover, the value of X is restricted by inequality (II.20) and existence condition (II.21) must be accounted. When this existence condition is fulfilled, the solution with a positive sign before the square root in expression (II.28) can satisfy equation (II.22).

It is also possible to evaluate the possibility of propagation of this new interfacial SH-wave along the common interface between two transversely isotropic piezoelectromagnetic composite materials listed in table I.1 from the previous chapter. It is necessary to choose PZT–Terfenol-D and $BaTiO_3$–$CoFe_2O_4$ as the first and second PEM half-spaces, respectively, because the SH-BAW speed for PEM1 is slower than that for PEM2. Therefore, the following parameters must be calculated for the two-layer system: $C^{\mathrm{I}}/C^{\mathrm{II}}$ ~ 0.33, $V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}$ ~ 0.59, $\left(K_{em}^{\mathrm{I}}\right)^2$ ~ 0.82, $\left(K_{em}^{\mathrm{II}}\right)^2$ ~ 0.16, $\alpha^{\mathrm{I}} = 0.01\sqrt{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}}$, and $\alpha^{\mathrm{II}} = 0.01\sqrt{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}}$. Using existence condition (II.21), it was found that the new interfacial SH-wave cannot propagate in the configuration consisting of PZT–Terfenol-D and $BaTiO_3$–$CoFe_2O_4$. This is true because the value of $V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}$ is significantly smaller than unity for the structure. In order that the new interfacial SH-wave can exist in this case, the value of $C^{\mathrm{I}}/C^{\mathrm{II}}$ must be significantly larger, namely $C^{\mathrm{I}}/C^{\mathrm{II}}$ > 1 or even $C^{\mathrm{I}}/C^{\mathrm{II}}$ >> 1. If the values of $\left(K_{em}^{\mathrm{I}}\right)^2$ and $\left(K_{em}^{\mathrm{II}}\right)^2$ are also very small in addition to such small value of $V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}$, the large value of $C^{\mathrm{I}}/C^{\mathrm{II}}$ must be also increased to compensate these small values of the other parameters. In configurations with suitable large values of $C^{\mathrm{I}}/C^{\mathrm{II}}$, the new interfacial SH-wave can propagate even in the case of $V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}$ ~ 0.1 or less when existence condition (II.20) is satisfied. In such cases, the value of the velocity V_{new1} of the new interfacial SH-wave can be very close to the value of the SH-BAW velocity V_{tem}^{I}: $V_{new1}/V_{tem}^{\mathrm{I}}$ ~ 0.999 or even 0.9999. This can actually result in such a situation when the PEM1 eigenvalue, which depends on the velocity V_{new1}, will be several orders smaller than the corresponding PEM2 eigenvalue. This can mean that the wave penetration depth in PEM1 can be significantly deeper than that in PEM2.

It is also possible to discuss some particular cases. Consider the case when the first half-space represents a pure piezomagnetics (e = 0) and the second is purely piezoelectric (h = 0). However, it is thought that it is possible to account the electromagnetic constant α, i.e. $\alpha \neq 0$ at the common interface between two media, because the piezoelectric phase is in a contact with the piezomagnetic phase. Also, it is assumed that the corresponding SH-BAW speed in the piezomagnetics is slower than that in the piezoelectrics. For this configuration, equation (II.22) for determination of the propagation velocity of the interfacial SH-wave reduces to the following equation:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}} b^{\mathrm{I}}\left[1+\left(K_{m\alpha}^{\mathrm{I}}\right)^2\right]+b^{\mathrm{II}}\left[1+\left(K_{e\alpha}^{\mathrm{II}}\right)^2\right]=\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\left(K_{m\alpha}^{\mathrm{I}}\right)^2+\left(K_{e\alpha}^{\mathrm{II}}\right)^2 \qquad \text{(II.29)}$$

where the corresponding non-dimensional coefficients denoted by $\left(K_{m\alpha}^{\mathrm{I}}\right)^2$ and $\left(K_{e\alpha}^{\mathrm{II}}\right)^2$ are expressed as follows:

$$\left(K_{m\alpha}^{\mathrm{I}}\right)^2=\frac{\varepsilon^{\mathrm{I}} h^{\mathrm{I}2}}{C^{\mathrm{I}}\left(\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\alpha^{\mathrm{I}2}\right)},\ \left(K_{e\alpha}^{\mathrm{II}}\right)^2=\frac{\mu^{\mathrm{II}} e^{\mathrm{II}2}}{C^{\mathrm{II}}\left(\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\alpha^{\mathrm{II}2}\right)} \qquad \text{(II.30)}$$

It is clearly seen in expressions (II.30) that these coefficients will be slightly larger for the case of non-zero values of the electromagnetic constants α^{I} and α^{II}. In equation (II.29), the following functions b^{I} and b^{II} depend on the corresponding SH-BAW velocities written below:

$$V_{tm\alpha}^{\mathrm{I}}=V_{t4}^{\mathrm{I}}\left(1+\left(K_{m\alpha}^{\mathrm{I}}\right)^2\right)^{1/2},\ V_{te\alpha}^{\mathrm{II}}=V_{t4}^{\mathrm{II}}\left(1+\left(K_{e\alpha}^{\mathrm{II}}\right)^2\right)^{1/2} \qquad \text{(II.31)}$$

where the SH-BAW velocities V_{t4}^{I} and V_{t4}^{II} uncoupled with both the electrical and magnetic potentials can be defined by expression (I.52) from the previous chapter when the corresponding superscripts "I" and "II" are used. It is apparent that the values of the velocities defined by expressions (II.31) will be also slightly higher due to the slightly larger values of the coefficients defined by expressions (II.30).

Consider the reverse case when the corresponding SH-BAW speed in the piezoelectrics is slower than that in the piezomagnetics. For this configuration, equation (II.22) reduces to the following equation:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}} b^{\mathrm{I}}\left[1+\left(K_{e\alpha}^{\mathrm{I}}\right)^2\right]+b^{\mathrm{II}}\left[1+\left(K_{m\alpha}^{\mathrm{II}}\right)^2\right]=\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\left(K_{e\alpha}^{\mathrm{I}}\right)^2+\left(K_{m\alpha}^{\mathrm{II}}\right)^2 \qquad \text{(II.32)}$$

where the corresponding coefficients and the corresponding SH-BAW velocities are defined by

$$\left(K_{e\alpha}^{\mathrm{I}}\right)^2 = \frac{\mu^{\mathrm{I}} e^{\mathrm{I}^2}}{C^{\mathrm{I}}\left(\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}} - \alpha^{\mathrm{I}^2}\right)},\ \left(K_{m\alpha}^{\mathrm{II}}\right)^2 = \frac{\varepsilon^{\mathrm{II}} h^{\mathrm{II}^2}}{C^{\mathrm{II}}\left(\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}} - \alpha^{\mathrm{II}^2}\right)} \qquad \text{(II.33)}$$

$$V_{te\alpha}^{\mathrm{I}} = V_{t4}^{\mathrm{I}}\left(1 + \left(K_{e\alpha}^{\mathrm{I}}\right)^2\right)^{1/2},\ V_{tm\alpha}^{\mathrm{II}} = V_{t4}^{\mathrm{II}}\left(1 + \left(K_{m\alpha}^{\mathrm{II}}\right)^2\right)^{1/2} \qquad \text{(II.34)}$$

In this case, the parameters given in expressions (II.33) and (II.34) will also have slightly larger values for non-zero values of the electromagnetic constants α^{I} and α^{II}.

It is worth noticing that when $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$ occur, equations (II.29) and (II.32) can reduce to the well-known equation for the case of the grounded interface obtained by Huang, Li, and Lee [100], see equation (20) in Ref. [100]. Huang, Li, and Lee [100] have studied the interfacial SH-wave propagation in a two-phase piezoelectric/piezomagnetic structure with an imperfect interface and introduced formula (20) for the case of the perfect bonding at the interface. It is thought that the interfacial SH-wave described by formula (20) in Ref. [100] can be called the interfacial Huang-Li-Lee wave or HLL-wave because this wave characteristic can be very important for the problems of wave propagation in laminated piezoelectric/piezomagnetic composite materials. Thus, it is necessary to distinguish such composites from the others and the interfacial HLL-wave can serve for this purpose. It is also noted that Ref. [100] has mentioned the results obtained by Soh and Liu [99], see formula (12) in Ref. [99]. However, Soh and Liu [99] have studied the case of interfacial SH-wave propagation along the non-metalized interface and did not receive the formula for the case of grounded interface introduced by formula (20) in Ref. [100]. As a result, the interfacial SH-wave propagating along the non-metalized interface between the hexagonal piezoelectrics and the hexagonal piezomagnetics can be called the interfacial Soh-Liu wave or SL-wave.

Consider the case of two dissimilar hexagonal (6 *mm*) piezoelectrics with the grounded interface. The media are perfectly bonded at the common interface. In this configuration, the piezomagnetic and magnetoelectric effects are absent, and therefore, the well-known interfacial Maerfeld-Tournois wave [98] can propagate.

Indeed, equation (II.22) reduces to the following equality for determination of the propagation speed of the interfacial MT-wave:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}} b^{\mathrm{I}}\left[1+\left(K_e^{\mathrm{I}}\right)^2\right]+b^{\mathrm{II}}\left[1+\left(K_e^{\mathrm{II}}\right)^2\right]=\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\left(K_e^{\mathrm{I}}\right)^2+\left(K_e^{\mathrm{II}}\right)^2 \quad \text{(II.35)}$$

where

$$K_e^{\mathrm{I}2}=\frac{e^{\mathrm{I}2}}{\varepsilon^{\mathrm{I}}C^{\mathrm{I}}},\ K_e^{\mathrm{II}2}=\frac{e^{\mathrm{II}2}}{\varepsilon^{\mathrm{II}}C^{\mathrm{II}}} \quad \text{(II.36)}$$

$$V_{te}^{\mathrm{I}}=V_{t4}^{\mathrm{I}}\left(1+K_e^{\mathrm{I}2}\right)^{1/2},\ V_{te}^{\mathrm{II}}=V_{t4}^{\mathrm{II}}\left(1+K_e^{\mathrm{II}2}\right)^{1/2} \quad \text{(II.37)}$$

In definitions (II.36), either of two parameters is called the coefficient of the electromechanical coupling (CEMC). The following functions b^{I} and b^{II} in equation (II.35) depend on the corresponding speeds of the SH-BAWs coupled with the electrical potential φ and are defined by expressions (II.37). It is also noted that formula (II.35) corresponds to formula (15) with existence condition (16) obtained by Maerfeld and Tournois in Ref. [98]. However, formula (15) in Ref. [98] was incorrectly written and it is vital to use the left-hand side of formula (9) instead of that in formula (15) to correct it.

Consider the case of two dissimilar transversely isotropic piezomagnetic materials of class 6 *mm*. They are also perfectly bonded at their common interface. In this structure, the piezoelectric and magnetoelectric effects are absent. For this case, equation (II.22) then reduces to the following formula for determination of the interfacial MT-wave speed:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}} b^{\mathrm{I}}\left[1+\left(K_m^{\mathrm{I}}\right)^2\right]+b^{\mathrm{II}}\left[1+\left(K_m^{\mathrm{II}}\right)^2\right]=\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\left(K_m^{\mathrm{I}}\right)^2+\left(K_m^{\mathrm{II}}\right)^2 \quad \text{(II.38)}$$

In expression (II.38), the corresponding parameters are defined as follows:

$$K_m^{\mathrm{I}\,2} = \frac{h^{\mathrm{I}^2}}{\mu^{\mathrm{I}} C^{\mathrm{I}}},\; K_m^{\mathrm{II}\,2} = \frac{h^{\mathrm{II}^2}}{\mu^{\mathrm{II}} C^{\mathrm{II}}} \tag{II.39}$$

$$V_{tm}^{\mathrm{I}} = V_{t4}^{\mathrm{I}}\left(1 + K_m^{\mathrm{I}\,2}\right)^{1/2},\; V_{tm}^{\mathrm{II}} = V_{t4}^{\mathrm{II}}\left(1 + K_m^{\mathrm{II}\,2}\right)^{1/2} \tag{II.40}$$

The parameters in expressions (II.39) are called the coefficients of the magnetomechanical coupling (CMMC) which characterize pure piezomagnetics. The parameters in expressions (II.40) represent the corresponding speeds of the SH-BAWs coupled with the magnetic potential ψ. Also, the SH-BAW velocities V_{t4}^{I} and V_{t4}^{II} uncoupled with the magnetic potential can be derived using expression (I.52) from the previous chapter.

Ref. [98] has discussed that formula (II.35) can reduce to the well-known formula for determination of the speed of the slower surface BG-wave guided by the metalized surface of pure piezoelectrics contacting with a vacuum. Indeed, the interfacial SH-wave can propagate with the BG-wave speed when the wave is guided by the common interface between two identical piezoelectrics. These piezoelectrics must be perfectly bonded at the interface and their corresponding symmetry axes are in opposite directions. The electrically closed (φ = 0) surface of the pure piezoelectrics can reveal the following formula for the speed of the slower surface BG-wave [13, 14, 45, 46, 48, 98, 100]:

$$V_{BGEC} = V_{te}\left[1 - \left(\frac{K_e^2}{1 + K_e^2}\right)^2\right]^{1/2} \tag{II.41}$$

The magnetically open (ψ = 0) surface of the pure piezomagnetics can also support the propagation of the slower surface BG-wave [46, 48]. Also, it is obvious that this BG-wave can propagate along the interface of two similar piezomagnetics when they are perfectly bonded and their corresponding symmetry axes are oppositely directed. Consequently, formula (II.38) can reduce to the following formula for determination of the slower surface BG-wave speed [46, 48]:

$$V_{BGMO} = V_{tm}\left[1-\left(\frac{K_m^2}{1+K_m^2}\right)^2\right]^{1/2} \tag{II.42}$$

It is possible to state that this subsection has acquainted the reader with the new result. The velocity V_{new1} of the new interfacial SH-wave was obtained and the obtained result was compared with the previous findings. However, the problem of the theoretical investigations of the SH-wave propagation is more complicated in the two-phase materials. Indeed, the second possibility exists which must be also treated. This possibility relates to the second set of the eigenvector components. Therefore, the following subsection describes the second possibility for the wave propagation in the transversely isotropic piezoelectromagnetics.

II.2. *The second sets of the eigenvector components*

For the first PEM half-space, the second sets of the eigenvector components are given by expressions (I.74) and (I.75) from the first chapter. They correspond to the eigenvalues defined by expressions (I.69) and (I.70). Also, the coupling between these two eigenvectors is demonstrated by equality (I.76). For the second PEM half-space, the explicit forms of the eigenvalues are determined in equations (I.77) and (I.78) and the second eigenvectors are defined by expressions (I.82) and (I.83). Also, useful equality (I.84) demonstrates that these two eigenvectors are not independent. Using all the definitions for the eigenvalues and the second eigenvectors mentioned above, the six homogeneous equations composed in expressions from (II.1) to (II.6) can be rewritten in the following forms:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \tag{II.43}$$

$$F^{\mathrm{I}}\left[e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]+F^{\mathrm{I2}}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]$$
$$+F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]+F^{\mathrm{II2}}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]=0 \quad \text{(II.44)}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\mu^{\mathrm{I}}-F^{\mathrm{I2}}\left(\frac{e^{\mathrm{I}}\left(h^{\mathrm{I}}\right)^2}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-e^{\mathrm{I}}\mu^{\mathrm{I}}\right)=0 \quad \text{(II.45)}$$

$$F^{\mathrm{II}}e^{\mathrm{II}}\mu^{\mathrm{II}}-F^{\mathrm{II2}}\left(\frac{e^{\mathrm{II}}\left(h^{\mathrm{II}}\right)^2}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}-e^{\mathrm{II}}\mu^{\mathrm{II}}\right)=0 \quad \text{(II.46)}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\alpha^{\mathrm{I}}+F^{\mathrm{I2}}\left(\frac{e^{\mathrm{I}}\left(h^{\mathrm{I}}\right)^2}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right)=0 \quad \text{(II.47)}$$

$$-F^{\mathrm{II}}h^{\mathrm{II}}\alpha^{\mathrm{II}}+F^{\mathrm{II2}}\left(\frac{e^{\mathrm{II}}\left(h^{\mathrm{II}}\right)^2}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right)=0 \quad \text{(II.48)}$$

It is blatant that these sex homogeneous equations written above also lead to the result obtained in the previous subsection. Indeed, the procedure for the determination of the phase velocity is the same. It is necessary and enough to cope with equations (II.43) and (II.44), where the second equation must be modified by a successive subtraction of all equations from (II.45) to (II.48). As a result, the velocity V_{new1} of the first new interracial SH-wave can be also obtained. Accounting existence condition (II.21), the value of the velocity V_{new1} can be also calculated with formula (II.22) or (II.28). It is also possible to find explicit forms for the weight factors F^{I}, F^{I2}, F^{II}, and F^{II2}. It is natural to express the first tree weight factors as some dependencies on the fourth. It is obvious that expression (II.43) can reveal the relationship between F^{I2} and F^{II2}. Using equations from (II.43) to (II.46), it is possible to obtain the explicit forms of the weight factors F^{I} and F^{II} as dependencies on both F^{II2} and V_{new1}. So, they can be written in the following non-dimensional forms:

$$F^{\mathrm{I2}}=F^{\mathrm{II2}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}\frac{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2} \quad \text{(II.49)}$$

$$F^{\mathrm{I}}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}=-F^{\mathrm{II2}}\left(\frac{\left(K_m^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}-1\right)-F^{\mathrm{II2}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}b^{\mathrm{I}}\left[\frac{1}{\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{\left(K_{em}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}\right]-F^{\mathrm{II2}}b^{\mathrm{II}}\left[\frac{1}{\left(K_{em}^{\mathrm{II}}\right)^2}+1\right]$$

(II.50)

$$F^{\mathrm{II}}=-F^{\mathrm{II2}}\frac{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}\left(\frac{\left(K_m^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}-1\right)-F^{\mathrm{II2}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}b^{\mathrm{I}}\left[\frac{1}{\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{\left(K_{em}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}\right]-F^{\mathrm{II2}}b^{\mathrm{II}}\left[\frac{1}{\left(K_{em}^{\mathrm{II}}\right)^2}+1\right] \quad \text{(II.51)}$$

The weight factors written above can be normalized by the factor of $\left(\left(F^{\mathrm{I}}\right)^2+\left(F^{\mathrm{I2}}\right)^2+\left(F^{\mathrm{II}}\right)^2+\left(F^{\mathrm{II2}}\right)^2\right)^{-1/2}$ to get the following vector (F^{I}, F^{I2}, F^{II}, F^{II2}) of the unit length.

There is also the third case when the first and second sets of the eigenvector components are mixed. The purpose of the following subsection is to demonstrate for the reader that this third case can agree with the first and second cases treated in this and the previous subsections.

II.3. *The combination of both the sets of the eigenvector components*

Consider the third possibility in the problem of the SH-wave propagation guided by the electrically closed ($\varphi = 0$) and magnetically open ($\psi = 0$) interface. This is the case when the first sets of the eigenvector components are used for the first PEM half-space and the second sets are chosen for the second PEM half-space. So, it is indispensable to utilize PEM1 eigenvectors (I.71) and (I.72) for PEM1 eigenvalues (I.69) and (I.70) and PEM2 eigenvectors (I.82) and (I.83) for PEM2 eigenvalues (I.77) and (I.78). In this case, the six homogeneous equations read as follows:

$$F^{\mathrm{I2}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II2}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad \text{(II.52)}$$

$$F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]$$
$$+F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]=0 \quad \text{(II.53)}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\alpha^{\mathrm{I}}-F^{\mathrm{I}2}\left(\frac{\left(e^{\mathrm{I}}\right)^2h^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-e^{\mathrm{I}}\alpha^{\mathrm{I}}\right)=0 \quad \text{(II.54)}$$

$$F^{\mathrm{II}}e^{\mathrm{II}}\mu^{\mathrm{II}}-F^{\mathrm{II}2}\left(\frac{e^{\mathrm{II}}\left(h^{\mathrm{II}}\right)^2}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}-e^{\mathrm{II}}\mu^{\mathrm{II}}\right)=0 \quad \text{(II.55)}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\varepsilon^{\mathrm{I}}+F^{\mathrm{I}2}\left(\frac{\left(e^{\mathrm{I}}\right)^2h^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)=0 \quad \text{(II.56)}$$

$$-F^{\mathrm{II}}h^{\mathrm{II}}\alpha^{\mathrm{II}}+F^{\mathrm{II}2}\left(\frac{e^{\mathrm{II}}\left(h^{\mathrm{II}}\right)^2}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right)=0 \quad \text{(II.57)}$$

Exploiting the same procedures described in the previous subsections, one can find that the phase velocity representing the velocity V_{new1} of the first new interracial SH-wave can be also calculated with formula (II.22) or (II.28), see also existence condition (II.21). Also, it is blatant that equation (II/52) can reveal the relationship between $F^{\mathrm{I}2}$ and $F^{\mathrm{II}2}$. Using this relationship and equations (II.55) and (II.56), it is possible to find the other two dependencies of F^{I} and F^{II} on $F^{\mathrm{II}2}$. It was also mentioned in the previous subsections that all the weight factors can be normalized by the factor of $\left(\left(F^{\mathrm{I}}\right)^2+\left(F^{\mathrm{I}2}\right)^2+\left(F^{\mathrm{II}}\right)^2+\left(F^{\mathrm{II}2}\right)^2\right)^{-1/2}$ to get the following vector (F^{I}, $F^{\mathrm{I}2}$, F^{II}, $F^{\mathrm{II}2}$) of the unit length. As a result, all the weight factors can have the following definite values different from those obtained in the previous two subsections:

$$F^{\mathrm{I}2}=F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2} \quad \text{(II.58)}$$

$$F^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}=-F^{\mathrm{II}2}\left(\frac{\left(K_{m}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}-1\right)-F^{\mathrm{II}2}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}b^{\mathrm{I}}\left[\frac{1}{\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{\left(K_{em}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}\right]-F^{\mathrm{II}2}b^{\mathrm{II}}\left[\frac{1}{\left(K_{em}^{\mathrm{II}}\right)^2}+1\right] \quad \text{(II.59)}$$

$$F^{\mathrm{II}} = -F^{\mathrm{II}2}\frac{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}\left(\frac{\left(K_e^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}-1\right)-F^{\mathrm{II}2}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}b^{\mathrm{I}}\left[\frac{1}{\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{\left(K_{em}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}\right] - F^{\mathrm{II}2}b^{\mathrm{II}}\left[\frac{1}{\left(K_{em}^{\mathrm{II}}\right)^2}+1\right] \quad \text{(II.60)}$$

It is possible to treat the other possible electrical and magnetic boundary conditions at the interface $x_3 = 0$. This is the main purpose of the following chapters. The following chapter studies the case of $D_3^{\mathrm{I}} = D_3^{\mathrm{II}} = 0$ and $B_3^{\mathrm{I}} = B_3^{\mathrm{II}} = 0$ at the common interface.

CHAPTER III

The Case of $D_3^{\mathrm{I}} = D_3^{\mathrm{II}} = 0$ and $B_3^{\mathrm{I}} = B_3^{\mathrm{II}} = 0$ at the Interface

Concerning the problem of interfacial SH-wave propagation along the electrically open ($D_3^{\mathrm{I}} = 0$ and $D_3^{\mathrm{II}} = 0$) and magnetically closed ($B_3^{\mathrm{I}} = 0$ and $B_3^{\mathrm{II}} = 0$) interface between two dissimilar PEM half-spaces, the mechanical boundary conditions such as $U^{\mathrm{I}} = U^{\mathrm{II}}$ and $(\sigma_{32})^{\mathrm{I}} = (\sigma_{32})^{\mathrm{II}}$ at the common interface $x_3 = 0$ can be borrowed from the previous chapter, see equations (II.1) and (II.2). They are written below in equations (III.1) and (III.2). The electrical displacements D_3^{I} and D_3^{II} are defined by expressions (I.100) and (I.101) and the magnetic flux components such as B_3^{I} and B_3^{II} are defined by expressions (I.106) and (I.107) from Chapter I, respectively. Therefore, the corresponding six homogeneous equations can be written for the case as follows:

$$F^{\mathrm{I}2}U^{0\mathrm{I}(5)} - F^{\mathrm{II}2}U^{0\mathrm{II}(6)} = 0 \tag{III.1}$$

$$\begin{aligned} &F^{\mathrm{I}}\left[e^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] + F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}U^{0\mathrm{I}(5)} + e^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] \\ &+ F^{\mathrm{II}}\left[e^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] + F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}U^{0\mathrm{II}(6)} + e^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \end{aligned} \tag{III.2}$$

$$F^{\mathrm{I}}\left[\varepsilon^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + \alpha^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] - F^{\mathrm{I}2}b^{\mathrm{I}}\left[e^{\mathrm{I}}U^{0\mathrm{I}(5)} - \varepsilon^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} - \alpha^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] = 0 \tag{III.3}$$

$$-F^{\mathrm{II}}\left[\varepsilon^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + \alpha^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] + F^{\mathrm{II}2}b^{\mathrm{II}}\left[e^{\mathrm{II}}U^{0\mathrm{II}(6)} - \varepsilon^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} - \alpha^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \tag{III.4}$$

$$F^{\mathrm{I}}\left[\alpha^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + \mu^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] - F^{\mathrm{I}2}b^{\mathrm{I}}\left[h^{\mathrm{I}}U^{0\mathrm{I}(5)} - \alpha^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} - \mu^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] = 0 \tag{III.5}$$

$$-F^{\mathrm{II}}\left[\alpha^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + \mu^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] + F^{\mathrm{II}2}b^{\mathrm{II}}\left[h^{\mathrm{II}}U^{0\mathrm{II}(6)} - \alpha^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} - \mu^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \tag{III.6}$$

These equations written above can reveal the phase velocity of the interfacial SH-wave in this electrically open and magnetically closed case. Three possibilities, namely the first eigenvectors, the second ones, and the mixture of them must be also considered. This is the main aim of this chapter.

III.1. *The first sets of the eigenvector components*

Consider the case of the PEM1 first eigenvectors defined by expressions (I.71) and (I.72) and the PEM2 first eigenvectors defined by expressions (I.79) and (I.80) from Chapter I. It is apparent that two equations corresponding to the mechanical boundary conditions can be borrowed from equations (II.7) and (II.8) written in the previous chapter. Utilizing the corresponding first eigenvectors, all equations from (III.3) to (III.6) can be significantly simplified. As a result, the six homogeneous equations for determination of the phase velocity of the interfacial SH-wave read:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^{2}}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^{2}}=0 \tag{III.7}$$

$$\begin{aligned}&F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^{2}}+e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]\\&+F^{\mathrm{II}}\left[e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^{2}}+e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]=0\end{aligned} \tag{III.8}$$

$$F^{\mathrm{I}}\times 0-F^{\mathrm{I}2}b^{\mathrm{I}}\times 0=0 \tag{III.9}$$

$$F^{\mathrm{II}}\times 0-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \tag{III.10}$$

$$F^{\mathrm{I}}\left(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)+F^{\mathrm{I}2}b^{\mathrm{I}}\times 0=0 \tag{III.11}$$

$$F^{\mathrm{II}}\left(e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right)+F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \tag{III.12}$$

where equations (III.11) and (III.12) were multiplied by the factors of $\left(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)/\left(\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^{2}\right)$ and $\left(e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right)/\left(\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^{2}\right)$, respectively. Using equations (III.11) and (III.12), two weight factors such as F^{I} and F^{II} must be equal to zero. Also, $F^{\mathrm{I}2}$ and $F^{\mathrm{II}2}$ are defined by expression (III.7).

Consequently, equations (III.7) and (III.8) with $F^{\mathrm{I}} = F^{\mathrm{II}} = 0$ can reveal the following equation for determination of the phase velocity of the interfacial SH-wave:

$$\frac{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2} b^{\mathrm{I}}\left[\frac{1}{\left(K_{em}^{\mathrm{I}}\right)^2}+1\right] + b^{\mathrm{II}}\left[\frac{1}{\left(K_{em}^{\mathrm{II}}\right)^2}+1\right] = 0 \qquad \text{(III.13)}$$

It is flagrant that equation (III.13) cannot have solutions because all the parameters can have definitely positive values. However there is a small probability that either $\left(K_{em}^{\mathrm{I}}\right)^2$ or $\left(K_{em}^{\mathrm{II}}\right)^2$ can have a negative sign due to a suitably large value of α where $\alpha > 0$. Also, it is well-known that for bulk metamaterials, $\varepsilon < 0$ and $\mu < 0$ resulting in $\varepsilon\mu > 0$ can occur. This can also lead to a negative sign for $\left(K_{em}^{\mathrm{I}}\right)^2$ when PEM1 is treated as a bulk metamaterial. In those cases, one can check that the equation cannot also have solutions.

III.2. *The second sets of the eigenvector components*

This is the case when the PEM1 second eigenvectors defined by expressions (I.74) and (I.75) and the PEM2 second eigenvectors defined by expressions (I.82) and (I.83) from Chapter I are employed. The equations corresponding to the mechanical boundary conditions can be written following equations (II.43) and (II.44) from the previous chapter. Utilizing the corresponding second eigenvectors, equations from (III.3) to (III.6) can be also written in significantly simplified forms. Therefore, the corresponding six homogeneous equations can be introduced for the case as follows:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}} - h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2} - F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2} = 0 \qquad \text{(III.14)}$$

$$F^{\mathrm{I}}\left[e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]+F^{\mathrm{I2}}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]$$
$$+F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]+F^{\mathrm{II2}}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]=0 \tag{III.15}$$

$$F^{\mathrm{I}}\left(e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right)-F^{\mathrm{I2}}b^{\mathrm{I}}\times 0=0 \tag{III.16}$$

$$F^{\mathrm{II}}\left(e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right)-F^{\mathrm{II2}}b^{\mathrm{II}}\times 0=0 \tag{III.17}$$

$$F^{\mathrm{I}}\times 0-F^{\mathrm{I2}}b^{\mathrm{I}}\times 0=0 \tag{III.18}$$

$$F^{\mathrm{II}}\times 0-F^{\mathrm{II2}}b^{\mathrm{II}}\times 0=0 \tag{III.19}$$

For convenience, the factors of $\left(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)/\left(\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2\right)$ and $\left(e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right)/\left(\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2\right)$ were respectively used for equations (III.16) and (III.17). Exploiting equations (III.16) and (III.17), weight factors F^{I} and F^{II} must equal to zero. Also, F^{I2} and F^{II2} can be defined by expression (III.14). Accounting these facts, it is obvious that the phase velocity of the interfacial SH-wave can be also determined by equation (III.13) which certainly has no solutions. Therefore, the interfacial SH-wave guided by electrically open and magnetically closed interface cannot propagate. This fact was found and briefly discussed in the previous subsection.

III.3. *The combination of both the sets of the eigenvector components*

For the combination of two possible eigenvectors, equations (III.1) and (III.2) corresponding to the mechanical boundary conditions can be also transformed into equations (II.52) and (II.53) from the previous chapter. Using the first eigenvectors for the first PEM half-space, the electrical and magnetic boundary conditions such as $D_3^{\mathrm{I}} = 0$ and $B_3^{\mathrm{I}} = 0$ are defined by equations (III.9) and (III.11), correspondingly. Exploiting the second eigenvectors for the second PEM half-space, $D_3^{\mathrm{II}} = 0$ and $B_3^{\mathrm{II}} = 0$ are defined by equations (III.17) and (III.19). The reader can check that this case also leads to equation (III.13) which has no solution for the phase velocity of the interfacial SH-wave.

CHAPTER IV

The Case of $\varphi^{\mathrm{I}} = \varphi^{\mathrm{II}} = 0$ and $B_3^{\mathrm{I}} = B_3^{\mathrm{II}} = 0$ at the Interface

Consider the problem of interfacial SH-wave propagation along the electrically closed ($\varphi^{\mathrm{I}} = 0$ and $\varphi^{\mathrm{II}} = 0$) and magnetically closed ($B_3^{\mathrm{I}} = 0$ and $B_3^{\mathrm{II}} = 0$) interface at $x_3 = 0$. For this case, equations (III.1) and (III.2) describe the mechanical boundary conditions such as $U^{\mathrm{I}} = U^{\mathrm{II}}$ and $(\sigma_{32})^{\mathrm{I}} = (\sigma_{32})^{\mathrm{II}}$, equations (II.3) and (II.4) represent the electrical boundary conditions, and equations (III.5) and (III.6) correspond to the magnetic boundary conditions. They can be written as follows:

$$F^{\mathrm{I}2}U^{0\mathrm{I}(5)} - F^{\mathrm{II}2}U^{0\mathrm{II}(6)} = 0 \qquad \text{(IV.1)}$$

$$\begin{aligned} &F^{\mathrm{I}}\left[e^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] + F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}U^{0\mathrm{I}(5)} + e^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] \\ &+ F^{\mathrm{II}}\left[e^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] + F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}U^{0\mathrm{II}(6)} + e^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \end{aligned} \qquad \text{(IV.2)}$$

$$F^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + F^{\mathrm{I}2}\varphi^{0\mathrm{I}(5)} = 0 \qquad \text{(IV.3)}$$

$$F^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + F^{\mathrm{II}2}\varphi^{0\mathrm{II}(6)} = 0 \qquad \text{(IV.4)}$$

$$F^{\mathrm{I}}\left[\alpha^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + \mu^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] - F^{\mathrm{I}2}b^{\mathrm{I}}\left[h^{\mathrm{I}}U^{0\mathrm{I}(5)} - \alpha^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} - \mu^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] = 0 \qquad \text{(IV.5)}$$

$$-F^{\mathrm{II}}\left[\alpha^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + \mu^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] + F^{\mathrm{II}2}b^{\mathrm{II}}\left[h^{\mathrm{II}}U^{0\mathrm{II}(6)} - \alpha^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} - \mu^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \qquad \text{(IV.6)}$$

These six homogeneous equations written above can actually reveal the phase velocity of the interfacial SH-wave. Two-phase materials such as piezoelectromagnetics can possess several possibilities to treat the problem. Indeed, the first PEM half-space has two different sets of the components and the second PEM half-space also has its own two different eigenvectors. These different cases are treated below in this chapter.

IV.1. *The first sets of the eigenvector components*

In the case of the first sets of the eigenvector components for both the PEM1 and PEM2, the six homogeneous equations written above can respectively transformed into equations (III.7), (III.8), (II.9), (II.10), (III.11), and (III.12). For convenience, they are written here below:

$$F^{I2}\frac{e^{I}\alpha^{I}-h^{I}\varepsilon^{I}}{C^{I}\left(K_{em}^{I}\right)^{2}}-F^{II2}\frac{e^{II}\alpha^{II}-h^{II}\varepsilon^{II}}{C^{II}\left(K_{em}^{II}\right)^{2}}=0 \tag{IV.7}$$

$$\begin{aligned}&F^{I}\left[e^{I}\alpha^{I}-h^{I}\varepsilon^{I}\right]+F^{I2}b^{I}\left[C^{I}\frac{e^{I}\alpha^{I}-h^{I}\varepsilon^{I}}{C^{I}\left(K_{em}^{I}\right)^{2}}+e^{I}\alpha^{I}-h^{I}\varepsilon^{I}\right]\\&+F^{II}\left[e^{II}\alpha^{II}-h^{II}\varepsilon^{II}\right]+F^{II2}b^{II}\left[C^{II}\frac{e^{II}\alpha^{II}-h^{II}\varepsilon^{II}}{C^{II}\left(K_{em}^{II}\right)^{2}}+e^{II}\alpha^{II}-h^{II}\varepsilon^{II}\right]=0\end{aligned} \tag{IV.8}$$

$$F^{I}e^{I}\alpha^{I}-F^{I2}\left(\frac{\left(e^{I}\right)^{2}h^{I}}{C^{I}\left(K_{em}^{I}\right)^{2}}-e^{I}\alpha^{I}\right)=0 \tag{IV.9}$$

$$F^{II}e^{II}\alpha^{II}-F^{II2}\left(\frac{\left(e^{II}\right)^{2}h^{II}}{C^{II}\left(K_{em}^{II}\right)^{2}}-e^{II}\alpha^{II}\right)=0 \tag{IV.10}$$

$$-F^{I}h^{I}\varepsilon^{I}+F^{I2}b^{I}\times 0=0 \tag{IV.11}$$

$$-F^{II}h^{II}\varepsilon^{II}+F^{II2}b^{II}\times 0=0 \tag{IV.12}$$

where the corresponding factors such as $-h^{I}\varepsilon^{I}/\left(\varepsilon^{I}\mu^{I}-\left(\alpha^{I}\right)^{2}\right)$ and $-h^{II}\varepsilon^{II}/\left(\varepsilon^{II}\mu^{II}-\left(\alpha^{II}\right)^{2}\right)$ were utilized for equations (IV.11) and (IV.12). Using equations (IV.11) and (IV.12), it is possible that two weight factors such as F^{I} and F^{II} must be equal to zero and this is not obligatory. Also, F^{I2} and F^{II2} can be defined by expression (IV.7).

For further theoretical treatments, it is possible to use only equations (IV.7) and (IV.8) written in the following modified forms:

$$F^{I2}=F^{II2}\frac{e^{II}\alpha^{II}-h^{II}\varepsilon^{II}}{e^{I}\alpha^{I}-h^{I}\varepsilon^{I}}\frac{C^{I}\left(K_{em}^{I}\right)^{2}}{C^{II}\left(K_{em}^{II}\right)^{2}} \tag{IV.13}$$

$$F^{\mathrm{I}2}e^{\mathrm{I}}\alpha^{\mathrm{I}}\left(\frac{\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}-1\right)+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]$$
$$+F^{\mathrm{II}2}e^{\mathrm{II}}\alpha^{\mathrm{II}}\left(\frac{\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}-1\right)+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]=0 \quad \text{(IV.14)}$$

where the second equation was obtained by a successive subtraction of four equations from (IV.9) to (IV.12). In equation (IV.14), the non-dimensional coefficients such as $\left(K_{\alpha}^{\mathrm{I}}\right)^2$ and $\left(K_{\alpha}^{\mathrm{II}}\right)^2$ are respectively defined by the following formulae:

$$\left(K_{\alpha}^{\mathrm{I}}\right)^2=\frac{e^{\mathrm{I}}h^{\mathrm{I}}}{C^{\mathrm{I}}\alpha^{\mathrm{I}}}=\frac{\alpha^{\mathrm{I}}e^{\mathrm{I}}h^{\mathrm{I}}}{C^{\mathrm{I}}\left(\alpha^{\mathrm{I}}\right)^2} \quad \text{(IV.15)}$$

$$\left(K_{\alpha}^{\mathrm{II}}\right)^2=\frac{e^{\mathrm{II}}h^{\mathrm{II}}}{C^{\mathrm{II}}\alpha^{\mathrm{II}}}=\frac{\alpha^{\mathrm{II}}e^{\mathrm{II}}h^{\mathrm{II}}}{C^{\mathrm{II}}\left(\alpha^{\mathrm{II}}\right)^2} \quad \text{(IV.16)}$$

These coefficients were first introduced in Ref. [46] and connect two terms containing α^{I} and α^{II} in the CMEMCs $\left(K_{em}^{\mathrm{I}}\right)^2$ and $\left(K_{em}^{\mathrm{II}}\right)^2$, respectively.

Exploiting the relationship between $F^{\mathrm{I}2}$ and $F^{\mathrm{II}2}$ defined by expression (IV.13), equation (IV.14) can certainly reveal the following relatively compact equation for the determination of the velocity V_{new2} of the second new interfacial SH-wave:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new2}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new2}}{V_{tem}^{\mathrm{II}}}\right)^2}$$
$$=\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2} \quad \text{(IV.17)}$$

where the CMEMCs $\left(K_{em}^{\mathrm{I}}\right)^2$ and $\left(K_{em}^{\mathrm{II}}\right)^2$ and the SH-BAW velocities V_{tem}^{I} and V_{tem}^{II} can be defined by using the corresponding superscripts "I" and "II" in expressions (I.53) and (I.55) from the first chapter.

It is natural that equation (IV.17) can have the following existence conditions:

$$Y > 0 \text{ and } \left(V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}\right)^2 > 1 - Y^2 \qquad \text{(IV.18)}$$

where

$$Y = \frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2 - \left(K_{\alpha}^{\mathrm{I}}\right)^2}{1 + \left(K_{em}^{\mathrm{II}}\right)^2} + \frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}} - h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2 - \left(K_{\alpha}^{\mathrm{II}}\right)^2}{1 + \left(K_{em}^{\mathrm{II}}\right)^2} \qquad \text{(IV.19)}$$

It is blatant that the piezoelectric effect is responsible for the existence of such new interfacial SH-wave because $e^{\mathrm{I}} = 0$ and $e^{\mathrm{II}} = 0$ in equation (IV.17) certainly result in the case treated in the previous chapter when no solutions can exist. For the case of both $h^{\mathrm{I}} = 0$ and $h^{\mathrm{II}} = 0$ or both $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$, one can check that the interfacial SH-wave can exist because $Y > 0$ occurs. Also, it is well-known that the values of the electromagnetic constants α^{I} and α^{II} can be very small resulting in $Y > 0$. However, both existence conditions (IV.18) must be satisfied. For some larger values of both α^{I} and α^{II}, a small probability can exist for some piezoelectromagnetics that $Y < 0$ can occur, see expression (IV.19). For very large values of α^{I} and α^{II}, it is possible that $Y > 0$ occurs again. This situation discussed above relates to positive values of the electromagnetic constants. Also, $e^{\mathrm{I}}\alpha^{\mathrm{I}} = h^{\mathrm{I}}\varepsilon^{\mathrm{I}}$ or $e^{\mathrm{II}}\alpha^{\mathrm{II}} = h^{\mathrm{II}}\varepsilon^{\mathrm{II}}$ in expression (IV.19) can result in $Y \to \infty$ and such new interfacial SH-wave cannot propagate.

Exploiting the composite materials listed in table I.1 from Chapter I, one can evaluate the possibility of propagation of such new interfacial SH-wave. It was mentioned in Chapter II that PZT–Terfenol-D and $BaTiO_3$–$CoFe_2O_4$ must be used as the first and second PEM half-spaces, respectively, because the SH-BAW speed for PEM1 is slower than that for PEM2. Therefore, the calculated parameters are as follows: $C^{\mathrm{I}}/C^{\mathrm{II}}$ ~ 0.33, $V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}$ ~ 0.59, $\left(K_{em}^{\mathrm{I}}\right)^2$ ~ 0.82, $\left(K_{em}^{\mathrm{II}}\right)^2$ ~ 0.16, $\left(K_{\alpha}^{\mathrm{I}}\right)^2$ ~ 7.27, $\left(K_{\alpha}^{\mathrm{II}}\right)^2$ ~ 5.36, $e^{\mathrm{I}}\alpha^{\mathrm{I}}/\left(e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)$ ~ – 0.10, and $e^{\mathrm{II}}\alpha^{\mathrm{II}}/\left(e^{\mathrm{II}}\alpha^{\mathrm{II}} - h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right)$ ~ – 0.03 for $\alpha^{\mathrm{I}} = 0.01\sqrt{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}}$ and $\alpha^{\mathrm{II}} = 0.01\sqrt{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}}$. It was numerically found that for these relatively small values of α^{I} and α^{II}, existence conditions (IV.18) cannot be satisfied. On the other hand, using a very large value of α^{II} such as $\alpha^{\mathrm{II}} = 0.98\sqrt{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}}$ and all the recalculated values of the corresponding parameters, existence conditions (IV.18) can be satisfied and therefore, such new interfacial SH-wave can propagate. However, such extremely large value of α^{II} for $BaTiO_3$–$CoFe_2O_4$ is questionable.

IV.2. *The second sets of the eigenvector components*

Consider the other possible case when the corresponding second eigenvectors for both the first and second PEM half-spaces are used. For this case, the mechanical, electrical, and magnetic boundary conditions are defined by equations (III.14) and (III.15), equations (II.45) and (II.46), and equations (III.18) and (III.19), respectively. For convenience, the corresponding six homogeneous equations are written below:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \tag{IV.20}$$

$$\begin{aligned}&F^{\mathrm{I}}\left[e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]\\&+F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]=0\end{aligned} \tag{IV.21}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\mu^{\mathrm{I}}-F^{\mathrm{I}2}\left(\frac{e^{\mathrm{I}}\left(h^{\mathrm{I}}\right)^2}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-e^{\mathrm{I}}\mu^{\mathrm{I}}\right)=0 \tag{IV.22}$$

$$F^{\mathrm{II}}e^{\mathrm{II}}\mu^{\mathrm{II}}-F^{\mathrm{II}2}\left(\frac{e^{\mathrm{II}}\left(h^{\mathrm{II}}\right)^2}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}-e^{\mathrm{II}}\mu^{\mathrm{II}}\right)=0 \tag{IV.23}$$

$$F^{\mathrm{I}}\times 0-F^{\mathrm{I}2}b^{\mathrm{I}}\times 0=0 \tag{IV.24}$$

$$F^{\mathrm{II}}\times 0-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \tag{IV.25}$$

It is flagrant that equations (IV.24) and (IV.25) can be surely excluded from the further analysis. So, the rest four homogeneous equations can certainly define three weight factors such as F^{I}, F^{II}, $F^{\mathrm{I}2}$ for $F^{\mathrm{II}2} = 1$ and the phase velocity of the interfacial SH-wave. The following equation can determine the velocity V_{new3} of the third new interfacial SH-wave:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new3}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new3}}{V_{tem}^{\mathrm{II}}}\right)^2}=\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{m}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2} \quad \text{(IV.26)}$$

The existence conditions for the third new SH-wave are those defined by expression (IV.18) from the previous subsection. However, the value of Y for equation (IV.26) differs from that used for equation (IV.17). For the case of this subsection, the value of Y reads:

$$Y=\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{m}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2} \quad \text{(IV.27)}$$

It is necessary to state that in expression (IV.27), $e^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{I}} = 0$ together with $e^{\mathrm{II}} = 0$ and $\alpha^{\mathrm{II}} = 0$ can result in $Y = 0$, and therefore, the new interfacial SH-wave cannot propagate. This fact can mean that the piezoelectric and magnetoelectric effects can define the existence of such new interfacial SH-wave.

The following subsection studies the case of the mixture of the first and second sets of the eigenvector components. This is the third possibility that must be also treated for these mechanical, electrical, and magnetic boundary conditions.

IV.3. ***The combination of both the sets of the eigenvector components***

Consider the combination of the eigenvectors when the first sets of the eigenvector components are used for the first PEM half-space and the second ones are utilized for the second PEM half-space. It is obvious that six homogeneous equations from (IV.1) to (IV.6) must be properly transformed for this case of the mechanical, electrical, and magnetic boundary conditions. It is also natural that the transformed equations can be readily borrowed from the previously studied cases. For instance, the mechanical and electrical boundary conditions can be written following equations from (II.52) to (II.55) from Chapter II and the magnetic boundary conditions are described by equations (III.11) and (III.19) from Chapter III. As a result, the following transformed forms of the six homogeneous equations must be used:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \qquad \text{(IV.28)}$$

$$\begin{aligned}&F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]\\&+F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]=0\end{aligned} \qquad \text{(IV.29)}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\alpha^{\mathrm{I}}-F^{\mathrm{I}2}\left(\frac{\left(e^{\mathrm{I}}\right)^2h^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-e^{\mathrm{I}}\alpha^{\mathrm{I}}\right)=0 \qquad \text{(IV.30)}$$

$$F^{\mathrm{II}}e^{\mathrm{II}}\mu^{\mathrm{II}}-F^{\mathrm{II}2}\left(\frac{e^{\mathrm{II}}\left(h^{\mathrm{II}}\right)^2}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}-e^{\mathrm{II}}\mu^{\mathrm{II}}\right)=0 \qquad \text{(IV.31)}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\varepsilon^{\mathrm{I}}+F^{\mathrm{I}2}b^{\mathrm{I}}\times 0=0 \qquad \text{(IV.32)}$$

$$F^{\mathrm{II}}\times 0-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \qquad \text{(IV.33)}$$

It was found that these six homogeneous equations written above can reveal the velocity V_{new4} of the fourth new interfacial SH-wave which can propagate along the common interface between two dissimilar piezoelectromagnetic half-spaces. The obtained equation for the determination of the value of the velocity V_{new4} is expressed as follows:

$$\begin{aligned}&\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new4}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new4}}{V_{tem}^{\mathrm{II}}}\right)^2}\\&=\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\end{aligned} \qquad \text{(IV.34)}$$

where the coefficient $\left(K_{\alpha}^{\mathrm{I}}\right)^2$ is defined by expression (IV.15).

It is clearly seen in equation (IV.34) that when $e^{\mathrm{I}} = 0$ occurs, the first term on the right-hand side vanishes and the second term on the same side can also vanish as soon as $e^{\mathrm{II}} = 0$ and $\alpha^{\mathrm{II}} = 0$. These facts can mean that the piezoelectric properties can define the existence of such new interfacial SH-wave. The existence condition is defined by expressions (IV.18), where the value of Y for the case is as follows:

$$Y = \frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}} \frac{C^{\mathrm{I}}}{C^{\mathrm{II}}} \frac{\left(K_{em}^{\mathrm{I}}\right)^2 - \left(K_{\alpha}^{\mathrm{I}}\right)^2}{1 + \left(K_{em}^{\mathrm{II}}\right)^2} + \frac{\left(K_{em}^{\mathrm{II}}\right)^2 - \left(K_{m}^{\mathrm{II}}\right)^2}{1 + \left(K_{em}^{\mathrm{II}}\right)^2} \tag{IV.35}$$

Also, $e^{\mathrm{I}}\alpha^{\mathrm{I}} = h^{\mathrm{I}}\varepsilon^{\mathrm{I}}$ in expressions (IV.34) and (IV.35) can actually lead to $Y \rightarrow \infty$ and such new interfacial SH-wave cannot propagate.

CHAPTER V

The Case of $D_3^{\mathrm{I}} = D_3^{\mathrm{II}} = 0$ and $\psi^{\mathrm{I}} = \psi^{\mathrm{II}} = 0$ at the Interface

Consider the interfacial SH-wave propagation guided by the electrically open ($D_3^{\mathrm{I}} = 0$ and $D_3^{\mathrm{II}} = 0$) and magnetically open ($\psi^{\mathrm{I}} = 0$ and $\psi^{\mathrm{II}} = 0$) interface at $x_3 = 0$. For this case, it is possible to borrow two equations corresponding to the mechanical boundary conditions such as $U^{\mathrm{I}} = U^{\mathrm{II}}$ and $(\sigma_{32})^{\mathrm{I}} = (\sigma_{32})^{\mathrm{II}}$ from the previous chapter. It is obvious that the electrical and magnetic boundary conditions are those used in Chapters III and II, respectively. Therefore, the corresponding six homogeneous equations read:

$$F^{\mathrm{I2}}U^{0\mathrm{I}(5)} - F^{\mathrm{II2}}U^{0\mathrm{II}(6)} = 0 \tag{V.1}$$

$$\begin{aligned} &F^{\mathrm{I}}\left[e^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] + F^{\mathrm{I2}}b^{\mathrm{I}}\left[C^{\mathrm{I}}U^{0\mathrm{I}(5)} + e^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] \\ &+ F^{\mathrm{II}}\left[e^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] + F^{\mathrm{II2}}b^{\mathrm{II}}\left[C^{\mathrm{II}}U^{0\mathrm{II}(6)} + e^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \end{aligned} \tag{V.2}$$

$$F^{\mathrm{I}}\left[\varepsilon^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + \alpha^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] - F^{\mathrm{I2}}b^{\mathrm{I}}\left[e^{\mathrm{I}}U^{0\mathrm{I}(5)} - \varepsilon^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} - \alpha^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] = 0 \tag{V.3}$$

$$-F^{\mathrm{II}}\left[\varepsilon^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + \alpha^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] + F^{\mathrm{II2}}b^{\mathrm{II}}\left[e^{\mathrm{II}}U^{0\mathrm{II}(6)} - \varepsilon^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} - \alpha^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \tag{V.4}$$

$$F^{\mathrm{I}}\psi^{0\mathrm{I}(3)} + F^{\mathrm{I2}}\psi^{0\mathrm{I}(5)} = 0 \tag{V.5}$$

$$F^{\mathrm{II}}\psi^{0\mathrm{II}(4)} + F^{\mathrm{II2}}\psi^{0\mathrm{II}(6)} = 0 \tag{V.6}$$

These six homogeneous equations written above must be transformed into corresponding convenient forms for further analysis. Using different sets of the eigenvector components there are three possibility to transform these equations. It is possible to treat the first case when the first eigenvectors are applied to get the phase velocity of the interfacial SH-wave and the corresponding existence conditions.

V.1. *The first sets of the eigenvector components*

It is usual for this work to start the further analysis with the case when the corresponding first eigenvectors are used for both the first and second PEM half-spaces. Following the same case studied in Chapter III, the mechanical and electrical boundary conditions are given in equations from (III.7) to (III.10). Also, equations (II.11) and (II.12) can be borrowed from Chapter II to describe the corresponding magnetic boundary conditions. It is natural to use these transformed six homogeneous equations mentioned above for this case. They read:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \tag{V.7}$$

$$\begin{aligned}&F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]\\&+F^{\mathrm{II}}\left[e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]=0\end{aligned} \tag{V.8}$$

$$F^{\mathrm{I}}\times 0-F^{\mathrm{I}2}b^{\mathrm{I}}\times 0=0 \tag{V.9}$$

$$F^{\mathrm{II}}\times 0-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \tag{V.10}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\varepsilon^{\mathrm{I}}+F^{\mathrm{I}2}\left(\frac{\left(e^{\mathrm{I}}\right)^2h^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)=0 \tag{V.11}$$

$$-F^{\mathrm{II}}h^{\mathrm{II}}\varepsilon^{\mathrm{II}}+F^{\mathrm{II}2}\left(\frac{\left(e^{\mathrm{II}}\right)^2h^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right)=0 \tag{V.12}$$

It is clearly seen in equations from (V.7) to (V.12) that one actually deals here with four homogeneous equations because equations (V.9) and (V.10) become negligible. Therefore, these four equations can be readily used to determine the explicit forms for the weight factors, see Chapter II. Also, these four equations have revealed the following equation for the calculation of the velocity V_{new5} of the fifth new interfacial SH-wave:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new5}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new5}}{V_{tem}^{\mathrm{II}}}\right)^2}=\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2} \quad (\mathrm{V}.13)$$

Indeed, equation (V.13) can also have the existence conditions written in the following form used in the previous chapter:

$$Y>0 \text{ and } \left(V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}\right)^2>1-Y^2 \quad (\mathrm{V}.14)$$

where

$$Y=\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2} \quad (\mathrm{V}.15)$$

It is possible to discuss the existence conditions for this case. It is assumed that usually $Y > 0$ always occurs. However, this not obligatory and there can be some cases when Y can have a negative sign. Also, $h = 0$ and $\alpha = 0$ for the first and second piezoelectromagnetics certainly results in $Y = 0$. As a result, such interfacial SH-waves cannot propagate.

V.2. *The second sets of the eigenvector components*

Using the corresponding second eigenvectors for the first and second PEM half-spaces, it is also essential to write the corresponding transformed equations. For this case, equations from (V.1) to (V.2) can be properly transformed into equations (III.14), (III.15), (III.16), (III.17), (II.47), and (II.48). Therefore, the suitable six homogeneous equations are composed as follows:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad (\mathrm{V}.16)$$

$$
\begin{aligned}
&F^{\mathrm{I}}\left[e^{\mathrm{I}}\mu^{\mathrm{I}} - h^{\mathrm{I}}\alpha^{\mathrm{I}}\right] + F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}} - h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2} + e^{\mathrm{I}}\mu^{\mathrm{I}} - h^{\mathrm{I}}\alpha^{\mathrm{I}}\right] \\
&+ F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}\right] + F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2} + e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}\right] = 0
\end{aligned}
\tag{V.17}
$$

$$
F^{\mathrm{I}}e^{\mathrm{I}}\mu^{\mathrm{I}} - F^{\mathrm{I}2}b^{\mathrm{I}} \times 0 = 0 \tag{V.18}
$$

$$
F^{\mathrm{II}}e^{\mathrm{II}}\mu^{\mathrm{II}} - F^{\mathrm{II}2}b^{\mathrm{II}} \times 0 = 0 \tag{V.19}
$$

$$
-F^{\mathrm{I}}h^{\mathrm{I}}\alpha^{\mathrm{I}} + F^{\mathrm{I}2}\left(\frac{e^{\mathrm{I}}\left(h^{\mathrm{I}}\right)^2}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2} - h^{\mathrm{I}}\alpha^{\mathrm{I}}\right) = 0 \tag{V.20}
$$

$$
-F^{\mathrm{II}}h^{\mathrm{II}}\alpha^{\mathrm{II}} + F^{\mathrm{II}2}\left(\frac{e^{\mathrm{II}}\left(h^{\mathrm{II}}\right)^2}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2} - h^{\mathrm{II}}\alpha^{\mathrm{II}}\right) = 0 \tag{V.21}
$$

It is apparent that it is necessary to successively subtract equations from (V.18) to (V.21) from equation (V.17) and then to use the relationship between the weight factors $F^{\mathrm{I}2}$ and $F^{\mathrm{II}2}$ defined in equation (V.16) for equation (V.17). The resulting equation can be further simplified. So, the final equation for the calculation of the velocity V_{new6} of the sixth new interfacial SH-wave can be introduced in the following form:

$$
\begin{aligned}
&\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new6}}{V_{tem}^{\mathrm{I}}}\right)^2} + \sqrt{1-\left(\frac{V_{new6}}{V_{tem}^{\mathrm{II}}}\right)^2} \\
&= -\frac{h^{\mathrm{I}}\alpha^{\mathrm{I}}}{e^{\mathrm{I}}\mu^{\mathrm{I}} - h^{\mathrm{I}}\alpha^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2 - \left(K_{\alpha}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2} - \frac{h^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2 - \left(K_{\alpha}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}
\end{aligned}
\tag{V.22}
$$

where the coefficients $\left(K_{\alpha}^{\mathrm{I}}\right)^2$ and $\left(K_{\alpha}^{\mathrm{II}}\right)^2$ are defined by expressions (IV.15) and (IV.16), respectively.

Existence conditions (V.14) can be also utilized here. However, it is apparent that for this case, the value of Y in existence conditions (V.14) is different from that defined in equation (V.15). For this case, the parameter Y is equal to the right-hand side in equation (V.22). It is clearly seen in equation (V.22) that $Y = 0$ occurs as soon as $h^{\mathrm{I}} = 0$ and $h^{\mathrm{II}} = 0$ and therefore, existence conditions (V.14) cannot be satisfied. Thus, it is possible to state that the presence of the piezomagnetic effect in the two-phase composites can cause the propagation of such new interfacial SH-wave. Also, $e^{\mathrm{I}}\mu^{\mathrm{I}} = h^{\mathrm{I}}\alpha^{\mathrm{I}}$ or $e^{\mathrm{II}}\mu^{\mathrm{II}} = h^{\mathrm{II}}\alpha^{\mathrm{II}}$ in expression (V.22) can lead to $Y \to \infty$ and such new interfacial SH-wave cannot propagate.

V.3. *The combination of both the sets of the eigenvector components*

For comparison, it is indispensable to treat the third possible case that mixtures the first and second sets of the eigenvector components. This means that the first eigenvectors can be chosen for the first PEM half-space and the second ones for the second PEM half-space. Exploiting them, the mechanical, electrical, and magnetic boundary conditions can result in equations (II.52), (II.53), (V.9), (V.19), (V.11), and (V.21) which can be used for this case. These six homogeneous equations can be then written down as follows:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2} - F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2} = 0 \qquad \text{(V.23)}$$

$$\begin{aligned} &F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right] + F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2} + e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right] \\ &+ F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}\right] + F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2} + e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}\right] = 0 \end{aligned} \qquad \text{(V.24)}$$

$$F^{\mathrm{I}} \times 0 - F^{\mathrm{I}2}b^{\mathrm{I}} \times 0 = 0 \qquad \text{(V.25)}$$

$$F^{\mathrm{II}}e^{\mathrm{II}}\mu^{\mathrm{II}} - F^{\mathrm{II}2}b^{\mathrm{II}} \times 0 = 0 \qquad \text{(V.26)}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\varepsilon^{\mathrm{I}} + F^{\mathrm{I}2}\left(\frac{\left(e^{\mathrm{I}}\right)^2 h^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right) = 0 \qquad \text{(V.27)}$$

$$-F^{\mathrm{II}}h^{\mathrm{II}}\alpha^{\mathrm{II}} + F^{\mathrm{II}2}\left(\frac{e^{\mathrm{II}}\left(h^{\mathrm{II}}\right)^2}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2} - h^{\mathrm{II}}\alpha^{\mathrm{II}}\right) = 0 \qquad \text{(V.28)}$$

It is overt that equation (V.25) does not participate in the analysis. Also, it is needed to subtract equations (V.26) and (V.28) from equation (V.24). As a result, this modified equation together with equations (V.23) and (V.27) can be transform to the following equation:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new7}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new7}}{V_{tem}^{\mathrm{II}}}\right)^2}$$
$$=\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}-\frac{h^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2} \qquad \text{(V.29)}$$

where the coefficient $\left(K_{\alpha}^{\mathrm{II}}\right)^2$ is defined by expression (IV.16).

Equation (V.29) can definitely reveal the velocity V_{new7} of the seventh new interfacial SH-wave propagating along the common interface of two dissimilar piezoelectromagnetics when they are perfectly bonded. It is obvious that the existence conditions defined by inequalities (V.14) must be also used. Indeed, in order that such new interfacial SH-wave can propagate, the parameter Y representing the right-hand side in equation (V.29) must have a positive sign. Also, $e^{\mathrm{II}}\mu^{\mathrm{II}}=h^{\mathrm{II}}\alpha^{\mathrm{II}}$ in expression (V.29) can lead to $Y \to \infty$ and such new interfacial SH-wave cannot propagate. It is also possible to mention the situation when $Y = 0$ occurs. This situation can happen when $h^{\mathrm{I}} = 0$, $\alpha^{\mathrm{I}} = 0$, and $h^{\mathrm{II}} = 0$. This can also mean that the piezomagnetic effect is mainly responsible for the existence of such new interfacial SH-wave. Like the cases studied in the previous subsections of this chapter, $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$ can significantly simplify the form of the parameter Y which reads

$$Y=\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{m}^{\mathrm{I}}\right)^2}{1+\left(K_{e}^{\mathrm{II}}\right)^2+\left(K_{m}^{\mathrm{II}}\right)^2}+\frac{\left(K_{m}^{\mathrm{II}}\right)^2}{1+\left(K_{e}^{\mathrm{II}}\right)^2+\left(K_{m}^{\mathrm{II}}\right)^2} \qquad \text{(V.30)}$$

CHAPTER VI

The Case of $\varphi^{\mathrm{I}} = \varphi^{\mathrm{II}}$, $D_3^{\mathrm{I}} = D_3^{\mathrm{II}}$, $\psi^{\mathrm{I}} = \psi^{\mathrm{II}}$, and $B_3^{\mathrm{I}} = B_3^{\mathrm{II}}$ at $x_3 = 0$

In this chapter, the applied mechanical boundary conditions such as $U^{\mathrm{I}} = U^{\mathrm{II}}$ and $(\sigma_{32})^{\mathrm{I}} = (\sigma_{32})^{\mathrm{II}}$ are similar to those used in Chapters from II to V. The superscripts "I" and "II" are used for the first and second PEM half-spaces, respectively. The electrical boundary conditions at $x_3 = 0$ such as $\varphi^{\mathrm{I}} = \varphi^{\mathrm{II}}$ and $D_3^{\mathrm{I}} = D_3^{\mathrm{II}}$ are defined by equations from (I.96) to (I.101) in Chapter I. Also, the magnetic boundary conditions such as $\psi^{\mathrm{I}} = \psi^{\mathrm{II}}$ and $B_3^{\mathrm{I}} = B_3^{\mathrm{II}}$ are defined by equations from (I.102) to (I.107). As a result, one can compose the following six homogeneous equations:

$$F^{\mathrm{I}2}U^{0\mathrm{I}(5)} - F^{\mathrm{II}2}U^{0\mathrm{II}(6)} = 0 \tag{VI.1}$$

$$\begin{aligned} &F^{\mathrm{I}}\left[e^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] + F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}U^{0\mathrm{I}(5)} + e^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] \\ &+ F^{\mathrm{II}}\left[e^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] + F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}U^{0\mathrm{II}(6)} + e^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \end{aligned} \tag{VI.2}$$

$$F^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + F^{\mathrm{I}2}\varphi^{0\mathrm{I}(5)} - F^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} - F^{\mathrm{II}2}\varphi^{0\mathrm{II}(6)} = 0 \tag{VI.3}$$

$$\begin{aligned} &F^{\mathrm{I}}\left[\varepsilon^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + \alpha^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] - F^{\mathrm{I}2}b^{\mathrm{I}}\left[e^{\mathrm{I}}U^{0\mathrm{I}(5)} - \varepsilon^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} - \alpha^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] \\ &+ F^{\mathrm{II}}\left[\varepsilon^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + \alpha^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] - F^{\mathrm{II}2}b^{\mathrm{II}}\left[e^{\mathrm{II}}U^{0\mathrm{II}(6)} - \varepsilon^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} - \alpha^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \end{aligned} \tag{VI.4}$$

$$F^{\mathrm{I}}\psi^{0\mathrm{I}(3)} + F^{\mathrm{I}2}\psi^{0\mathrm{I}(5)} - F^{\mathrm{II}}\psi^{0\mathrm{II}(4)} - F^{\mathrm{II}2}\psi^{0\mathrm{II}(6)} = 0 \tag{VI.5}$$

$$\begin{aligned} &F^{\mathrm{I}}\left[\alpha^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + \mu^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] - F^{\mathrm{I}2}b^{\mathrm{I}}\left[h^{\mathrm{I}}U^{0\mathrm{I}(5)} - \alpha^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} - \mu^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] \\ &+ F^{\mathrm{II}}\left[\alpha^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + \mu^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] - F^{\mathrm{II}2}b^{\mathrm{II}}\left[h^{\mathrm{II}}U^{0\mathrm{II}(6)} - \alpha^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} - \mu^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \end{aligned} \tag{VI.6}$$

It is clearly seen that this system of the six homogeneous equations written above can be significantly more complicated compared with the cases treaded in the previous chapters. However, it is necessary to treat this case by the same way carried out earlier.

VI.1. *The first sets of the eigenvector components*

Employing the first eigenvectors for both the first and second PEM half-spaces, equations (VI.1) and (VI.2) can be readily transformed into equations (II.7) and (II.8), respectively. Therefore, it is natural to borrow these two equations from Chapter II. Using the corresponding first eigenvectors, equations from (VI.3) and (VI.6) can be properly transformed and introduced in the corresponding simplified forms. Subsequently, the explicit forms of the six homogeneous equations written above can be written as follows:

$$F^{\mathrm{I2}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II2}}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \tag{VI.7}$$

$$\begin{aligned}&F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]+F^{\mathrm{I2}}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]\\&+F^{\mathrm{II}}\left[e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]+F^{\mathrm{II2}}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]=0\end{aligned} \tag{VI.8}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\alpha^{\mathrm{I}}+F^{\mathrm{I2}}e^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}}e^{\mathrm{I}}\alpha^{\mathrm{II}}-F^{\mathrm{II2}}e^{\mathrm{I}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \tag{VI.9}$$

$$F^{\mathrm{I}}\times 0-F^{\mathrm{I2}}b^{\mathrm{I}}\times 0+F^{\mathrm{II}}\times 0-F^{\mathrm{II2}}b^{\mathrm{II}}\times 0=0 \tag{VI.10}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\varepsilon^{\mathrm{I}}-F^{\mathrm{I2}}h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}+F^{\mathrm{II}}h^{\mathrm{I}}\varepsilon^{\mathrm{II}}+F^{\mathrm{II2}}h^{\mathrm{I}}\varepsilon^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \tag{VI.11}$$

$$F^{\mathrm{II}}+F^{\mathrm{I}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}+F^{\mathrm{I2}}b^{\mathrm{I}}\times 0+F^{\mathrm{II2}}b^{\mathrm{II}}\times 0=0 \tag{VI.12}$$

It is blatant that equation (VI.10) can be readily neglected and equation (VI.12) gives the explicit relationship between the weight factors F^{I} and F^{II}. It is natural to use this relationship for equations (VI.8), (VI.9), and (VI.11) to properly transform these three equations. Next, the certain relationship between the weight factors F^{I2} and F^{II2} defined by equation (VI.7) can be used for these three transformed equations and the second and third equations can be subtracted from the first. The result of these several complicated mathematical operations is given below in equation (VI.13). It is certain that equation (VI.13) can reveal the velocity V_{new8} of the eighth new interfacial SH-wave.

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new8}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new8}}{V_{tem}^{\mathrm{II}}}\right)^2}=\frac{1-\dfrac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\dfrac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}}{1+\dfrac{e^{\mathrm{I}}\alpha^{\mathrm{II}}-h^{\mathrm{I}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\dfrac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}}$$
$$\times\left(\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{h^{\mathrm{I}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}-\frac{e^{\mathrm{I}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\right) \quad \text{(VI.13)}$$

where the coefficient $\left(K_{\alpha}^{\mathrm{II}}\right)^2$ is defined by expression (IV.16).

In equation (VI.13), it is natural that the complicated right-hand side represents the parameter denoted by Y. Consequently, the existence conditions can be written in the following forms used in the previous chapter:

$$Y>0 \text{ and } \left(V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}\right)^2>1-Y^2 \quad \text{(VI.14)}$$

Also, one can find that equation (VI.13) can be significantly simplified as soon as $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$. It is necessary to state that $e^{\mathrm{I}} = 0$ and $e^{\mathrm{II}} = 0$ can further simplify equation (VI.13). Also, $e^{\mathrm{II}}\alpha^{\mathrm{II}} = h^{\mathrm{II}}\varepsilon^{\mathrm{II}}$ in expression (VI.13) can lead to $Y \to \infty$ and such new interfacial SH-wave cannot propagate. Besides, it is also possible to treat the other cases.

VI.2. *The second sets of the eigenvector components*

Consider the second possible case when the corresponding second eigenvectors are used for both the first and second PEM half-spaces having the common interface at $x_3 = 0$, see figure I.1. For this case, equations (II.43) and (II.44) corresponding to the mechanical boundary conditions can be borrowed from Chapter II. Using the corresponding second eigenvectors, equations (VI.3) and (VI.4) corresponding to the electrical boundary conditions and equations (VI.5) and (VI.6) corresponding to the magnetic ones can be readily transformed and introduced in some suitable forms. Therefore, these six homogeneous equations can be represented in the following forms:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^{2}}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^{2}}=0 \tag{VI.15}$$

$$\begin{aligned}&F^{\mathrm{I}}\left[e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^{2}}+e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]\\&+F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^{2}}+e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]=0\end{aligned} \tag{VI.16}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\mu^{\mathrm{I}}+F^{\mathrm{I}2}e^{\mathrm{I}}\mu^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^{2}-\left(K_{m}^{\mathrm{I}}\right)^{2}}{\left(K_{em}^{\mathrm{I}}\right)^{2}}-F^{\mathrm{II}}e^{\mathrm{I}}\mu^{\mathrm{II}}-F^{\mathrm{II}2}e^{\mathrm{I}}\mu^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^{2}-\left(K_{m}^{\mathrm{II}}\right)^{2}}{\left(K_{em}^{\mathrm{II}}\right)^{2}}=0 \tag{VI.17}$$

$$F^{\mathrm{II}}+F^{\mathrm{I}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^{2}}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^{2}}-F^{\mathrm{I}2}b^{\mathrm{I}}\times 0-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \tag{VI.18}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\alpha^{\mathrm{I}}-F^{\mathrm{I}2}h^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^{2}-\left(K_{\alpha}^{\mathrm{I}}\right)^{2}}{\left(K_{em}^{\mathrm{I}}\right)^{2}}+F^{\mathrm{II}}h^{\mathrm{I}}\alpha^{\mathrm{II}}+F^{\mathrm{II}2}h^{\mathrm{I}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^{2}-\left(K_{\alpha}^{\mathrm{II}}\right)^{2}}{\left(K_{em}^{\mathrm{II}}\right)^{2}}=0 \tag{VI.19}$$

$$F^{\mathrm{I}}\times 0-F^{\mathrm{I}2}b^{\mathrm{I}}\times 0+F^{\mathrm{II}}\times 0-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \tag{VI.20}$$

It is clearly seen that equation (VI.20) can be excluded from the further analysis and equation (VI.18) provides the relationship between the weight factors F^{I} and F^{II}. Using this relationship and that given by equation (VI.15), it is possible to properly transform equations (VI.16), (VI.17), and (VI.19). After that, the latter two equations can be subtracted from equation (VI.16) which can be further transformed. After all the complicated transformations, the resulting equation for the determination of the velocity V_{new9} of the ninth new interfacial SH-wave can be represented in the following form:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new9}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new9}}{V_{tem}^{\mathrm{II}}}\right)^2}=\frac{1-\dfrac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}\dfrac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}}{1+\dfrac{e^{\mathrm{I}}\mu^{\mathrm{II}}-h^{\mathrm{I}}\alpha^{\mathrm{II}}}{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}\dfrac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}}$$
$$\times\left(\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{h^{\mathrm{I}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}-\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\right) \tag{VI.21}$$

where the coefficient $\left(K_{\alpha}^{\mathrm{II}}\right)^2$ is defined by expression (IV.16).

It is obvious that the right-hand side in equation (VI.21) represents the parameter Y and using this parameter, the existence conditions are those given by inequalities (VI.14) from the previous subsection. One can also find that equation (VI.21) can be significantly simplified as soon as $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$. Also, $e^{\mathrm{II}}\mu^{\mathrm{II}} = h^{\mathrm{II}}\alpha^{\mathrm{II}}$ in expression (VI.21) can lead to $Y \to \infty$ and such new interfacial SH-wave cannot propagate. It is also possible to treat the third case that mixes the first and second eigenvectors.

VI.3. *The combination of the first and second sets*

Consider the case when the first eigenvectors are exploited for the first PEM half-space and the second eigenvectors are employed for the second PEM half-space. For this case, it is useful to borrow equations (II.52) and (II.53) responsible for the mechanical boundary conditions. Using the corresponding eigenvectors, equations from (VI.3) to (VI.6) can be transformed again to get suitable equations corresponding to the electrical and magnetic boundary conditions. Thus, the explicit forms of the six homogeneous equations read:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \tag{VI.22}$$

$$F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]$$
$$+F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]=0 \tag{VI.23}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\alpha^{\mathrm{I}}+F^{\mathrm{I2}}e^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}}e^{\mathrm{I}}\mu^{\mathrm{II}}-F^{\mathrm{II2}}e^{\mathrm{I}}\mu^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad \text{(VI.24)}$$

$$F^{\mathrm{II}}e^{\mathrm{II}}\mu^{\mathrm{II}}-F^{\mathrm{II2}}b^{\mathrm{II}}\times 0+F^{\mathrm{I}}\times 0-F^{\mathrm{I2}}b^{\mathrm{I}}\times 0=0 \quad \text{(VI.25)}$$

$$-F^{\mathrm{II}}h^{\mathrm{II}}\alpha^{\mathrm{II}}-F^{\mathrm{II2}}h^{\mathrm{II}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}+F^{\mathrm{I}}h^{\mathrm{II}}\varepsilon^{\mathrm{I}}+F^{\mathrm{I2}}h^{\mathrm{II}}\varepsilon^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}=0 \quad \text{(VI.26)}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\varepsilon^{\mathrm{I}}-F^{\mathrm{I2}}b^{\mathrm{I}}\times 0+F^{\mathrm{II}}\times 0-F^{\mathrm{II2}}b^{\mathrm{II}}\times 0=0 \quad \text{(VI.27)}$$

It is natural to successively subtract equations (VI.24), (VI.25), (VI.26), and (VI.27) from equation (VI.23). Then, it is necessary to use relationship (VI.22) between the weight factors F^{I2} and F^{II2}. The resulting expression can also have a complicated form. For simplicity, it is possible however to use the following two equations instead of four ones:

$$F^{\mathrm{I}}\left(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)+F^{\mathrm{I2}}e^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}}e^{\mathrm{I}}\mu^{\mathrm{II}}-F^{\mathrm{II2}}e^{\mathrm{I}}\mu^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad \text{(VI.28)}$$

$$F^{\mathrm{II}}\left(e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right)-F^{\mathrm{II2}}h^{\mathrm{II}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}+F^{\mathrm{I}}h^{\mathrm{II}}\varepsilon^{\mathrm{I}}+F^{\mathrm{I2}}h^{\mathrm{II}}\varepsilon^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}=0 \quad \text{(VI.29)}$$

These two equations can be further modified to have only the weight factor F^{I} in the first and only the weight factor F^{II} in the second. As a result, they can have the following forms:

$$\begin{aligned}
&F^{\mathrm{I}}\left(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)\left(1+\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\right)\\
&-\left(F^{\mathrm{II2}}h^{\mathrm{II}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}-F^{\mathrm{I2}}h^{\mathrm{II}}\varepsilon^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}\right)\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\\
&+F^{\mathrm{I2}}e^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II2}}e^{\mathrm{I}}\mu^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0
\end{aligned} \quad \text{(VI.30)}$$

$$F^{\mathrm{II}}\left(e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right)\left(1+\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\right)$$
$$+\left(F^{\mathrm{II}2}e^{\mathrm{I}}\mu^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}-F^{\mathrm{I}2}e^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}\right)\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}} \qquad \text{(VI.31)}$$
$$-F^{\mathrm{II}2}h^{\mathrm{II}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}+F^{\mathrm{I}2}h^{\mathrm{II}}\varepsilon^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}=0$$

Therefore, these two modified equations must be subtracted from equation (VI.23) and the equation for the determination of the velocity V_{new10} of the tenth new interfacial SH-wave can be written in the following final form:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new10}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new10}}{V_{tem}^{\mathrm{II}}}\right)^2}=\left(1+\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\right)^{-1}$$
$$\times\left\{\left(\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}-\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\right)\left(1-\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\right)\right. \qquad \text{(VI.32)}$$
$$\left.+\left(1+\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\right)\left(\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}-\frac{h^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\right)\right\}$$

where the coefficients $\left(K_{\alpha}^{\mathrm{I}}\right)^2$ and $\left(K_{\alpha}^{\mathrm{II}}\right)^2$ are defined by expressions (IV.15) and (IV.16), respectively.

The existence conditions of such new interfacial SH-wave are those demonstrated in inequalities (VI.14), where the parameter Y for this case is equal to the right-hand side of equation (VI.32). According to the existence conditions, the value of the parameter Y must be larger than zero. However, $e^{\mathrm{I}} = 0$ and $h^{\mathrm{II}} = 0$ definitely results in $Y = 0$, see the equation (VI.32), and therefore, no solution can be found. This can mean that such interfacial SH-wave cannot propagate when the piezoelectric phase of the first PEM half-space and the piezomagnetic phase of the second PEM half-space vanish. On the other hand, $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$ cannot lead to $Y = 0$, see the explicit form of the parameter Y in expression (VI.33) written below.

$$Y=\left(1-\frac{h^{\mathrm{II}}}{h^{\mathrm{I}}}\frac{e^{\mathrm{I}}}{e^{\mathrm{II}}}\right)^{-1}\times\left\{\left(\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_e^{\mathrm{I}}\right)^2}{1+\left(K_e^{\mathrm{II}}\right)^2+\left(K_m^{\mathrm{II}}\right)^2}-\frac{e^{\mathrm{I}}}{e^{\mathrm{II}}}\frac{\left(K_e^{\mathrm{II}}\right)^2}{1+\left(K_e^{\mathrm{II}}\right)^2+\left(K_m^{\mathrm{II}}\right)^2}\right)\left(1+\frac{h^{\mathrm{II}}}{h^{\mathrm{I}}}\right)\right.$$
$$\left.+\left(1+\frac{e^{\mathrm{I}}}{e^{\mathrm{II}}}\right)\left(-\frac{h^{\mathrm{II}}}{h^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_m^{\mathrm{I}}\right)^2}{1+\left(K_e^{\mathrm{II}}\right)^2+\left(K_m^{\mathrm{II}}\right)^2}+\frac{\left(K_m^{\mathrm{II}}\right)^2}{1+\left(K_e^{\mathrm{II}}\right)^2+\left(K_m^{\mathrm{II}}\right)^2}\right)\right\} \qquad \text{(VI.33)}$$

CHAPTER VII

The Case of $\varphi^{\mathrm{I}} = \varphi^{\mathrm{II}}$, $D_3^{\mathrm{I}} = D_3^{\mathrm{II}}$, and $B_3^{\mathrm{I}} = B_3^{\mathrm{II}} = 0$ at $x_3 = 0$

For this case, the mechanical and electrical boundary conditions are those used in the previous chapter, see equations from (VI.1) to (VI.4). The magnetic boundary conditions represent the magnetically closed interface, namely $B_3^{\mathrm{I}} = 0$ and $B_3^{\mathrm{II}} = 0$ at $x_3 = 0$ and therefore, the corresponding equations such as equations (III.5) and (III.6) can be borrowed from Chapter III. These six homogeneous equations can be then written as follows:

$$F^{\mathrm{I}2} U^{0\mathrm{I}(5)} - F^{\mathrm{II}2} U^{0\mathrm{II}(6)} = 0 \tag{VII.1}$$

$$\begin{aligned} &F^{\mathrm{I}}\left[e^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] + F^{\mathrm{I}2} b^{\mathrm{I}}\left[C^{\mathrm{I}} U^{0\mathrm{I}(5)} + e^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] \\ &+ F^{\mathrm{II}}\left[e^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] + F^{\mathrm{II}2} b^{\mathrm{II}}\left[C^{\mathrm{II}} U^{0\mathrm{II}(6)} + e^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \end{aligned} \tag{VII.2}$$

$$F^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + F^{\mathrm{I}2}\varphi^{0\mathrm{I}(5)} - F^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} - F^{\mathrm{II}2}\varphi^{0\mathrm{II}(6)} = 0 \tag{VII.3}$$

$$\begin{aligned} &F^{\mathrm{I}}\left[\varepsilon^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + \alpha^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] - F^{\mathrm{I}2} b^{\mathrm{I}}\left[e^{\mathrm{I}} U^{0\mathrm{I}(5)} - \varepsilon^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} - \alpha^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] \\ &+ F^{\mathrm{II}}\left[\varepsilon^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + \alpha^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] - F^{\mathrm{II}2} b^{\mathrm{II}}\left[e^{\mathrm{II}} U^{0\mathrm{II}(6)} - \varepsilon^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} - \alpha^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \end{aligned} \tag{VII.4}$$

$$F^{\mathrm{I}}\left[\alpha^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + \mu^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] - F^{\mathrm{I}2} b^{\mathrm{I}}\left[h^{\mathrm{I}} U^{0\mathrm{I}(5)} - \alpha^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} - \mu^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] = 0 \tag{VII.5}$$

$$-F^{\mathrm{II}}\left[\alpha^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + \mu^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] + F^{\mathrm{II}2} b^{\mathrm{II}}\left[h^{\mathrm{II}} U^{0\mathrm{II}(6)} - \alpha^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} - \mu^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \tag{VII.6}$$

These six fundamental equations written above must be further transformed. There are three possibilities to transform them in different ways due to the existence of different sets of the eigenvector components. Therefore, three following subsections are responsible for the studies of these three possibilities.

VII.1. *The first sets of the eigenvector components*

In the case of the first eigenvectors for the first and second PEM half-spaces shown in figure I.1, it is convenient that the first four equations can be borrowed from the previous chapter, namely equations from (VI.7) to (VI.10). In the same manner, the last two equations can be borrowed from Chapter III, namely equations (III.11) and (III.12). Consequently, they can be rewritten as follows:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad \text{(VII.7)}$$

$$\begin{aligned}&F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]\\&+F^{\mathrm{II}}\left[e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]=0\end{aligned} \quad \text{(VII.8)}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\alpha^{\mathrm{I}}+F^{\mathrm{I}2}e^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}}e^{\mathrm{I}}\alpha^{\mathrm{II}}-F^{\mathrm{II}2}e^{\mathrm{I}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad \text{(VII.9)}$$

$$F^{\mathrm{I}}\times 0-F^{\mathrm{I}2}b^{\mathrm{I}}\times 0+F^{\mathrm{II}}\times 0-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \quad \text{(VII.10)}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\varepsilon^{\mathrm{I}}+F^{\mathrm{I}2}b^{\mathrm{I}}\times 0=0 \quad \text{(VII.11)}$$

$$F^{\mathrm{II}}\left(e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right)+F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \quad \text{(VII.12)}$$

It is usual in this theoretical study to successively subtract equations (VII.9), (VII.10), (VII.11), and (VII.12) from equation (VII.8). Also, it is crucial to account expression (VII.7) and the following equality $F^{\mathrm{I}} = F^{\mathrm{II}} = 0$, see equations (VII.11) and (VII.12). As a result, modified equation (VII.8) can be represented in the following form for determination of the velocity V_{new11} of the eleventh new interfacial SH-wave:

$$\begin{aligned}&\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new11}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new11}}{V_{tem}^{\mathrm{II}}}\right)^2}\\&=\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}-\frac{e^{\mathrm{I}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\end{aligned} \quad \text{(VII.13)}$$

where the coefficients $\left(K_\alpha^{\mathrm{I}}\right)^2$ and $\left(K_\alpha^{\mathrm{II}}\right)^2$ are defined by expressions (IV.15) and (IV.16), respectively.

Equation (VII.13) must also satisfy the following well-known existence conditions:

$$Y > 0 \text{ and } \left(V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}\right)^2 > 1 - Y^2 \tag{VII.14}$$

where the parameter Y can be defined by

$$Y = \frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2 - \left(K_\alpha^{\mathrm{I}}\right)^2}{1 + \left(K_{em}^{\mathrm{II}}\right)^2} - \frac{e^{\mathrm{I}}}{e^{\mathrm{II}}}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}} - h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2 - \left(K_\alpha^{\mathrm{II}}\right)^2}{1 + \left(K_{em}^{\mathrm{II}}\right)^2} \tag{VII.15}$$

It is clearly seen in expression (VII.15) that the value of the parameter Y is equal to zero as soon as the piezoelectric constant e^{I} of the first piezoelectromagnetics equals to zero. This can mean that the piezoelectric phase of the first piezoelectromagnetics, which should possesses the smaller value of SH-BAW velocity V_{tem}^{I}, can completely response for the existence of such new interfacial SH-wave. On the other hand, $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$ lead to the following simplified form of the parameter Y:

$$Y = \frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_e^{\mathrm{I}}\right)^2}{1 + \left(K_e^{\mathrm{II}}\right)^2 + \left(K_m^{\mathrm{II}}\right)^2} - \frac{e^{\mathrm{I}}}{e^{\mathrm{II}}}\frac{\left(K_e^{\mathrm{II}}\right)^2}{1 + \left(K_e^{\mathrm{II}}\right)^2 + \left(K_m^{\mathrm{II}}\right)^2} \tag{VII.16}$$

VII.2. *The second sets of the eigenvector components*

In this case of the second eigenvectors for the first and second PEM half-spaces, it is also convenient to borrow four equations from (VI.15) to (VI.18) from the previous chapter. These equations correspond to the mechanical and electrical boundary conditions for this case. The magnetic boundary conditions are described by equations (III.18) and (III.19) from Chapter III and therefore, they can be also borrowed. Thus, the corresponding system of the six homogeneous equations is composed as follows:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad \text{(VII.17)}$$

$$\begin{aligned}&F^{\mathrm{I}}\left[e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]\\&+F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]=0\end{aligned} \quad \text{(VII.18)}$$

$$F^{\mathrm{I}}+F^{\mathrm{I}2}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{m}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}}\frac{\mu^{\mathrm{II}}}{\mu^{\mathrm{I}}}-F^{\mathrm{II}2}\frac{\mu^{\mathrm{II}}}{\mu^{\mathrm{I}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad \text{(VII.19)}$$

$$F^{\mathrm{II}}+F^{\mathrm{I}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}-F^{\mathrm{I}2}b^{\mathrm{I}}\times 0-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \quad \text{(VII.20)}$$

$$F^{\mathrm{I}}\times 0-F^{\mathrm{I}2}b^{\mathrm{I}}\times 0=0 \quad \text{(VII.21)}$$

$$F^{\mathrm{II}}\times 0-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \quad \text{(VII.22)}$$

It is obvious that equations (VII.21) and (VII.22) vanish and equation (VII.20) defines the relationship between the weight factors F^{I} and F^{II}. Employing this relationship and the other relationship defined by expression (VII.17), equation (VII.18) can be transformed into the following relatively compact form which can determine the velocity V_{new12} of the twelfth new interfacial SH-wave:

$$\begin{aligned}&\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new12}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new12}}{V_{tem}^{\mathrm{II}}}\right)^2}=\left(\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}-\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\right)\\&\times\left\{\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}-\frac{\mu^{\mathrm{I}}}{\mu^{\mathrm{II}}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{m}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\right\}\Bigg/\left(\frac{\mu^{\mathrm{I}}}{\mu^{\mathrm{II}}}+\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}\right)\end{aligned} \quad \text{(VII.23)}$$

The existence conditions for equation (VII.23) can be also determined in the similar manner. Indeed, the existence conditions can be also defined by inequalities (VII.14) written in the previous subsection of this chapter. Also, $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$ results in the following relatively simple form of the parameter Y:

$$Y = \frac{\varepsilon^{\mathrm{I}} e^{\mathrm{II}} - \varepsilon^{\mathrm{II}} e^{\mathrm{I}}}{\varepsilon^{\mathrm{I}} + \varepsilon^{\mathrm{II}}} \left\{ \frac{\left(K_e^{\mathrm{II}}\right)^2 / e^{\mathrm{II}}}{1 + \left(K_e^{\mathrm{II}}\right)^2 + \left(K_m^{\mathrm{II}}\right)^2} - \frac{C^{\mathrm{I}}}{C^{\mathrm{II}}} \frac{\left(K_e^{\mathrm{I}}\right)^2 / e^{\mathrm{I}}}{1 + \left(K_e^{\mathrm{II}}\right)^2 + \left(K_m^{\mathrm{II}}\right)^2} \right\} \tag{VII.24}$$

It is clearly seen in expression (VII.24) that $Y = 0$ occurs for $e^{\mathrm{I}} = 0$ and $e^{\mathrm{II}} = 0$, but $h^{\mathrm{I}} = 0$ and $h^{\mathrm{II}} = 0$ cannot give $Y = 0$. So, the piezoelectric and magnetoelectric effects can be responsible for the existence of such new interfacial SH-wave.

VII.3. *The combination of the first and second sets*

Indeed, it is indispensable to theoretically investigate the third possible case when the first eigenvectors are utilized for the first PEM half-space and the second ones are chosen for the second PEM half-space. For this situation, the suitable equations for the mechanical and electrical boundary conditions are equations from (VI.22) to (VI.25) given in the previous chapter. Also, the suitable magnetic boundary conditions are written down in equations (VII.11) and (VII.22) from the first and second subsections of this chapter, respectively. Therefore, one can write the following six homogeneous equations:

$$F^{\mathrm{I}2} \frac{e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2} - F^{\mathrm{II}2} \frac{e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2} = 0 \tag{VII.25}$$

$$\begin{aligned} &F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right] + F^{\mathrm{I}2} b^{\mathrm{I}} \left[C^{\mathrm{I}} \frac{e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2} + e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}} \right] \\ &+ F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}\right] + F^{\mathrm{II}2} b^{\mathrm{II}} \left[C^{\mathrm{II}} \frac{e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2} + e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}} \right] = 0 \end{aligned} \tag{VII.26}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\alpha^{\mathrm{I}}+F^{\mathrm{I}2}e^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}}e^{\mathrm{I}}\mu^{\mathrm{II}}-F^{\mathrm{II}2}e^{\mathrm{I}}\mu^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad \text{(VII.27)}$$

$$F^{\mathrm{II}}\left(e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right)-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0+F^{\mathrm{I}}\times 0-F^{\mathrm{I}2}b^{\mathrm{I}}\times 0=0 \quad \text{(VII.28)}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\varepsilon^{\mathrm{I}}+F^{\mathrm{I}2}b^{\mathrm{I}}\times 0=0 \quad \text{(VII.29)}$$

$$F^{\mathrm{II}}\times 0-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \quad \text{(VII.30)}$$

It is natural to account relationship (VII.25) for equations (VII.26) and (VII.27) and then to successively subtract equations (VII.27), (VII.28), and (VII.29) from equation (VII.26). As a result, the following final form can be obtained, with which one can calculate the velocity V_{new13} of the thirteenth new interfacial SH-wave:

$$\begin{aligned}&\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new13}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new13}}{V_{tem}^{\mathrm{II}}}\right)^2}\\ &=\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}-\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\end{aligned} \quad \text{(VII.31)}$$

where the coefficient $\left(K_{\alpha}^{\mathrm{I}}\right)^2$ is defined by expression (IV.15).

The right-hand side of equation (VII.31) can be denoted by Y. The complicated parameter Y plays an important role in equalities (VII.14) which represent the existence conditions. It is blatant that $e^{\mathrm{I}} = 0$ certainly results in $Y = 0$ and therefore, such interfacial SH-wave cannot propagate because $Y > 0$ must occur due to a definitely positive sign of the left-hand side of equation (VII.31). Besides, the value of the parameter Y in the case of both the material parameters $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$ is defined by equation (VII.16) from the first subsection of this chapter.

CHAPTER VIII

The Case of $\varphi^{I} = \varphi^{II}$, $D_3^{I} = D_3^{II}$, and $\psi^{I} = \psi^{II} = 0$ at $x_3 = 0$

Consider the mechanical boundary conditions such as $U^{I} = U^{II}$ and $(\sigma_{32})^{I} = (\sigma_{32})^{II}$, the electrical ones such as $\varphi^{I} = \varphi^{II}$ and $D_3^{I} = D_3^{II}$, and the magnetic ones such as $\psi^{I} = 0$ and $\psi^{II} = 0$ representing the magnetically open interface at $x_3 = 0$. The corresponding equations are those from (VI.1) to (VI.4) written in Chapter VI and equations (II.5) and (II.6) written in Chapter II. For this case, these six homogeneous equations can be formed as follows:

$$F^{I2}U^{0I(5)} - F^{II2}U^{0II(6)} = 0 \tag{VIII.1}$$

$$\begin{aligned} &F^{I}\left[e^{I}\varphi^{0I(3)} + h^{I}\psi^{0I(3)}\right] + F^{I2}b^{I}\left[C^{I}U^{0I(5)} + e^{I}\varphi^{0I(5)} + h^{I}\psi^{0I(5)}\right] \\ &+ F^{II}\left[e^{II}\varphi^{0II(4)} + h^{II}\psi^{0II(4)}\right] + F^{II2}b^{II}\left[C^{II}U^{0II(6)} + e^{II}\varphi^{0II(6)} + h^{II}\psi^{0II(6)}\right] = 0 \end{aligned} \tag{VIII.2}$$

$$F^{I}\varphi^{0I(3)} + F^{I2}\varphi^{0I(5)} - F^{II}\varphi^{0II(4)} - F^{II2}\varphi^{0II(6)} = 0 \tag{VIII.3}$$

$$\begin{aligned} &F^{I}\left[\varepsilon^{I}\varphi^{0I(3)} + \alpha^{I}\psi^{0I(3)}\right] - F^{I2}b^{I}\left[e^{I}U^{0I(5)} - \varepsilon^{I}\varphi^{0I(5)} - \alpha^{I}\psi^{0I(5)}\right] \\ &+ F^{II}\left[\varepsilon^{II}\varphi^{0II(4)} + \alpha^{II}\psi^{0II(4)}\right] - F^{II2}b^{II}\left[e^{II}U^{0II(6)} - \varepsilon^{II}\varphi^{0II(6)} - \alpha^{II}\psi^{0II(6)}\right] = 0 \end{aligned} \tag{VIII.4}$$

$$F^{I}\psi^{0I(3)} + F^{I2}\psi^{0I(5)} = 0 \tag{VIII.5}$$

$$F^{II}\psi^{0II(4)} + F^{II2}\psi^{0II(6)} = 0 \tag{VIII.6}$$

It is overt that these equations can reveal the suitable phase velocity of the interfacial SH-wave. However, the problem of the existence of the interfacial SH-wave guided by the common interface between two dissimilar piezoelectromagnetics actually splits into three possibilities which must be recorded. Indeed, the problem is significantly complicated because of the existence of two different eigenvectors for either of the piezoelectromagnetics. Therefore, the following three subsections have the purpose to provide the appropriate theory for each case.

VIII.1. *The first sets of the eigenvector components*

It is natural to choose the corresponding first eigenvectors for the first and second PEM half-spaces to commence the analysis. For this purpose, it is convenient to use already transformed equations from (VI.7) to (VI.10) which describe the mechanical and electrical boundary conditions. Also, the suitable magnetic boundary conditions for this case are described by expressions (II.11) and (II.12) in Chapter II. For this case, the corresponding six homogeneous equations can e composed in the following forms:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \qquad \text{(VIII.7)}$$

$$\begin{gathered}F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]\\+F^{\mathrm{II}}\left[e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]=0\end{gathered} \qquad \text{(VIII.8)}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\alpha^{\mathrm{I}}+F^{\mathrm{I}2}e^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}}e^{\mathrm{I}}\alpha^{\mathrm{II}}-F^{\mathrm{II}2}e^{\mathrm{I}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \qquad \text{(VIII.9)}$$

$$F^{\mathrm{I}}\times 0-F^{\mathrm{I}2}b^{\mathrm{I}}\times 0+F^{\mathrm{II}}\times 0-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \qquad \text{(VIII.10)}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\varepsilon^{\mathrm{I}}-F^{\mathrm{I}2}h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}=0 \qquad \text{(VIII.11)}$$

$$F^{\mathrm{II}}\left(e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right)+F^{\mathrm{II}2}\left(e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right)\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \qquad \text{(VIII.12)}$$

It is blatant that equations (VIII.9) and (VIII.11) can be subtracted from equation (VIII.8) and the last equation can be further modified by using relationships (VIII.7) and (VIII.12). Consequently, the modified equation can be further simplified. The final form which can reveal the velocity V_{new14} of the fourteenth new interfacial SH-wave can be written as follows:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new14}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new14}}{V_{tem}^{\mathrm{II}}}\right)^2}$$
$$=\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{e^{\mathrm{I}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\frac{\left(K_{\alpha}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2} \quad \text{(VIII.13)}$$

where the coefficient $\left(K_{\alpha}^{\mathrm{II}}\right)^2$ is defined by expression (IV.16).

It is obvious that equation (VIII.13) must also have the existence conditions. These conditions can be written as the following inequalities:

$$Y>0 \text{ and } \left(V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}\right)^2>1-Y^2 \quad \text{(VIII.14)}$$

where the parameter Y is defined by the following equality:

$$Y=\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{e^{\mathrm{I}}}{e^{\mathrm{II}}}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\frac{\left(K_{\alpha}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2} \quad \text{(VIII.15)}$$

The the parameter Y represents the right-hand side of equation (VIII.13). Therefore, it is normal to require $Y > 0$ because it is clearly seen that the left-hand side of equation (VIII.13) cannot have a negative sign. This can be true because it is possible to assume that $\left(K_{em}^{\mathrm{I}}\right)^2>0$ and $\left(K_{em}^{\mathrm{II}}\right)^2>0$ usually occur. It is also possible to discuss the case of $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$. This can significantly simplify the form of the parameter Y. Therefore, the simplified parameter Y reads:

$$Y=\frac{\left(K_{m}^{\mathrm{II}}\right)^2-e^{\mathrm{I}}\left(K_{e}^{\mathrm{II}}\right)^2/e^{\mathrm{II}}}{1+\left(K_{e}^{\mathrm{II}}\right)^2+\left(K_{m}^{\mathrm{II}}\right)^2}+\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{e}^{\mathrm{I}}\right)^2+\left(K_{m}^{\mathrm{I}}\right)^2}{1+\left(K_{e}^{\mathrm{II}}\right)^2+\left(K_{m}^{\mathrm{II}}\right)^2} \quad \text{(VIII.16)}$$

One can check that the piezoelectric constants such as $e^{\mathrm{I}} = 0$ and $e^{\mathrm{II}} = 0$ can definitely lead to the fact that equation (VIII.13) can reduce to equation (II.38) from

Chapter II. It is necessary to mention that equation (II.38) determines the speed of the corresponding interfacial MT-wave [98] guided by the common interface of two dissimilar piezomagnetics. However, $h^{\mathrm{I}} = 0$ and $h^{\mathrm{II}} = 0$ in expression (VIII.16) cannot lead to the corresponding interfacial MT-wave solution.

VIII.2. *The second sets of the eigenvector components*

Utilizing the corresponding second eigenvectors, the mechanical and electrical boundary conditions can be written following equations (VI.15), (VI.16), (VI.17), and (VI.18) from Chapter VI. Besides, the magnetic boundary conditions for this case such as $\psi^{\mathrm{I}} = 0$ and $\psi^{\mathrm{II}} = 0$ can be also borrowed from the second chapter. The suitable equations are equations (II.47) and (II.48). So, the following forms of the six homogeneous equations can be used for the further transformations:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \qquad \text{(VIII.17)}$$

$$\begin{aligned}&F^{\mathrm{I}}\left[e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]\\&+F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]=0\end{aligned} \qquad \text{(VIII.18)}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\mu^{\mathrm{I}}+F^{\mathrm{I}2}e^{\mathrm{I}}\mu^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{m}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}}e^{\mathrm{I}}\mu^{\mathrm{II}}-F^{\mathrm{II}2}e^{\mathrm{I}}\mu^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \qquad \text{(VIII.19)}$$

$$F^{\mathrm{II}}e^{\mathrm{II}}\mu^{\mathrm{II}}+F^{\mathrm{I}}e^{\mathrm{II}}\mu^{\mathrm{II}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}-F^{\mathrm{I}2}b^{\mathrm{I}}\times 0-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \qquad \text{(VIII.20)}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\alpha^{\mathrm{I}}-F^{\mathrm{I}2}h^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}=0 \qquad \text{(VIII.21)}$$

$$-F^{\mathrm{II}}h^{\mathrm{II}}\alpha^{\mathrm{II}}-F^{\mathrm{II}2}h^{\mathrm{II}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \qquad \text{(VIII.22)}$$

Equations (VIII.19), (VIII.20), (VIII.21), and (VIII.22) must be subtracted from equation (VIII.18) to get a modified equation. Also, it is necessary to use relationship (VIII.17) to get a complicate form of the modified equation. However, it is possible to obtain the following two equations instead of the four ones:

$$F^{\mathrm{I}}\left(e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right)+F^{\mathrm{I2}}\left(e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right)-F^{\mathrm{II}}e^{\mathrm{I}}\mu^{\mathrm{II}}-F^{\mathrm{II2}}e^{\mathrm{I}}\mu^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad \text{(VIII.23)}$$

$$F^{\mathrm{II}}\left(e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right)-F^{\mathrm{II2}}h^{\mathrm{II}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}+F^{\mathrm{I}}e^{\mathrm{II}}\mu^{\mathrm{II}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}=0 \quad \text{(VIII.24)}$$

These two new equations can be properly transformed before a subtraction from equation (VIII.18). As a result, they read:

$$\begin{aligned}&F^{\mathrm{I}}\left(e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right)\left(1+\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}}{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}\right)+F^{\mathrm{I2}}\left(e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right)\\&-F^{\mathrm{II2}}\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}h^{\mathrm{II}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}-F^{\mathrm{II2}}e^{\mathrm{I}}\mu^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0\end{aligned} \quad \text{(VIII.25)}$$

$$\begin{aligned}&F^{\mathrm{II}}\left(e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right)\left(1+\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}}{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}\right)-F^{\mathrm{II2}}h^{\mathrm{II}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}\\&+\left(F^{\mathrm{II2}}e^{\mathrm{I}}\mu^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}-F^{\mathrm{I2}}\left(e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right)\right)\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}}{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}=0\end{aligned} \quad \text{(VIII.26)}$$

Finally, they can be subtracted from equation (VIII.18) and several further transformations can lead to the following simplified form for determination of the velocity V_{new15} of the fifteenth new interfacial SH-wave:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new15}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new15}}{V_{tem}^{\mathrm{II}}}\right)^2}=\left\{\left(1-\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}}{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}\right)\right.$$
$$\times\left(\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}-\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\right)-\left(1+\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\right)$$
$$\left.\times\frac{h^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\right/\left(1+\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}}{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}\right) \qquad \text{(VIII.27)}$$

where the coefficient $\left(K_{\alpha}^{\mathrm{II}}\right)^2$ is defined by expression (IV.16).

For this case, the parameter Y representing the right-hand side of equation (VIII.27) is quite complicated. It is mentioned that the parameter Y is used in the existence conditions defined by inequalities (VIII.14) from the previous subsection. Also, $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$ can be realized and one can readily perform this simplification to get a simple form of the parameter Y. For comparison, the following subsection studies the third case which mixes the eigenvectors.

VIII.3. *The combination of the first and second sets*

It is also vital to investigate the third possibility when the first eigenvectors are exploited for the first PEM half-space and the second ones are employed for the second PEM half-space. The corresponding six homogeneous equations for the case can be also borrowed from the previous studied carried out in this work. For instance, equations (VI.22), (VI.23), (VI.24), and (VI.25) from Chapter VI describe the mechanical and electrical boundary conditions. In addition, equations (VIII.11) and (VIII.22) from the first and second subsections of this chapter, respectively, can be responsible for the magnetic boundary conditions. Thus, the corresponding six homogeneous equations read:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \qquad \text{(VIII.28)}$$

$$F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]+F^{\mathrm{I2}}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]$$
$$+F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]+F^{\mathrm{II2}}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]=0 \quad \text{(VIII.29)}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\alpha^{\mathrm{I}}+F^{\mathrm{I2}}e^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}}e^{\mathrm{I}}\mu^{\mathrm{II}}-F^{\mathrm{II2}}e^{\mathrm{I}}\mu^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad \text{(VIII.30)}$$

$$F^{\mathrm{II}}e^{\mathrm{II}}\mu^{\mathrm{II}}-F^{\mathrm{II2}}b^{\mathrm{II}}\times 0+F^{\mathrm{I}}\times 0-F^{\mathrm{I2}}b^{\mathrm{I}}\times 0=0 \quad \text{(VIII.31)}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\varepsilon^{\mathrm{I}}-F^{\mathrm{I2}}h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}=0 \quad \text{(VIII.32)}$$

$$-F^{\mathrm{II}}h^{\mathrm{II}}\alpha^{\mathrm{II}}-F^{\mathrm{II2}}h^{\mathrm{II}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad \text{(VIII.33)}$$

It is usual procedure to successively subtract the last four equations from the second expression. Using relationship (VIII.28), the final form of the equation for the determination of the velocity V_{new16} of the sixteenth new interfacial SH-wave can be inscribed as follows:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new16}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new16}}{V_{tem}^{\mathrm{II}}}\right)^2}$$
$$=\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}-\frac{h^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}$$
$$-\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{h^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}-\frac{e^{\mathrm{I}}\mu^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2} \quad \text{(VIII.34)}$$

where the coefficient $\left(K_{\alpha}^{\mathrm{II}}\right)^2$ is defined by expression (IV.16).

The existence conditions are defined by inequalities (VIII.14) given in the first subsection. The existence conditions contain the parameter Y which is defined by the right-hand side in equation (VIII.34). For the case of $\alpha^{\mathrm{I}}=0$ and $\alpha^{\mathrm{II}}=0$, the parameter Y can be significantly simplified as follows:

$$Y = \frac{C^{\mathrm{I}}}{C^{\mathrm{II}}} \frac{\left(K_e^{\mathrm{I}}\right)^2 + \left(K_m^{\mathrm{I}}\right)^2}{1 + \left(K_e^{\mathrm{II}}\right)^2 + \left(K_m^{\mathrm{II}}\right)^2} + \frac{\left(K_m^{\mathrm{II}}\right)^2}{1 + \left(K_e^{\mathrm{II}}\right)^2 + \left(K_m^{\mathrm{II}}\right)^2} + \frac{e^{\mathrm{I}}}{e^{\mathrm{II}}} \frac{\left(K_m^{\mathrm{II}}\right)^2 - \left(K_e^{\mathrm{II}}\right)^2}{1 + \left(K_e^{\mathrm{II}}\right)^2 + \left(K_m^{\mathrm{II}}\right)^2} \quad \text{(VIII.35)}$$

In equation (VIII.35), one can also apply $e^{\mathrm{I}} = 0$ and $e^{\mathrm{II}} = 0$ or $h^{\mathrm{I}} = 0$ and $h^{\mathrm{II}} = 0$.

CHAPTER IX

The Case of $D_3^{\mathrm{I}} = D_3^{\mathrm{II}} = 0$, $\psi^{\mathrm{I}} = \psi^{\mathrm{II}}$, and $B_3^{\mathrm{I}} = B_3^{\mathrm{II}}$ at $x_3 = 0$

These mechanical, electrical, and magnetic boundary conditions can be also treated in this theoretical work. The mechanical boundary conditions at the common interface $x_3 = 0$ shown in figure I.1 from the first chapter are $U^{\mathrm{I}} = U^{\mathrm{II}}$ and $(\sigma_{32})^{\mathrm{I}} = (\sigma_{32})^{\mathrm{II}}$, see equations (VI.1) and (VI.2) from Chapter VI. The electrical boundary conditions represent the electrically open interface, namely $D_3^{\mathrm{I}} = 0$ and $D_3^{\mathrm{II}} = 0$ respectively defined by equations (III.3) and (III.4) from Chapter III. Besides, the magnetic boundary conditions are $\psi^{\mathrm{I}} = \psi^{\mathrm{II}}$ and $B_3^{\mathrm{I}} = B_3^{\mathrm{II}}$, see equations (VI.5) and (VI.6). Therefore, one can write the following six homogenous equations:

$$F^{\mathrm{I}2}U^{0\mathrm{I}(5)} - F^{\mathrm{II}2}U^{0\mathrm{II}(6)} = 0 \tag{IX.1}$$

$$\begin{aligned} &F^{\mathrm{I}}\left[e^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] + F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}U^{0\mathrm{I}(5)} + e^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] \\ &+ F^{\mathrm{II}}\left[e^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] + F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}U^{0\mathrm{II}(6)} + e^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \end{aligned} \tag{IX.2}$$

$$F^{\mathrm{I}}\left[\varepsilon^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + \alpha^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] - F^{\mathrm{I}2}b^{\mathrm{I}}\left[e^{\mathrm{I}}U^{0\mathrm{I}(5)} - \varepsilon^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} - \alpha^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] = 0 \tag{IX.3}$$

$$-F^{\mathrm{II}}\left[\varepsilon^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + \alpha^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] + F^{\mathrm{II}2}b^{\mathrm{II}}\left[e^{\mathrm{II}}U^{0\mathrm{II}(6)} - \varepsilon^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} - \alpha^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \tag{IX.4}$$

$$F^{\mathrm{I}}\psi^{0\mathrm{I}(3)} + F^{\mathrm{I}2}\psi^{0\mathrm{I}(5)} - F^{\mathrm{II}}\psi^{0\mathrm{II}(4)} - F^{\mathrm{II}2}\psi^{0\mathrm{II}(6)} = 0 \tag{IX.5}$$

$$\begin{aligned} &F^{\mathrm{I}}\left[\alpha^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + \mu^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] - F^{\mathrm{I}2}b^{\mathrm{I}}\left[h^{\mathrm{I}}U^{0\mathrm{I}(5)} - \alpha^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} - \mu^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] \\ &+ F^{\mathrm{II}}\left[\alpha^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + \mu^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] - F^{\mathrm{II}2}b^{\mathrm{II}}\left[h^{\mathrm{II}}U^{0\mathrm{II}(6)} - \alpha^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} - \mu^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \end{aligned} \tag{IX.6}$$

The equations written above are valid for each of three cases which will be studied in this chapter. These three cases represent three possibilities which can be realized because the first and second PEM half-spaces can possess their own two different eigenvectors. Therefore, three different combinations of the eigenvectors can exist. It is possible to consider the first combination.

IX.1. *The first sets of the eigenvector components*

Consider the corresponding first eigenvectors used for the first and second PEM half-spaces. For this case, the corresponding six transformed homogeneous equations can be written following equations (VI.7), (VI.8), (III.9), (III.10), (VI.11), and (VI.12). For this case, they can be rewritten as follows:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \tag{IX.7}$$

$$\begin{gathered}F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]\\+F^{\mathrm{II}}\left[e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]=0\end{gathered} \tag{IX.8}$$

$$F^{\mathrm{I}}\times 0-F^{\mathrm{I}2}b^{\mathrm{I}}\times 0=0 \tag{IX.9}$$

$$F^{\mathrm{II}}\times 0-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \tag{IX.10}$$

$$F^{\mathrm{I}}+F^{\mathrm{I}2}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}}\frac{\varepsilon^{\mathrm{II}}}{\varepsilon^{\mathrm{I}}}-F^{\mathrm{II}2}\frac{\varepsilon^{\mathrm{II}}}{\varepsilon^{\mathrm{I}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \tag{IX.11}$$

$$F^{\mathrm{II}}+F^{\mathrm{I}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}+F^{\mathrm{I}2}b^{\mathrm{I}}\times 0+F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \tag{IX.12}$$

Employing relationships (IX.7) and (IX.12) for equations (IX.8) and (IX.11), the last two equation can be properly transformed and then subtracted from each other to exclude the weight factor F^{I}. The resulting equation can demonstrate the dependence of the velocity V_{new17} of the seventeenth new interfacial SH-wave on the material parameters. This equation reads:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new17}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new17}}{V_{tem}^{\mathrm{II}}}\right)^2}=\left(\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}-\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\right)$$
$$\times\left(\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}-\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{\varepsilon^{\mathrm{I}}}{\varepsilon^{\mathrm{II}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\right)\Bigg/\left(\frac{\varepsilon^{\mathrm{I}}}{\varepsilon^{\mathrm{II}}}+\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}\right) \quad \text{(IX.13)}$$

This equation must satisfy the following existence conditions used in the previous chapters:

$$Y>0 \text{ and } \left(V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}\right)^2>1-Y^2 \quad \text{(IX.14)}$$

The reader can check that $\alpha^{\mathrm{I}}=0$ and $\alpha^{\mathrm{II}}=0$ significantly simplifies the form of the parameter Y which represents the right-hand side of equation (IX.13).

IX.2. *The second sets of the eigenvector components*

The theoretical consideration of the corresponding second eigenvectors for the first and second PEM half-spaces can be also realized. It is certain that equations (VI.15) and (VI.16), (III.16) and (III.17), (VI.19) and (VI.20) can correspond to the mechanical, electrical, and magnetic boundary conditions used in this case. So, the corresponding six homogeneous equations can be composed as follows:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad \text{(IX.15)}$$

$$F^{\mathrm{I}}\left[e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]$$
$$+F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]=0 \quad \text{(IX.16)}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\mu^{\mathrm{I}}-F^{\mathrm{I}2}b^{\mathrm{I}}\times 0=0 \quad \text{(IX.17)}$$

$$F^{\mathrm{II}}\left(e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right)-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \quad \text{(IX.18)}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\alpha^{\mathrm{I}}-F^{\mathrm{I2}}h^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}+F^{\mathrm{II}}h^{\mathrm{I}}\alpha^{\mathrm{II}}+F^{\mathrm{II2}}h^{\mathrm{I}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad \text{(IX.19)}$$

$$F^{\mathrm{I}}\times 0-F^{\mathrm{I2}}b^{\mathrm{I}}\times 0+F^{\mathrm{II}}\times 0-F^{\mathrm{II2}}b^{\mathrm{II}}\times 0=0 \quad \text{(IX.20)}$$

It is apparent that the last equation cannot take part in the further theoretical analysis. It is natural that equations (IX.17), (IX.18), and (IX.19) must be subtracted from expression (IX.16). Utilizing relationship (IX.15) and equation (IX.18), the latter equation can be further transformed to get the following explicit form for the determination of the velocity V_{new18} of the eighteenth new interfacial SH-wave:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new18}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new18}}{V_{tem}^{\mathrm{II}}}\right)^2}$$
$$=-\frac{h^{\mathrm{I}}\alpha^{\mathrm{I}}}{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{h^{\mathrm{I}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2} \quad \text{(IX.21)}$$

where the coefficients $\left(K_{\alpha}^{\mathrm{I}}\right)^2$ and $\left(K_{\alpha}^{\mathrm{II}}\right)^2$ are defined by expressions (IV.15) and (IV.16), respectively.

The right-hand side of expression (IX.21) represents the parameter Y which must satisfy existence conditions (IX.14) given in the previous subsection. It is clearly seen in expression (IX.21) that $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$ can simplify the form of the parameter Y. Thus, this parameter reads:

$$Y=\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{m}^{\mathrm{I}}\right)^2}{1+\left(K_{e}^{\mathrm{II}}\right)^2+\left(K_{m}^{\mathrm{II}}\right)^2}-\frac{h^{\mathrm{I}}}{h^{\mathrm{II}}}\frac{\left(K_{m}^{\mathrm{II}}\right)^2}{1+\left(K_{e}^{\mathrm{II}}\right)^2+\left(K_{m}^{\mathrm{II}}\right)^2} \quad \text{(IX.22)}$$

IX.3. *The combination of the first and second sets*

It is also possible to utilize the first eigenvectors for the first PEM half-space and the second ones for the second PEM half-space. The equations describing the mechanical boundary conditions can be also borrowed from Chapter VI. They are equations (VI.22) and (VI.23). For this mixed case, the electrical boundary conditions can be represented by expressions (III.9) and (III.17) from Chapter III. The magnetic boundary conditions can be also borrowed from Chapter VI, see equations (VI.26) and (VI.27). Consequently, the six homogeneous equations are as follows.

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad \text{(IX.23)}$$

$$\begin{aligned}&F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]\\&+F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]=0\end{aligned} \quad \text{(IX.24)}$$

$$F^{\mathrm{I}}\times 0-F^{\mathrm{I}2}b^{\mathrm{I}}\times 0=0 \quad \text{(IX.25)}$$

$$F^{\mathrm{II}}e^{\mathrm{II}}\mu^{\mathrm{II}}-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \quad \text{(IX.26)}$$

$$-F^{\mathrm{II}}h^{\mathrm{II}}\alpha^{\mathrm{II}}-F^{\mathrm{II}2}h^{\mathrm{II}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}+F^{\mathrm{I}}h^{\mathrm{II}}\varepsilon^{\mathrm{I}}+F^{\mathrm{I}2}h^{\mathrm{II}}\varepsilon^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}=0 \quad \text{(IX.27)}$$

$$F^{\mathrm{I}}\left(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)+F^{\mathrm{I}2}b^{\mathrm{I}}\times 0-F^{\mathrm{II}}\times 0+F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \quad \text{(IX.28)}$$

Equation (IX.25) can be neglected and equation (IX.28) demonstrates that the new weight factor such as $F^{\mathrm{I}}\left(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)$ can be equal to zero due to $F^{\mathrm{I}}=0$. It is needed to subtract equations (IX.26), (IX.27), and (IX.28) from equation (IX.24). Therefore, the velocity V_{new19} of the nineteenth new interfacial SH-wave can be obtained from the following final form:

$$\begin{aligned}&\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new19}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new19}}{V_{tem}^{\mathrm{II}}}\right)^2}\\&=\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}-\frac{h^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\end{aligned} \quad \text{(IX.29)}$$

where the coefficient $\left(K_{\alpha}^{\mathrm{II}}\right)^2$ is defined by expression (IV.16).

Like the investigations performed in the previous subsections, the existence conditions are defined by inequalities (IX.14) and the parameter Y represents the right-hand side of equation (IX.29). Exploiting $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$, equation (IX.29) can be simplified and written in the following form:

$$Y = -\frac{h^{\mathrm{II}}}{h^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_m^{\mathrm{I}}\right)^2}{1+\left(K_e^{\mathrm{II}}\right)^2+\left(K_m^{\mathrm{II}}\right)^2}+\frac{\left(K_m^{\mathrm{II}}\right)^2}{1+\left(K_e^{\mathrm{II}}\right)^2+\left(K_m^{\mathrm{II}}\right)^2} \qquad \text{(IX.30)}$$

CHAPTER X

The Case of $\varphi^{\mathrm{I}} = \varphi^{\mathrm{II}} = 0$, $\psi^{\mathrm{I}} = \psi^{\mathrm{II}}$, and $B_3^{\mathrm{I}} = B_3^{\mathrm{II}}$ at $x_3 = 0$

Finally, it is reasonable to theoretically investigate the case of $U^{\mathrm{I}} = U^{\mathrm{II}}$, $(\sigma_{32})^{\mathrm{I}} = (\sigma_{32})^{\mathrm{II}}$, $\varphi^{\mathrm{I}} = 0$, $\varphi^{\mathrm{II}} = 0$, $\psi^{\mathrm{I}} = \psi^{\mathrm{II}}$, and $B_3^{\mathrm{I}} = B_3^{\mathrm{II}}$ at $x_3 = 0$. Therefore, equations (VI.1) and (VI.2), equations (II.3) and (II.4), and equations (VI.5) and (VI.6) can be used for the mechanical, electrical, and magnetic boundary conditions, respectively. In this case, $\varphi^{\mathrm{I}} = 0$ and $\varphi^{\mathrm{II}} = 0$ determine the electrically closed interface. Thus, the six homogeneous equations can be composed as follows:

$$F^{\mathrm{I}2}U^{0\mathrm{I}(5)} - F^{\mathrm{II}2}U^{0\mathrm{II}(6)} = 0 \tag{X.1}$$

$$\begin{aligned} &F^{\mathrm{I}}\left[e^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] + F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}U^{0\mathrm{I}(5)} + e^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} + h^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] \\ &+ F^{\mathrm{II}}\left[e^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] + F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}U^{0\mathrm{II}(6)} + e^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} + h^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \end{aligned} \tag{X.2}$$

$$F^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + F^{\mathrm{I}2}\varphi^{0\mathrm{I}(5)} = 0 \tag{X.3}$$

$$F^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + F^{\mathrm{II}2}\varphi^{0\mathrm{II}(6)} = 0 \tag{X.4}$$

$$F^{\mathrm{I}}\psi^{0\mathrm{I}(3)} + F^{\mathrm{I}2}\psi^{0\mathrm{I}(5)} - F^{\mathrm{II}}\psi^{0\mathrm{II}(4)} - F^{\mathrm{II}2}\psi^{0\mathrm{II}(6)} = 0 \tag{X.5}$$

$$\begin{aligned} &F^{\mathrm{I}}\left[\alpha^{\mathrm{I}}\varphi^{0\mathrm{I}(3)} + \mu^{\mathrm{I}}\psi^{0\mathrm{I}(3)}\right] - F^{\mathrm{I}2}b^{\mathrm{I}}\left[h^{\mathrm{I}}U^{0\mathrm{I}(5)} - \alpha^{\mathrm{I}}\varphi^{0\mathrm{I}(5)} - \mu^{\mathrm{I}}\psi^{0\mathrm{I}(5)}\right] \\ &+ F^{\mathrm{II}}\left[\alpha^{\mathrm{II}}\varphi^{0\mathrm{II}(4)} + \mu^{\mathrm{II}}\psi^{0\mathrm{II}(4)}\right] - F^{\mathrm{II}2}b^{\mathrm{II}}\left[h^{\mathrm{II}}U^{0\mathrm{II}(6)} - \alpha^{\mathrm{II}}\varphi^{0\mathrm{II}(6)} - \mu^{\mathrm{II}}\psi^{0\mathrm{II}(6)}\right] = 0 \end{aligned} \tag{X.6}$$

It is vital to demonstrate that the system of these homogeneous equations can have some solutions. This is the main purpose of this study. Like the theoretical treatments of the previous chapters, it is possible to start the analysis using the first eigenvectors for the first and second PEM half-spaces.

X.1. *The first sets of the eigenvector components*

Exploiting the first eigenvectors for the first and second PEM half-spaces, the six homogenous equations written above can be further transformed. It is also natural to borrow them from the previous studies. For example, equations (VI.7) and (VI.8) describing the mechanical boundary conditions can be borrowed from Chapter VI. Equations (II.9) and (II.10) from the second chapter can describe the electrical boundary conditions. The magnetic boundary conditions can be defined by equations (VI.11) and (VI.12). As a result, the six modified homogeneous equations can be written in the following forms:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \tag{X.7}$$

$$\begin{aligned}&F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right]\\&+F^{\mathrm{II}}\left[e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right]=0\end{aligned} \tag{X.8}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\alpha^{\mathrm{I}}+F^{\mathrm{I}2}e^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}=0 \tag{X.9}$$

$$F^{\mathrm{II}}e^{\mathrm{II}}\alpha^{\mathrm{II}}+F^{\mathrm{II}2}e^{\mathrm{II}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \tag{X.10}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\varepsilon^{\mathrm{I}}-F^{\mathrm{I}2}h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}+F^{\mathrm{II}}h^{\mathrm{I}}\varepsilon^{\mathrm{II}}+F^{\mathrm{II}2}h^{\mathrm{I}}\varepsilon^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \tag{X.11}$$

$$-F^{\mathrm{II}}h^{\mathrm{II}}\varepsilon^{\mathrm{II}}-F^{\mathrm{I}}h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}+F^{\mathrm{I}2}b^{\mathrm{I}}\times 0+F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \tag{X.12}$$

It is obvious that it is probably convenient to work with the new weight factors such as $F^{\mathrm{I}}(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}})$ and $F^{\mathrm{II}}(e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}})$ instead of the weight factors F^{I} and F^{II} because equation (X.8) contains them. For this purpose, equation (X.9) must be added to equation (X.11) and equation (X.10) must be added to equation (X.12). As a result, the following two equations together with equation (X.8) can form a new system of three homogeneous equations instead of the six ones:

$$F^{\mathrm{I}}\left(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)+F^{\mathrm{I}2}\left(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)$$
$$+F^{\mathrm{II}}\left(e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right)\frac{h^{\mathrm{I}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}+F^{\mathrm{II}2}h^{\mathrm{I}}\varepsilon^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \quad \text{(X.13)}$$

$$F^{\mathrm{II}}\left(e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right)+F^{\mathrm{II}2}e^{\mathrm{II}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}$$
$$-F^{\mathrm{I}}\left(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}=0 \quad \text{(X.14)}$$

Equations (X.13) and (X.14) contain only the new weight factors $F^{\mathrm{I}}(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}})$ and $F^{\mathrm{II}}(e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}})$. Using these equations, it is possible to properly transform them in order to have $F^{\mathrm{I}}(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}})$ in one equation and $F^{\mathrm{II}}(e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}})$ in the other. As a result, the weight factors $F^{\mathrm{I}}(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}})$ and $F^{\mathrm{II}}(e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}})$ can be defined as follows:

$$F^{\mathrm{II}}\left(e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\right)\left(1+\frac{h^{\mathrm{I}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}\right)$$
$$+\left(F^{\mathrm{I}2}+F^{\mathrm{II}2}\frac{h^{\mathrm{I}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}\right)h^{\mathrm{II}}\varepsilon^{\mathrm{II}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2} \quad \text{(X.15)}$$
$$+F^{\mathrm{II}2}e^{\mathrm{II}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0$$

$$F^{\mathrm{I}}\left(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)\left(1+\frac{h^{\mathrm{I}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}\right)$$
$$+F^{\mathrm{I}2}\left(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)-F^{\mathrm{II}2}h^{\mathrm{I}}\varepsilon^{\mathrm{II}}\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2} \quad \text{(X.16)}$$
$$+F^{\mathrm{II}2}h^{\mathrm{I}}\varepsilon^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0$$

Subsequently, equations (X.15) and (X.15) are ready to use in equation (X.8) in order to exclude the weight factors $F^{\mathrm{I}}(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}})$ and $F^{\mathrm{II}}(e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}})$. Employing the relationship between the weight factors F^{I2} and F^{II2}, the final form of the equation for the determination of the velocity V_{new20} of the twentieth new interfacial SH-wave can be introduced as follows:

$$\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new20}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new20}}{V_{tem}^{\mathrm{II}}}\right)^2}=\left\{\left(1+\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}\right)\right.$$
$$\times\left(\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{h^{\mathrm{I}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{e}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\right)+\left(1-\frac{h^{\mathrm{I}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\right) \quad \text{(X.17)}$$
$$\left.\times\frac{e^{\mathrm{II}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\right\}\Bigg/\left(1+\frac{h^{\mathrm{I}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}{e^{\mathrm{II}}\alpha^{\mathrm{II}}-h^{\mathrm{II}}\varepsilon^{\mathrm{II}}}\frac{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}-\left(\alpha^{\mathrm{I}}\right)^2}{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}-\left(\alpha^{\mathrm{II}}\right)^2}\right)$$

where the coefficient $\left(K_{\alpha}^{\mathrm{II}}\right)^2$ is defined by expression (IV.16).

The existence conditions can be written as follows:

$$Y>0 \text{ and } \left(V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}\right)^2>1-Y^2 \quad \text{(X.18)}$$

where the parameter Y is equal to the right-hand side of equation (X.17). It can be also written in the following simplified form as soon as $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$:

$$Y=\left\{\left(\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{e}^{\mathrm{I}}\right)^2+\left(K_{m}^{\mathrm{I}}\right)^2}{1+\left(K_{e}^{\mathrm{II}}\right)^2+\left(K_{m}^{\mathrm{II}}\right)^2}-\frac{h^{\mathrm{I}}}{h^{\mathrm{II}}}\frac{\left(K_{m}^{\mathrm{II}}\right)^2}{1+\left(K_{e}^{\mathrm{II}}\right)^2+\left(K_{m}^{\mathrm{II}}\right)^2}\right)\left(1-\frac{h^{\mathrm{II}}}{h^{\mathrm{I}}}\frac{\mu^{\mathrm{I}}}{\mu^{\mathrm{II}}}\right)\right.$$
$$\left.+\left(1+\frac{h^{\mathrm{I}}}{h^{\mathrm{II}}}\right)\frac{\left(K_{e}^{\mathrm{II}}\right)^2}{1+\left(K_{e}^{\mathrm{II}}\right)^2+\left(K_{m}^{\mathrm{II}}\right)^2}\right\}\Bigg/\left(1+\frac{\mu^{\mathrm{I}}}{\mu^{\mathrm{II}}}\right) \quad \text{(X.19)}$$

It is flagrant that $h^{\mathrm{I}} = 0$ and $h^{\mathrm{II}} = 0$ cannot be realized because $Y \to \infty$. However, $e^{\mathrm{I}} = 0$ and $e^{\mathrm{II}} = 0$ can further simplify the form of the parameter Y. This can mean that the piezomagnetic phases of the first and second PEM half-spaces are responsible for the existence of such new interfacial SH-wave.

X.2. *The second sets of the eigenvector components*

It is essential to investigate the case when the corresponding second sets can be used for the first and second PEM half-spaces. Using equations (VI.15), (VI.16), (II.45), (II.46), (VI.19), and (VI.20), the six modified homogeneous equations can be written down as follows:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}-F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}=0 \tag{X.20}$$

$$\begin{aligned}&F^{\mathrm{I}}\left[e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]+F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2}+e^{\mathrm{I}}\mu^{\mathrm{I}}-h^{\mathrm{I}}\alpha^{\mathrm{I}}\right]\\&+F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]+F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2}+e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right]=0\end{aligned} \tag{X.21}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\mu^{\mathrm{I}}+F^{\mathrm{I}2}e^{\mathrm{I}}\mu^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{m}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}=0 \tag{X.22}$$

$$F^{\mathrm{II}}\left(e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right)+F^{\mathrm{II}2}\left(e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right)\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \tag{X.23}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\alpha^{\mathrm{I}}-F^{\mathrm{I}2}h^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}+F^{\mathrm{II}}h^{\mathrm{I}}\alpha^{\mathrm{II}}+F^{\mathrm{II}2}h^{\mathrm{I}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2}=0 \tag{X.24}$$

$$F^{\mathrm{I}}\times 0-F^{\mathrm{I}2}b^{\mathrm{I}}\times 0+F^{\mathrm{II}}\times 0-F^{\mathrm{II}2}b^{\mathrm{II}}\times 0=0 \tag{X.25}$$

It is blatant that equation (X.25) can be neglected and three equations (X.22), (X.23) and (X.24) must be subtracted from equation (X.21). As a result, the following equation can determine the velocity V_{new21} of the twenty-first new interfacial SH-wave:

$$\begin{aligned}&\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new21}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new21}}{V_{tem}^{\mathrm{II}}}\right)^2}\\&=\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{\left(K_{em}^{\mathrm{II}}\right)^2-\left(K_{m}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{h^{\mathrm{I}}\alpha^{\mathrm{II}}}{e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}}\frac{\left(K_{m}^{\mathrm{II}}\right)^2-\left(K_{\alpha}^{\mathrm{II}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\end{aligned} \tag{X.26}$$

where the coefficient $\left(K_{\alpha}^{\mathrm{II}}\right)^2$ is defined by expression (IV.16).

For equation (X.26), the existence conditions can be also defined by inequalities (X.18) which use the parameter Y. For this case, this parameter represents the right-hand side of equation (X.26). The reader can use $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$ to get a simplified form of Y.

X.3. *The combination of the first and second sets*

It is also possible to use the first eigenvectors for the first PEM half-space and the second eigenvectors for the second PEM half-space. Utilizing equations (VI.22), (VI.23), (II.9), (II.46), (VI.26), and (VI.27) for this case, the six homogeneous equations can be composed as follows:

$$F^{\mathrm{I}2}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2} - F^{\mathrm{II}2}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2} = 0 \tag{X.27}$$

$$\begin{aligned} &F^{\mathrm{I}}\left[e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right] + F^{\mathrm{I}2}b^{\mathrm{I}}\left[C^{\mathrm{I}}\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}{C^{\mathrm{I}}\left(K_{em}^{\mathrm{I}}\right)^2} + e^{\mathrm{I}}\alpha^{\mathrm{I}} - h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right] \\ &+ F^{\mathrm{II}}\left[e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}\right] + F^{\mathrm{II}2}b^{\mathrm{II}}\left[C^{\mathrm{II}}\frac{e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}}{C^{\mathrm{II}}\left(K_{em}^{\mathrm{II}}\right)^2} + e^{\mathrm{II}}\mu^{\mathrm{II}} - h^{\mathrm{II}}\alpha^{\mathrm{II}}\right] = 0 \end{aligned} \tag{X.28}$$

$$F^{\mathrm{I}}e^{\mathrm{I}}\alpha^{\mathrm{I}} + F^{\mathrm{I}2}e^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2 - \left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2} = 0 \tag{X.29}$$

$$F^{\mathrm{II}}e^{\mathrm{II}}\mu^{\mathrm{II}} + F^{\mathrm{II}2}e^{\mathrm{II}}\mu^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2 - \left(K_{m}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2} = 0 \tag{X.30}$$

$$-F^{\mathrm{II}}h^{\mathrm{II}}\alpha^{\mathrm{II}} - F^{\mathrm{II}2}h^{\mathrm{II}}\alpha^{\mathrm{II}}\frac{\left(K_{em}^{\mathrm{II}}\right)^2 - \left(K_{\alpha}^{\mathrm{II}}\right)^2}{\left(K_{em}^{\mathrm{II}}\right)^2} + F^{\mathrm{I}}h^{\mathrm{II}}\varepsilon^{\mathrm{I}} + F^{\mathrm{I}2}h^{\mathrm{II}}\varepsilon^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2 - \left(K_{e}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2} = 0 \tag{X.31}$$

$$-F^{\mathrm{I}}h^{\mathrm{I}}\varepsilon^{\mathrm{I}} - F^{\mathrm{I}2}b^{\mathrm{I}}\times 0 + F^{\mathrm{II}}\times 0 - F^{\mathrm{II}2}b^{\mathrm{II}}\times 0 = 0 \tag{X.32}$$

It is convenient to use the following two equations instead of equations (X.29), (X.30), (X.31), and (X.32):

$$F^{\mathrm{I}}\left(e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}\right)+F^{\mathrm{I2}}e^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}=0 \tag{X.33}$$

$$\begin{gathered}F^{\mathrm{II}}\left(e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right)+F^{\mathrm{II2}}\left(e^{\mathrm{II}}\mu^{\mathrm{II}}-h^{\mathrm{II}}\alpha^{\mathrm{II}}\right)\\-F^{\mathrm{I2}}e^{\mathrm{I}}\alpha^{\mathrm{I}}\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}+F^{\mathrm{I2}}h^{\mathrm{II}}\varepsilon^{\mathrm{I}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{\left(K_{em}^{\mathrm{I}}\right)^2}=0\end{gathered} \tag{X.34}$$

Equations (X.33) and (X.34) must be subtracted from equation (X.28). The last equation can be further transformed to get a simplified form. For this purpose, relationship (X.27) must be also used. As a result, the final form of the equation for the determination of the velocity V_{new22} of the twenty-second new interfacial SH-wave can be obtained as follows:

$$\begin{gathered}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{1+\left(K_{em}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\sqrt{1-\left(\frac{V_{new22}}{V_{tem}^{\mathrm{I}}}\right)^2}+\sqrt{1-\left(\frac{V_{new22}}{V_{tem}^{\mathrm{II}}}\right)^2}\\=1+\left(1-\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\right)\frac{e^{\mathrm{I}}\alpha^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{\alpha}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}+\frac{h^{\mathrm{II}}\varepsilon^{\mathrm{I}}}{e^{\mathrm{I}}\alpha^{\mathrm{I}}-h^{\mathrm{I}}\varepsilon^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{em}^{\mathrm{I}}\right)^2-\left(K_{e}^{\mathrm{I}}\right)^2}{1+\left(K_{em}^{\mathrm{II}}\right)^2}\end{gathered} \tag{X.35}$$

where the coefficient $\left(K_{\alpha}^{\mathrm{I}}\right)^2$ is defined by expression (IV.15).

It is obvious that some existence conditions must exist for the complicated case described by equation (X.35). They are given by inequalities (X.18) where the parameter Y is equal to the right-hand side of expression (X.35). Also, it is necessary to account that the following equality $e^{\mathrm{I}}\alpha^{\mathrm{I}} = h^{\mathrm{I}}\varepsilon^{\mathrm{I}}$ definitely gives an infinite value of the parameter Y. This equality must be also accounted because the left-hand side of equation (X.35) cannot be equal to a very large number. When $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$ occur, one can find that the parameter Y has the following form:

$$Y=1+\frac{h^{\mathrm{II}}}{h^{\mathrm{I}}}\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{e}^{\mathrm{I}}\right)^2-\left(K_{m}^{\mathrm{I}}\right)^2}{1+\left(K_{e}^{\mathrm{II}}\right)^2+\left(K_{m}^{\mathrm{II}}\right)^2}-\frac{C^{\mathrm{I}}}{C^{\mathrm{II}}}\frac{\left(K_{e}^{\mathrm{I}}\right)^2}{1+\left(K_{e}^{\mathrm{II}}\right)^2+\left(K_{m}^{\mathrm{II}}\right)^2} \tag{X.36}$$

It is also possible to estimate the possibility of propagation of this new interfacial SH-wave in the configuration consisting of PZT–Terfenol-D and $BaTiO_3$–$CoFe_2O_4$. For this two-layer system, the following parameters were calculated: $C^{\mathrm{I}}/C^{\mathrm{II}}$ ~ 0.33, $V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}$ ~ 0.59, $\left(K_{em}^{\mathrm{I}}\right)^2$ ~ 0.82, $\left(K_{em}^{\mathrm{II}}\right)^2$ ~ 0.16, $\alpha^{\mathrm{I}} = 0.01\sqrt{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}}$, and $\alpha^{\mathrm{II}} = 0.01\sqrt{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}}$. Using them, it was found the parameter Y has a negative sign. This illuminates that such new interfacial SH-wave cannot propagate.

CHAPTER XI

Discussion

It this theoretical work, each chapter from the fourth to the tenth provides the corresponding solutions for three new interfacial SH-waves. Also, Chapter II has demonstrated that only single new interfacial SH-wave can propagate guided by the eclectically closed and magnetically open interface because each of the three possibilities gives the same result. Besides, Chapter III has studied the case of the eclectically open and magnetically closed interface which cannot support any interfacial wave propagation. So, the expressions for the calculations of the speeds of all the new interfacial SH-waves and the corresponding existence conditions were obtained in the previous chapters of this work. However, the explicit forms of the complete mechanical displacement and the complete electrical and magnetic potentials were not demonstrated. This is so because the expressions can be very complicated and the size of the work can be significantly expanded. However, it is possible to discuss that and schematically demonstrate the common procedure for the determination of the parameters.

Based on the theories developed in the previous chapters, the reader can now state that one has to deal with the corresponding six homogeneous equations in each chapter beginning with Chapter II. However, it is thought that it is more convenient to deal with the corresponding three equations instead of the six ones. Indeed, the first equation of six provides the relationship between the weight factors F^{I2} and F^{II2}. Besides, it is obvious that the second equation is the main equation because it includes the phase velocity which must be determined. This second equation also depends on both $F^{I}\left(e^{I}\varphi^{I}-h^{I}\psi^{I}\right)$ and $F^{II}\left(e^{II}\varphi^{II}-h^{II}\psi^{II}\right)$ where $F^{I}\left(e^{I}\varphi^{I}-h^{I}\psi^{I}\right)$ can be equal to $F^{I}\left(e^{I}\alpha^{I}-h^{I}\varepsilon^{I}\right)$ or $F^{I}\left(e^{I}\mu^{I}-h^{I}\alpha^{I}\right)$, see equations (I.73) and (I.76), and $F^{II}\left(e^{II}\varphi^{II}-h^{II}\psi^{II}\right)$ can be equal to $F^{II}\left(e^{II}\alpha^{II}-h^{II}\varepsilon^{II}\right)$ or $F^{II}\left(e^{II}\mu^{II}-h^{II}\alpha^{II}\right)$, see equations

(I.81) and (I.84). Therefore, the last four equations of the six ones must be properly transformed into suitable two equations in order to cope with the new weight factors such as $F^{\mathrm{I}}\left(e^{\mathrm{I}}\varphi^{\mathrm{I}}-h^{\mathrm{I}}\psi^{\mathrm{I}}\right)$ and $F^{\mathrm{II}}\left(e^{\mathrm{II}}\varphi^{\mathrm{II}}-h^{\mathrm{II}}\psi^{\mathrm{II}}\right)$ instead of F^{I} and F^{II}. It is apparent that a subtraction of these four equation from the second one must lead to the new equation containing neither $F^{\mathrm{I}}\left(e^{\mathrm{I}}\varphi^{\mathrm{I}}-h^{\mathrm{I}}\psi^{\mathrm{I}}\right)$ nor $F^{\mathrm{II}}\left(e^{\mathrm{II}}\varphi^{\mathrm{II}}-h^{\mathrm{II}}\psi^{\mathrm{II}}\right)$. Therefore, the new two equations obtained from the four ones must lead to the same result after subtraction of them from the second equation. For some cases, both new equations can contain both $F^{\mathrm{I}}\left(e^{\mathrm{I}}\varphi^{\mathrm{I}}-h^{\mathrm{I}}\psi^{\mathrm{I}}\right)$ and $F^{\mathrm{II}}\left(e^{\mathrm{II}}\varphi^{\mathrm{II}}-h^{\mathrm{II}}\psi^{\mathrm{II}}\right)$. In spite of this situation, these two equations can be further transformed in order to obtain new equations, of which one will contain only $F^{\mathrm{I}}\left(e^{\mathrm{I}}\varphi^{\mathrm{I}}-h^{\mathrm{I}}\psi^{\mathrm{I}}\right)$ and the second will contain only $F^{\mathrm{II}}\left(e^{\mathrm{II}}\varphi^{\mathrm{II}}-h^{\mathrm{II}}\psi^{\mathrm{II}}\right)$. This procedure can significantly complicate the final result.

So, it is possible to schematically write these three homogeneous equations obtained from the six ones in the following matrix form:

$$\begin{pmatrix} 1 & 1 & d_1 \\ 1 & 0 & d_2 \\ 0 & 1 & d_3 \end{pmatrix}\begin{pmatrix} F^{\mathrm{I}}\left(e^{\mathrm{I}}\varphi^{\mathrm{I}}-h^{\mathrm{I}}\psi^{\mathrm{I}}\right) \\ F^{\mathrm{II}}\left(e^{\mathrm{II}}\varphi^{\mathrm{II}}-h^{\mathrm{II}}\psi^{\mathrm{II}}\right) \\ F^{\mathrm{II2}} \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{XI.1}$$

where the parameters such as d_1, d_2, and d_3 are only those terms which contain the weight factor F^{II2}. For simplicity, it is possible to use F^{II2} instead of $F^{\mathrm{II2}}\left(e^{\mathrm{II}}\varphi^{\mathrm{II}}-h^{\mathrm{II}}\psi^{\mathrm{II}}\right)$.

Consequently, the determinant of the coefficient matrix in equation (XI.1) can be readily transformed by the following way:

$$\begin{vmatrix} 1 & 1 & d_1 \\ 1 & 0 & d_2 \\ 0 & 1 & d_3 \end{vmatrix}=0\rightarrow\begin{vmatrix} 0 & 0 & d_1-d_2-d_3 \\ 1 & 0 & d_2 \\ 0 & 1 & d_3 \end{vmatrix}=0 \tag{XI.2}$$

It is clearly seen in expression (XI.2) that $d_1 - d_2 - d_3 = 0$ can definitely equal to zero this determinant. This equation determines the speed of the corresponding new interfacial SH-wave. Also, the parameter Y in the existence conditions is equal to $Y =$

$d_2 + d_3$. It is also flagrant that the new weight factors $F^{\mathrm{I}}\left(e^{\mathrm{I}}\varphi^{\mathrm{I}} - h^{\mathrm{I}}\psi^{\mathrm{I}}\right)$ and $F^{\mathrm{II}}\left(e^{\mathrm{II}}\varphi^{\mathrm{II}} - h^{\mathrm{II}}\psi^{\mathrm{II}}\right)$ can be defined by

$$F^{\mathrm{I}}\left(e^{\mathrm{I}}\varphi^{\mathrm{I}} - h^{\mathrm{I}}\psi^{\mathrm{I}}\right) = -d_2 \quad \text{(XI.3)}$$

$$F^{\mathrm{II}}\left(e^{\mathrm{II}}\varphi^{\mathrm{II}} - h^{\mathrm{II}}\psi^{\mathrm{II}}\right) = -d_3 \quad \text{(XI.4)}$$

As a result, the weight factors F^{I} and F^{II} are defined by

$$F^{\mathrm{I}} = -d_2 / \left(e^{\mathrm{I}}\varphi^{\mathrm{I}} - h^{\mathrm{I}}\psi^{\mathrm{I}}\right) \quad \text{(XI.5)}$$

$$F^{\mathrm{II}} = -d_3 / \left(e^{\mathrm{II}}\varphi^{\mathrm{II}} - h^{\mathrm{II}}\psi^{\mathrm{II}}\right) \quad \text{(XI.6)}$$

In addition, the weight factor $F^{\mathrm{II}2}$ can be naturally chosen to be equal to unity, $F^{\mathrm{II}2} = 1$.

For the case of Chapter III, $d_2 = d_3 = 0$ occurs and therefore there is no any suitable solution. Also, $d_2 = d_3 = 0$ results in $F^{\mathrm{I}}\left(e^{\mathrm{I}}\varphi^{\mathrm{I}} - h^{\mathrm{I}}\psi^{\mathrm{I}}\right) = F^{\mathrm{II}}\left(e^{\mathrm{II}}\varphi^{\mathrm{II}} - h^{\mathrm{II}}\psi^{\mathrm{II}}\right) = 0$ due to $F^{\mathrm{I}} = F^{\mathrm{II}} = 0$. In this case, $F^{\mathrm{I}2} \neq 0$ and $F^{\mathrm{II}2} \neq 0$ occur because F^{I} and F^{II} are uncoupled with $F^{\mathrm{I}2}$ and $F^{\mathrm{II}2}$. For the other cases, it is possible that one of the weight factors F^{I} and F^{II} can be equal to zero. For instance, this occurs in the second and third subsections of Chapter IX and in the first and third subsections of Chapter VII. It is necessary to state that the existence conditions for all the cases represent the requirements for the corresponding parameter Y which is equal to the right-hand sides of the corresponding expressions. For many cases, this parameter can approach an infinity, $Y \to \infty$ due to $e^{\mathrm{I}}\varphi^{\mathrm{I}} = h^{\mathrm{I}}\psi^{\mathrm{I}}$ and or $e^{\mathrm{II}}\varphi^{\mathrm{II}} = h^{\mathrm{II}}\psi^{\mathrm{II}}$. It is blatant that $Y \to \infty$ cannot support the wave propagation because the left-hand sides of the corresponding expressions are finite.

There are currently a few hexagonal (6 *mm*) piezoelectromagnetics to constitute suitable two-layer structures. However, it is possible to find two composites to evaluate some possibilities of the propagation of the new interfacial SH-waves. The hexagonal composites such as PZT–Terfenol-D and $BaTiO_3$–$CoFe_2O_4$ are well-known. It is possible to use them in the calculations. PZT–Terfenol-D and $BaTiO_3$–

$CoFe_2O_4$ were chosen as the first and second PEM half-spaces in the calculations, respectively, because the SH-BAW speed for PEM1 is slower than that for PEM2. This configuration has the following calculated parameters: $C^{\mathrm{I}}/C^{\mathrm{II}}$ ~ 0.33, $V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}$ ~ 0.59, $\left(K_{em}^{\mathrm{I}}\right)^2$ ~ 0.82, $\left(K_{em}^{\mathrm{II}}\right)^2$ ~ 0.16, $\alpha^{\mathrm{I}} = 0.01\sqrt{\varepsilon^{\mathrm{I}}\mu^{\mathrm{I}}}$, and $\alpha^{\mathrm{II}} = 0.01\sqrt{\varepsilon^{\mathrm{II}}\mu^{\mathrm{II}}}$. It was found that this configuration cannot support the propagation of the new interfacial SH-waves studied in Chapters II and X. This is obvious because it is preferable to constitute a configuration which will possess $C^{\mathrm{I}}/C^{\mathrm{II}} > 1$.

An additional problem is that a large number of suitable piezoelectromagnetics cannot be found in the literature to constitute various two-layer structures. It was stated in the introduction that to evaluate the propagation possibility for various structures is not the main purpose of this work. Also, the reader can perform such evaluations for the results obtained in Chapters from IV to IX. Indeed, it is also possible to vary the values of the electromagnetic constants α^{I} and α^{II}. Besides, this theoretical work relates to the propagation problems of the new interfacial SH-waves when two dissimilar piezoelectromagnetic half-spaces are perfectly bonded at the common interface. To treat some interfacial imperfections does not represent the main purpose of this work and these problems can be investigated in the future. However, it is possible to assume that the interfacial imperfection can support propagation of some interfacial SH-waves which cannot propagate along the perfectly bonded interface. Huang, Li, and Lee [100] have solidly demonstrated this fact for the problem of propagation of interfacial SH-waves along the common interface between single-phase materials such as pure piezoelectrics and pure piezomagnetics.

CONCLUSION

These theoretical investigations are concerned with the propagation problems of interfacial SH-waves. These SH-waves can be guided by the common interface between dissimilar hexagonal (6 *mm*) piezoelectromagnetic half-spaces. Piezoelectromagnetics are known as two-phase materials possessing the piezoelectric, piezomagnetic, and magnetoelectric effects. This simultaneous possession of the effects certainly complicates the theoretical treatments of the problems. It was found that as many as twenty two new interfacial SH-waves can propagate in such two-layer structures. The propagation of each of the found SH-waves must satisfy the corresponding mechanical, electrical, and magnetic boundary conditions. It is apparent that different sets of the boundary conditions at the interface $x_3 = 0$ (see figure I.1) result in the fact that so many interfacial SH-wave can be guided by the common interface. For the mechanically free interface, it is also necessary to require the equality of the mechanical displacements. These two mechanical boundary conditions were remained the same and the electrical and magnetic ones vary. The possible electrical ones are as follows: the electrically open ($D_3^{\mathrm{I}} = 0$ and $D_3^{\mathrm{II}} = 0$) or electrically closed ($\varphi^{\mathrm{I}} = 0$ and $\varphi^{\mathrm{II}} = 0$) interface, as well as the case of $\varphi^{\mathrm{I}} = \varphi^{\mathrm{II}}$ and $D_3^{\mathrm{I}} = D_3^{\mathrm{II}}$ where φ and D_3 are the electrical potential and electrical displacement component, respectively. Besides, the possible magnetic ones are as follows: the magnetically open ($\psi^{\mathrm{I}} = 0$ and $\psi^{\mathrm{II}} = 0$) or magnetically closed ($B_3^{\mathrm{I}} = 0$ and $B_3^{\mathrm{II}} = 0$) interface, as well as the case of $\psi^{\mathrm{I}} = \psi^{\mathrm{II}}$ and $B_3^{\mathrm{I}} = B_3^{\mathrm{II}}$ where ψ and B_3 are the magnetic potential and magnetic flux component. It is convenient to utilize the superscripts "I" and "II" to distinguish the first and second half-spaces from each other.

The speed of each of the twenty two new interfacial SH-waves can be calculated using the corresponding expression obtained in an explicit form and some obtained forms can be quite complicated. Also, the corresponding existence conditions must be accounted for each case in order to be sure that such new interfacial SH-waves can propagate in a certain configuration consisting of chosen dissimilar piezoelectromagnetics. All the formulae for the calculation of the wave speed are valid when the SH-BAW speed for the first PEM half-space is smaller than that for the second PEM half-space, see figure I.1. However, all the obtained formulae are also valid for the reverse case because one can always rearrange the configuration, namely PEM1 → PEM2 and PEM2 → PEM1.

The obtained results can be also useful for the case when the single-phase material such as the pure piezoelectrics or pure piezomagnetics is used instead of one of the piezoelectromagnetics. It was found that for certain cases only suitable piezoelectrics can contact with the piezoelectromagnetics to satisfy the existence conditions. For the other certain cases, only suitable piezomagnetics can contact with the piezoelectromagnetics to allow the wave propagation satisfying the existence conditions. For the configuration of two dissimilar piezoelectromagnetics, all the formulae for the determination of the wave speed and the existence conditions can be significantly simplified when $\alpha^{\mathrm{I}} = 0$ and $\alpha^{\mathrm{II}} = 0$ are exploited. It is well-known that in general, the values of the electromagnetic constant α can be very small. However, it couples the other material constants that can significantly complicate the results.

The sample calculations were performed for some cases when PZT–Terfenol-D and $BaTiO_3$–$CoFe_2O_4$ composites are used as the first and second PEM half-spaces, respectively. These two piezoelectromagnetic composite materials are well-known. They relate to the hexagonal materials of class 6 *mm*. This work has used only these two composites because there are no investigations of the other possible hexagonal (6 *mm*) composites. Using these two composites, it was found that they are not the best solution to utilize them together because the SH-BAW speed for the $BaTiO_3$–$CoFe_2O_4$ composite is approximately two times larger than that for the PZT–Terfenol-D composite, see table I.1. In the calculations, PZT–Terfenol-D must be therefore used as the first PEM half-space because $V_{tem}^{\mathrm{I}}/V_{tem}^{\mathrm{II}}$ ~ 0.59. Also, this configuration of two composite has a very small value of $C^{\mathrm{I}}/C^{\mathrm{II}}$ ~ 0.33. As a result, this configuration cannot support the interfacial SH-wave propagation. However, it was also demonstrated that when the values of electromagnetic constants α^{I} and α^{II}

are properly changed, some new interfacial SH-waves can exist even in such configuration. This can mean that the magnetoelectric effect can significantly affect on the existence of the interfacial SH-waves. It is well-known that the electromagnetic constant can significantly depend on the applied magnetic field. Therefore, it is expected that these results can be useful for the creation of various novel technical devices, for instance, sensors, switchers, etc.

REFERENCES

[1] A.E.H. Love, *Some problems of Geodynamics*, Cambridge University Press, London, England, UK, 1911, 180 pages.

[2] G.W. Farnell, E.L. Adler, "Elastic wave propagation in thin layers" in: W.P. Mason and R.N. Thurston (Eds.), *Physical Acoustics: Principles and Methods*, **Vol. 9,** Academic Press, New York, 1972, pp. 35 – 127.

[3] G.W. Farnell, "Types and properties of surface acoustical waves" in: A.A. Oliner (Ed.), *Acoustic Surface Waves. Topics in Applied Physics*, **Vol. 24**, Springer Verlag, Berlin–Heidelberg–New York, 1978, pp. 26 – 96.

[4] G.W. Farnell, "Properties of elastic surface waves" in: W.P. Mason and R.N. Thurston (Eds.), *Physical Acoustics*, **Vol. 6,** Academic Press, New York, 1970, pp. 109 – 166.

[5] E. Dieulesaint, D. Royer, *Elastic Waves in Solids: Applications to Signal Processing* / translated by A. Bastin and M. Motz, Chichester [English]; New York, J. Wiley, 1980, 511 pages.

[6] D. Royer, E. Dieulesaint, *Elastic Waves in Solids I: Free and Guided Propagation* (Advanced Texts in Physics) / translated by D.P. Morgan [English]; Springer Verlag, Berlin-Heidelberg-New York-Barcelona-Hong Kong-London-Milan-Paris-Singapore-Tokyo, 2000, 369 pages.

[7] D. Royer, E. Dieulesaint, *Elastic Waves in Solids II: Generation, Acousto-optic Interaction, Applications* (Advanced Texts in Physics) / translated by D.P. Morgan [English]; Springer Verlag, Berlin-Heidelberg-New York-Barcelona-Hong Kong-London-Milan-Paris-Singapore-Tokyo, 2000, 441 pages.

[8] S.V. Biryukov, Yu.V. Gulyaev, V.V. Krylov, V.P. Plessky, *Surface Acoustic Waves in Inhomogeneous Media*, L.M. Brekhovskikh (Edr.), Springer Series on Wave Phenomena, **Vol. 20**, Springer-Verlag, Berlin-Heidelberg-New York, 1995, 388 pages.

[9] Lord Rayleigh (J.W. Strutt), "On waves propagated along the plane surfaces of an elastic solid" *Proceedings of the London Mathematical Society* **17**, 4 – 11 (1885).

[10] A.A. Zakharenko, "On cubic crystal anisotropy for waves with Rayleigh-wave polarization" *Non-destructive Testing and Evaluation* **21** (2) 61 – 77 (2006).

[11] A.A. Zakharenko, "Dispersive Rayleigh type waves in layered systems consisting of piezoelectric crystals bismuth silicate and bismuth germanate" *Acta Acustica united with Acustica* **91** (4) 708 – 715 (2005).

[12] A.A. Zakharenko, "Acoustic waves with the in-plane polarization in piezoelectric cubic structures" *Canadian Journal of Pure & Applied Sciences* (*SENRA Academic Publishers, Burnaby, British Columbia, Canada*) **3** (1) 675 – 690 (2009); ISSN: 1715-9997 (open access at http://www.cjpas.org).

[13] J.L. Bleustein, "A new surface wave in piezoelectric materials" *Applied Physics Letters* **13** (12) 412 – 413 (1968).

[14] Yu.V. Gulyaev, "Electroacoustic surface waves in solids" *Soviet Physics Journal of Experimental and Theoretical Physics Letters* **9**, 37 – 38 (1969).

[15] D.A. Berlincourt, D.R. Curran, H. Jaffe, "Piezoelectric and piezomagnetic materials and their function in transducers" in *Physical Acoustics: Principles and Methods*, edited by W.P. Mason, **Vol. 1**, Part A, Academic Press, New York, 1964, pages 169 – 270.

[16] H. Liu, Z.B. Kuang, Z.M. Cai, "Propagation of Bleustein-Gulyaev waves in a prestressed layered piezoelectric structures" *Ultrasonics* **41**, 397 – 405 (2003).

[17] C. Lardat, C. Maerfeld, P. Tournois, "Theory and performance of acoustical dispersive surface wave delay lines" *Proceedings of the IEEE* **59** (3) 355 – 364 (1971).

[18] G.G. Kessenikh, V.N. Lyubimov, L.A. Shuvalov, "On surface Love waves in piezoelectrics" *Soviet Kristallografiya* (Crystallography Reports, Moscow) **27** (3) 437 – 443 (1982), in Russian.

[19] A.A. Zakharenko, "Love type waves in layered systems consisting of two piezoelectric cubic crystals" *Journal of Sound and Vibration* **285** (4–5) 877 – 886 (2005).

[20] A.A. Zakharenko, *Dispersive SAWs in Layered Systems Consisting of Cubic Piezoelectrics*, Saarbruecken – Krasnoyarsk, LAP LAMBERT Academic Publishing GmbH & Co. KG, 72 pages, 2010, ISBN: 978-3-8433-7523-8.

[21] G. Kovacs, A. Venema, "Theoretical comparison of sensitivities of acoustic shear wave modes for biochemical sensing in liquids" *Applied Physics Letters* **61** (6) 639 – 641 (1992).

[22] E. Gizeli, M. Liley, C.R. Lowe, H. Vogel, "Antibody binding to a functionalized supported lipid layer: A direct acoustic immunosensor" *Analytical Chemistry* **69**, 4808 – 4813 (1997).

[23] M. Vellekoop, B. Jakoby, "A Love-wave ice detector" *Proceedings of the IEEE Ultrasonic Symposium*, **Vol. 1**, 453 – 456 (1999).

[24] J. Grate, S. Martine, Q. White, "Acoustic wave sensors" *Analytical Chemistry* **65** (21) 940 – 948 (1993).

[25] Yu.V. Gulyaev, "Review of shear surface acoustic waves in solids" *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control* **45** (4) 935 – 938 (1998).

[26] Yu.V. Gulyaev, F.S. Hickernell, "Acoustoelectronics: history, present state and new ideas for a new Era" *Acoustical Physics* **51** (1) 81 – 88 (2005) [*Akusticheskii Zhurnal* (Moscow) **51** (1) 101 – 110 (2005)].

[27] S.S. Rao, M. Sunar, "Piezoelectricity and its use in disturbance sensing and control of flexible structures: a survey" *Applied Mechanics Review* **47**, 113 – 123 (1994).

[28] Yu.V. Gulyaev, I.E. Dikshtein, V.G. Shavrov, "Magnetoacoustic surface waves in magnetic crystals at reorientation phase transitions" *Uspekhi Phizicheskikh Nauk* (Moscow) **167** (7) 735 – 750 (1997).

[29] F.S. Hickernell, "Shear horizontal BG surface acoustic waves on piezoelectrics: a historical note" *The IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control* **52**, 809 – 811 (2005).

[30] M. Yamaguchi, "Early days of SH–type surface acoustic wave research" *Japanese Journal of Applied Physics* **42**, Part 1, № 5B, 2909 – 2917 (2003).

[31] J.F. Nye, *Physical Properties of Crystals. Their Representation by Tensors and Matrices*, Oxford, Clarendon Press, 1989, 385 pages.

[32] R.E. Newnham, *Properties of Materials: Anisotropy, Symmetry, Structure* (Kindle edition), Oxford University Press Inc., Oxford-New York, 2005 (reprinted 2008), 391 pages.

[33] V.I. Al'shits, V.N. Lyubimov, "On existence of piezoelectric and piezomagnetic properties of anisotropic media" *Soviet Kristallografia* (Moscow) **35** (2) 483 – 484 (1990).

[34] J. Aboudi, "Micromechanical analysis of fully coupled electro-magneto-thermo-elastic multiphase composites" *Smart Materials and Structures* **10**, 867 – 877 (2001).

[35] A.R. Annigeri, N. Ganesan, S. Swarnamani, "Free vibrations of simply supported layered and multiphase magneto-electro-elastic cylindrical shells" *Smart Materials and Structures* **15**, 459 – 467 (2006).

[36] M. Fiebig, "Revival of the magnetoelectric effect" *Journal of Physics D: Applied Physics* **38** (8) R123 – R152 (2005).

[37] J.-P. Rivera, "The linear magnetoelectric effect in $LiCoPO_4$ revisited" *Ferroelectrics* **161** (1) 147 – 164 (1994).

[38] G.T. Rado, J.M. Ferrari, W.G. Maisch, "Magnetoelectric susceptibility and magnetic symmetry of magnetoelectrically annealed $TbPO_4$" *Physical Review B* **29** (7) 4041 – 4048 (1984).

[39] H. Schmid, "Some symmetry aspects of ferroics and single phase multiferroics" *Journal of Physics: Condensed Matter* **20**, 434201 (2008) 24 pages.

[40] V.I. Al'shits, A.N. Darinskii, J. Lothe, "On the existence of surface waves in half-infinite anisotropic elastic media with piezoelectric and piezomagnetic properties" *Wave Motion* **16** (3) 265 – 283 (1992).

[41] X.-F. Li, "Dynamic analysis of a cracked magnetoelectroelastic medium under anti-plane mechanical and in-plane electric and magnetic impacts" *International Journal of Solids and Structures* **42** (11-12) 3185 – 3205 (2005).

[42] C.W. Nan, "Magnetoelectric effect in composites of piezoelectric and piezomagnetic phases" *Physical Review B* **50** (9) 6082 – 6088 (1994).

[43] K. Mori, M. Wuttig, "Magnetoelectric coupling in Terfenol-D/polyvinylidenedifluoride composites" *Applied Physics Letters* **81** (1) 100 – 101 (2002).

[44] P. Yang, K. Zhao, Y. Yin, J.G. Wan, J.S. Zhu, "Magnetoelectric effect in magnetostrictive/piezoelectric laminate composite Terfenol-D/$LiNbO_3$ [(*zxtw*)−129°/30°]" *Applied Physics Letters* **88**, 172903 (2006) 3 pages.

[45] A. Melkumyan, "Twelve shear surface waves guided by clamped/free boundaries in magneto-electro-elastic materials" *International Journal of Solids and Structures* **44** (10) 3594 – 3599 (2007).

[46] A.A. Zakharenko, *Seven new SH-SAWs in cubic piezoelectromagnetics*, Saarbruecken – Krasnoyarsk, LAP LAMBERT Academic Publishing GmbH & Co. KG, 172 pages, 2011, ISBN: 978-3-8473-3485-9.

[47] A.A. Zakharenko, "Analytical investigation of surface wave characteristics of piezoelectromagnetics of class 6 *mm*" *International Scholarly Research Network (ISRN) Applied Mathematics* (India), **Volume 2011**, Article ID 408529, 8 pages (2011); DOI:10.5402/2011/408529 (free on-line access).

[48] A.A. Zakharenko, *Propagation of seven new SH-SAWs in piezoelectromagnetics of class 6 mm*, Saarbruecken – Krasnoyarsk, LAP LAMBERT Academic Publishing GmbH & Co. KG, 84 pages, 2010, ISBN: 978-3-8433-6403-4.

[49] A.K. Kaw, *Mechanics of Composite Materials* (Second edition, Mechanical and Aerospace Engineering Series), CRC Press, 2005, 466 pages.

[50] *Handbook of Polymer Composites for Engineers* (Edited by L. Hollaway), Woodhead Publishing Ltd., England, 1994.

[51] *Engineered Materials Handbook*: *Composites*, **Volume 1**, ASM International Metals Park OH, 1988.

[52] *International Encyclopedia of Composites* (Edited by S.M. Lee), **Volumes 1-6**, VCH, New York, 1990.

[53] *Delaware Composites Design Encyclopedia* (Edited by L.A. Carlsson and J.W. Gillespie), **Volumes 1-6**, Technomic, Lancaster, PA, 1989.

[54] F.L. Matthews, R.D. Rawlings, *Composite Materials: Engineering and Science*, Boca Raton: CRC Press, 1999, 480 pages.

[55] W. Kleber, *An introduction to Crystallography*, Veb–Verlag Technik, Berlin, 1971, 336 pages.

[56] Yu.I. Sirotin, M.P. Shaskol'skaya, *Basics of Crystallophysics*, Nauka Publishers, Moscow, 1979, 640 pages.

[57] M. Bichurin, V. Petrov, A. Zakharov, D. Kovalenko, S.Ch. Yang, D. Maurya, V. Bedekar, Sh. Priya, "Magnetoelectric interactions in lead-based and lead-free composites" *Materials* **2011** (4) 651 – 702 (2011).

[58] G. Srinivasan, "Magnetoelectric composites" *Annual Review of Materials Research* **40**, 153 – 178 (2010).

[59] Ü. Özgür, Ya. Alivov, H. Morkoç, "Microwave ferrites, part 2: passive components and electrical tuning" *Journal of Materials Science: Materials in Electronics* **20** (10) 911 – 952 (2009).

[60] J. Zhai, Z.-P. Xing, Sh.-X. Dong, J.-F. Li, D. Viehland, "Magnetoelectric laminate composites: an overview" *Journal of the American Ceramic Society* **91** (2) 351 – 358 (2008).

[61] W. Eerenstein, N.D. Mathur, J.F. Scott, "Multiferroic and magnetoelectric materials" *Nature* **442**, 759 – 765 (2006).

[62] D. Fang, Yo-P. Wan, X. Feng, A.K. Soh, "Deformation and fracture of functional ferromagnetics" *ASME Applied Mechanics Review* **61**, 020803-1 – 020803-23 (2008).

[63] A. Sihvola, "Metamaterials in electromagnetics" *Metamaterials* **1** (1) 2 – 11 (2007).

[64] N.A. Hill (N.A. Spaldin), "Why are there so few magnetoelectric materials?" *Journal of Physical Chemistry B* **104**, 6697 – 6709 (2000).

[65] W. Prellier, M.P. Singh, P. Murugavel, "The single-phase multiferroic oxides – from bulk to thin film" *Journal of Physics: Condensed Matter* **17** (30) R803 (2005).

[66] N.A. Spaldin, M. Fiebig, "The renaissance of magnetoelectric multiferroics" *Science* **309** (5733) 391 – 392 (2005).

[67] D.I. Khomskii, "Multiferroics: Different ways to combine magnetism and ferroelectricity" *Journal of Magnetism and Magnetic Materials* **306** (1) 1 – 8 (2006).

[68] G.A. Smolenskii, I.E. Chupis, "Ferroelectromagnets" *Soviet Physics Uspekhi* (*Uspekhi Phizicheskikh Nauk*, Moscow) **25** (7) 475 – 493 (1982).

[69] S.-W. Cheong, M. Mostovoy, "Multiferroics: A magnetic twist for ferroelectricity" *Nature Materials* **6**, 13 – 20 (2007).

[70] R. Ramesh, N.A. Spaldin, "Multiferroics: Progress and prospects in thin films" *Nature Materials* **6**, 21 – 29 (2007).

[71] T. Kimura, "Spiral magnets as magnetoelectrics" *Annual Review of Materials Research* **37**, 387 – 413 (2007).

[72] T. Kimura, T. Goto, H. Shintani, K. Ishizaka, T. Arima, Y. Tokura, "Magnetic control of ferroelectric polarization" *Nature* **426**, 55 – 58 (2003).

[73] K.F. Wang , J.-M. Liu, Z.F. Ren, "Multiferroicity: The coupling between magnetic and polarization orders" *Advances in Physics* **58** (4) 321 – 448 (2009).

[74] R. Ramesh, "Materials science: Emerging routes to multiferroics" *Nature* **461**, 1218 – 1219 (2009).

[75] K.T. Delaney, M. Mostovoy, N.A. Spaldin, "Superexchange-driven magnetoelectricity in magnetic vertices" *Physical Review Letters* **102** (15) 157203, 4 pages (2009).

[76] A. Fert, "Origin, development, and future of spintronics (Nobel lectures)" *Reviews of Modern Physics* **80** (4) 1517 – 1530 (2008).

[77] A. Fert, "Origin, development, and future of spintronics (Nobel lectures)" *Physics – Uspekhi* **51** (12) 1336 – 1348 (2008) [*Uspekhi Phizicheskikh Nauk*, Moscow, **178** (12) 1336 – 1348 (2008)].

[78] C. Chappert, J.-V. Kim, "Metal spintronics: Electronics free of charge" *Nature Physics* **4**, 837 – 838 (2008).

[79] M. Bibes, A. Barthélémy, "Multiferroics: Towards a magnetoelectric memory" *Nature Materials* **7**, 425 – 426 (2008).

[80] M.I. Bichurin, V.M. Petrov, D.A. Filippov, G. Srinivasan, S.V. Nan, *Magnetoelectric materials*, Academia Estestvoznaniya Publishers, Moscow (2006).

[81] Y.K. Fetisov, A.A. Bush, K.E. Kamentsev, A.Y. Ostashchenko, G. Srinivasan, "Ferrite-piezoelectric multilayers for magnetic field sensors" *The IEEE Sensor Journal* **6** (4) 935 – 938 (2006).

[82] G. Srinivasan, Y.K. Fetisov, "Microwave magnetoelectric effects and signal processing devices" *Integrated Ferroelectrics* **83** (1) 89 – 98 (2006).

[83] S. Priya, R.A. Islam, S.X. Dong, D. Viehland, "Recent advancements in magnetoelectric particulate and laminate composites" *Journal of Electroceramics* **19**, 147 – 164 (2007).

[84] R. Grossinger, G.V. Duong, R. Sato-Turtelli, "The physics of magnetoelectric composites" *Journal of Magnetism and Magnetic Materials* **320**, 1972 – 1977 (2008).

[85] C.W. Ahn, D. Maurya, C.S. Park, S. Nahm, S. Priya, "A generalized rule for large piezoelectric response in perovskite oxide ceramics and its application for design of lead-free compositions" *Journal of Applied Physics* **105**, 114108 (2009) 6 pages.

[86] V.M. Petrov, M.I. Bichurin, V.M. Laletin, N. Paddubnaya, G. Srinivasan, "Modeling of magnetoelectric effects in ferromagnetic/piezoelectric bulk composites" in *Proceedings of the 5th International Conference on Magnetoelectric Interaction Phenomena in Crystals*, MEIPIC-5, Sudak, Ukraine, 21–24 September, 2003; arXiv:cond-mat/0401645.

[87] G. Harshe, J.P. Dougherty, R.E. Newnham, "Theoretical modelling of 3-0/0-3 magnetoelectric composites" *International Journal of Applied Electromagnetics in Materials* **4** (1) 161 – 171 (1993).

[88] Y.H. Chu, L.W. Martin, M.B. Holcomb, R. Ramesh, "Controlling magnetism with multiferroics" *Materials Today* **10** (10) 16 – 23 (2007).

[89] C.W. Nan, M.I. Bichurin, S.X. Dong, D. Viehland, G. Srinivasan, "Multiferroic magnetoelectric composites: Historical perspective, status, and future directions" *Journal of Applied Physics* **103** (3) 031101 (2008).

[90] H. Schmid, "Magnetic ferroelectric materials" *Bulletin of Materials Science* **17** (7) 1411 – 1414 (1994).

[91] J. Ryu, Sh. Priya, K. Uchino, H.-E. Kim, "Magnetoelectric effect in composites of magnetostrictive and piezoelectric materials" *Journal of Electroceramics* **8**, 107 – 119 (2002).

[92] J. van Suchtelen, "Product properties: a new application of composite materials" *Philips Research Reports* **27** (1) 28 – 37 (1972).

[93] J. van den Boomgaard, D.R. Terrell, R.A.J. Born, H.F.J.I. Giller, "*In-situ* grown eutectic magnetoelectric composite-material. 1. Composition and unidirectional solidification" *Journal of Materials Science* **9** (10) 1705 – 1709 (1974).

[94] A.M.J.G. van Run, D.R. Terrell, J.H. Scholing, "*In-situ* grown eutectic magnetoelectric composite-material. 2. Physical properties" *Journal of Materials Science* **9** (10) 1710 – 1714 (1974).

[95] J. van den Boomgaard, A.M.J.G. van Run, J. van Suchtelen, "Piezoelectric-piezomagnetic composites with magnetoelectric effect" *Ferroelectrics* **14** 727 – 728 (1976).

[96] V.E. Wood, A.E. Austin, in *Magnetoelectric Interaction Phenomena in Crystals*, A.J. Freeman, H. Schmid, Eds., Gordon and Breach Science Publishers, Newark, New York, 1975, pp. 181 – 194.

[97] Y.N. Venevtsev, V.V. Gagulin, I.D. Zhitomirsky, "Material science aspects of seignette-magnetism problem" *Ferroelectrics* **73**, 221 – 248 (1987).

[98] C. Maerfeld, P. Tournois, "Pure shear elastic surface wave guided by the interface of two semi-infinite media" *Applied Physics Letters* **19** (4) 117 – 118 (1971).

[99] A.-K. Soh, J.-X. Liu, "Interfacial shear horizontal waves in a piezoelectric-piezomagnetic bi-material" *Philosophical Magazine Letters* **86** (1) 31 – 35 (2006).

[100] Y. Huang, X.-F. Li, K.Y. Lee, "Interfacial shear horizontal (SH) waves propagating in a two-phase piezoelectric/piezomagnetic structure with an imperfect interface" *Philosophical Magazine Letters* **89** (2) 95 – 103 (2009).

[101] S.I. Rokhlin, Y.J. Wang, "Analysis of boundary conditions for elastic wave interaction with an interface between two solids" *Journal of the Acoustical Society of America* **89** (2) 503 – 515 (1991).

[102] M.X. Deng, "Analysis of second-harmonic generation of Lamb waves propagating in layered planar structures with imperfect interfaces"*Applied Physics Letters* **88** (22) 221902 (2006) 3 pages.

[103] H. Fan, J.S. Yang, L.M. Xu, "Piezoelectric waves near an imperfectly bonded interface between two half-spaces" *Applied Physics Letters* 88 (20) 203509 (2006) 3 pages.

[104] A. Melkumyan, Y.-W. Mai, "Influence of imperfect bonding on interface waves guided by piezoelectric/piezomagnetic composites" *Philosophical Magazine* **88** (23) 2965 – 2977 (2008).

[105] J.A. Otero, H. Calas, R. Rodríguez-Ramos, J. Bravo-Castillero, A.R. Aguiar, G. Monsivais, "Dispersion relations for SH waves on a magnetoelectroelastic heterostructure with imperfect interfaces" *Journal of Mechanics of Materials and Structures* **6** (7-8) 969 – 993 (2011).

[106] J. Chen, E. Pan, H. Chen, "Wave propagation in magneto-electro-elastic multilayered plates" *International Journal of Solids and Structures* **44** (3-4) 1073 – 1085 (2007).

[107] M.I. Bichurin, I.A. Kornev, V.M. Petrov, A.S. Tatarenko, Yu.V. Kiliba, G. Srinivasan, "Theory of magnetoelectric effects at microwave frequencies in a piezoelectric/magnetostrictive multilayer composite" *Physical Review B* **64**, 094409 (2001).

[108] S.X. Dong, J.-F. Li, D. Viehland, "Ultrahigh magnetic field sensitivity in laminates of Terfenol-D and $Pb(Mg_{1/3}Nb_{2/3})O_3$-$PbTiO_3$ crystals" *Applied Physics Letters* **83** (11) 2265 (2003) 3 pages.

[109] S.X. Dong, J.Y. Zhai, N.G. Wang, F.M. Bai, J.-F. Li, D. Viehland, T.A. Lograsso, "Fe-Ga/$Pb(Mg_{1/3}Nb_{2/3})O_3$-$PbTiO_3$ magnetoelectric laminate composites" *Applied Physics Letters* **87** (22) 222504 (2005) 3 pages.

[110] M.I. Bichurin, D.A. Filippov, V.M. Petrov, V.M. Laletin, N. Paddubnaya, G. Srinivasan, "Resonance magnetoelectric effects in layered magnetostrictive-piezoelectric composites" *Physical Review B* **68** (13) 132408 (2003) 4 pages.

[111] J.Y. Zhai, S.X. Dong, Z.P. Xing, J.-F. Li, D. Viehland, "Giant magnetoelectric effect in Metglas/Polyvinylidene-Fluoride laminates" *Applied Physics Letters* **89** (8) 083507 (2006) 3 pages.

[112] S.X. Dong, J.Y. Zhai, J.-F. Li, D. Viehland, "Near-ideal magnetoelectricity in high-permeability magnetostrictive/piezofiber laminates with a (2-1) connectivity" *Applied Physics Letters* **89** (25) 252904 (2006) 3 pages.

[113] J.Y. Zhai, N. Cai, Z. Shi, Y.H. Lin, C.-W. Nan, "Coupled magnetodielectric properties of laminated $PbZr_{0.53}Ti_{0.47}O_3/NiFe_2O_4$ ceramics" *Journal of Applied Physics* **95** (10) 5685 (2004) 6 pages.

[114] J. Liu, Y. Zhang, Y.H. Lin, C.W. Nan, "Magnetoelectric coupling in $BaTiO_3/(NiFe_2O_4/BaTiO_3)_n$ (n = 1,2,3,4) multilayered thin films" *Journal of Applied Physics* **105** (8) 083915 (2009) 4 pages.

[115] W. Chen, Z.H. Wang, W. Zhu, O.K. Tan, "Ferromagnetic, ferroelectric and dielectric properties of $Pb(Zr_{0.53}Ti_{0.47})O_3/CoFe_2O_4$ multiferroic composite thick films" *Journal of Physics D: Applied Physics* 42 (7) 075421 (2009).

[116] Y.-D. Xu, L. Wang, M. Shi, H.-L. Su, G. Wu, "Magnetostatic coupling in $CoFe_2O_4/Pb(Zr_{0.53}Ti_{0.47})O_3$ magnetoelectric composite thin films of 2-2 type structure" *Chinese Journal of Chemical Physics* **25** (1) 115 – 119 (2012).

[117] S.X. Dong, J.Y. Zhai, Z.P. Xing, J.-F. Li, D. Viehland, "Giant magnetoelectric effect (under a dc magnetic bias of 2 Oe) in laminate composites of FeBSiC alloy ribbons and $Pb(Zn_{1/3},Nb_{2/3})O_3$–7%$PbTiO_3$ fibers" *Applied Physics Letters* **91** (2) 022915 (2007) 3 pages.

[118] R. Malmhäll, G. Bäckström, K.V. Rao, S.M. Bhagat, M. Meichle, M.B. Salamon, "Metglas 2826B - transport, magnetic and thermal properties" *Journal of Applied Physics* **49** (3) 1727 (1978) 3 pages.

[119] S.X. Dong, J.Y. Zhai, F.M. Bai, J.-F. Li, D. Viehland, T.A. Lograsso, "Magnetostrictive and magnetoelectric behavior of Fe-20 at. % Ga/$Pb(Zr,Ti)O_3$ laminates" *Journal of Applied Physics* **97** (10) 103902 (2005) 6 pages.

[120] S.X. Dong, J.-F. Li, D. Viehland, "A longitudinal-longitudinal mode Terfenol-D/Pb($Mg_{1/3}Nb_{2/3}$)O_3-$PbTiO_3$ laminate composite" *Applied Physics Letters* **85** (22) 5305 (2004) 2 pages.

[121] Z.P. Xing, S.X. Dong, J.Y. Zhai, L. Yan, J.-F. Li, D. Viehland, "Resonant bending mode of Terfenol-D/steel/Pb(Zr,Ti)O_3 magnetoelectric laminate composites" *Applied Physics Letters* **89** (11) 112911 (2006) 3 pages.

[122] S.X. Dong, J.Y. Zhai, J.-F. Li, D. Viehland, M.I. Bichurin, "Magnetoelectric gyration effect in $Tb_{1-x}Dy_xFe_{2-y}$/Pb(Zr,Ti)O_3 laminated composites at the electromechanical resonance" *Applied Physics Letters* **89** (24) 243512 (2006) 3 pages.

[123] W.H. Sun, G.L. Ju, J.W. Pan, Y.D. Li, "Effects of the imperfect interface and piezoelectric/piezomagnetic stiffening on the SH wave in a multiferroic composite" *Ultrasonics* **51** (7) 831 – 838 (2011).

[124] J. Du, K. Xian, J. Wang, "SH surface acoustic wave propagation in a cylindrically layered piezomagnetic/piezoelectric structure" *Ultrasonics* **49** (1) 131 – 138 (2009).

[125] S.X. Dong, J.-F. Li, D. Viehland, "Voltage gain effect in a ring-type magnetoelectric laminate" *Applied Physics Letters* **84** (21) 4188 (2004) 3 pages.

[126] S.X. Dong, J. Bai, J.Y. Zhai, G.Q. Lu, J.-F. Li, D. Viehland, S.J. Zhang, T.R. Shrout, "Circumferential-mode, quasi-ring-type, manetoelectric laminate composite - a highly sensitive electric current and/or vortex magnetic field sensor" *Applied Physics Letters* **86** (18) 182506 (2005) 3 pages.

[127] S.X. Dong, J.-F. Li, D. Viehland, "Vortex magnetic field sensor based on ring-type magnetoelectric laminate" *Applied Physics Letters* **85** (12) 2307 (2004) 3 pages.

[128] D.N. Astrov, "The magnetoelectric effect in antiferromagnets" *Soviet Physics – Journal of Experimental and Theoretical Physics* (*JETP*) **11**, 708 – 709 (1960) [Zhurnal Experientalnoi i Teoreticheskoi Fiziki (USSR) **38**, 984 – 985 (1960)].

[129] D.N. Astrov, "Magnetoelectric effect in antiferromagnetic Cr_2O_3" *Soviet Physics – Journal of Experimental and Theoretical Physics* (*JETP*) **13** (4) 729 – 733 (1961).

[130] G.T. Rado, V.J. Folen, "Observation of the magnetically induced magnetoelectric effect and evidence for antiferromagnetic domains" *Physical Review Letters* **7** (8) 310 – 311 (1961).

[131] V.E. Lyamov, *Polarization effects and interaction anisotropy of acoustic waves in crystals*, Moscow, MSU Publishing, 1983, 224 pages; in Russian.

[132] Y.-Z. Wang, F.-M. Li, W.-H. Huang, X. Jiang, Yu.-Sh. Wang, K. Kishimoto, "Wave band gaps in two-dimensional piezoelectric/piezomagnetic phononic crystals" *International Journal of Solids and Structures* **45**, 4203 – 4210 (2008).

[133] J.X. Liu, X.L. Liu, Y.B. Zhao, "Green functions for anisotropic magnetoelectroelastic solids with an elliptical cavity or a crack" *International Journal of Engineering Science* **39**, 1405 – 1418 (2001).

[134] W.Y. Tian, U. Gabbert, "Macrocrack–microcrack interaction problem in magnetoelectroelastic solids" *Mechanics of Materials* 37, 565 – 592 (2005).

[135] R. Rojas-Diaz, A. Saez, F. Garcia-Sanchez, Ch. Zhang, "Time-harmonic Green's functions for anisotropic magnetoelectroelasticity" *International Journal of Solids and Structures* **45**, 144 – 158 (2008).

[136] E. Pan, "Exact solution for simply supported and multilayered magneto-electro-elastic plates" *Journal of Applied Mechanics* **68**, 608 – 618 (2001).

[137] D.R. Lovett, *Tensor properties of crystals*, Second edition, Taylor & Francis, 1999, 480 pages.

[138] C. Hammond, *The basics of crystallography and diffraction*, Third edition, Oxford University Press, USA, 2009, 448 pages.

[139] B.A. Auld, *Acoustic Fields and Waves in Solids*, **volume 1**, John Wiley and Sons, New York, 1973.

[140] F.I. Fedorov, *Theory of Elastic Waves in Crystals*, Plenum Press, New York, 1968.

[141] W.A. Wooster. *Tensors and Group Theory for the Physical Properties of Crystals*, Clarendon Press, Oxford, 1973.

[142] M.A. Jaswon, M.A. Rose, *Crystal Symmetry: Theory of Colour Crystallography*, Ellis Horwood Limited, Chichester, 1983.

[143] A.H. Nayfeh, *Wave Propagation in Layered Anisotropic Media with Applications to Composites*, Elsevier, Amsterdam, 1995.

[144] S.G. Lekhnitskii, *Theory of Elasticity of an Anisotropic Elastic Body*, Holden-Day, San Francisco, 1963.

[145] P.K. Mittal, *Oscillations, Waves and Acoustics*, Publisher: I.K. International Publishing House Pvt. Ltd., New Delhi, India, 2010, 568 pages.

[146] I.N. Bronstein, K.A. Semendyaev, *The reference-book on mathematics for engineers and higher school students*, Science, Moscow, 2000, 608 pages.

[147] J.-X. Liu, D.-N. Fang, X.-L. Liu, "A shear horizontal surface wave in magnetoelectric materials" *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control* **54** (7) 1287 – 1289 (2007).

[148] B.L. Wang, Y.-W. Mai, O.P. Niraula, "A horizontal shear surface wave in magnetoelectroelastic materials" *Philosophical Magazine Letters* **87** (1) 53 – 58 (2007).

[149] W.-Y. Wei, J.-X. Liu, D.-N. Fang, "Existence of shear horizontal surface waves in a magneto-electro-elastic material" *Chinese Physics Letters* **26** (10) 104301 (2009) 3 pages.

[150] M. Avellaneda, G. Harshe, "Magnetoelectric Effect in Piezoelectric/Magnetostrictive Multilayer (2-2) Composites" *Journal of Intelligent Material Systems and Structures* **5**, 501 – 513 (1994); DOI: 10.1177/1045389X9400500406.

[151] P. Chen, Ya.-P. Shen, "Propagation of axial shear magneto–electro-elastic waves in piezoelectric–piezomagnetic composites with randomly distributed cylindrical inhomogeneities" *International Journal of Solids and Structures* **44**, 1511 – 1532 (2007).

[152] T.J.-Ch. Liu, Ch.-H. Chue, "On the singularities in a bimaterial magneto-electro-elastic composite wedge under antiplane deformation" *Composite Structures* **72**, 254 – 265 (2006).

[153] B.A. Bednarcyk, "A fully coupled micro/macro theory for thermo-electro-magneto-elasto-plastic composite laminates" NASA/CR—2002-211468 (2002) 46 pages; available electronically at http://gltrs.grc.nasa.gov/GLTRS.

[154] M. Sunar, A.Z. Al-Gami, M.H. Ali, R. Kahraman, "Finite element modeling of thermopiezomagnetic smart structures" *AIAA Journal* **40** (9) 1846 – 1851 (2002).