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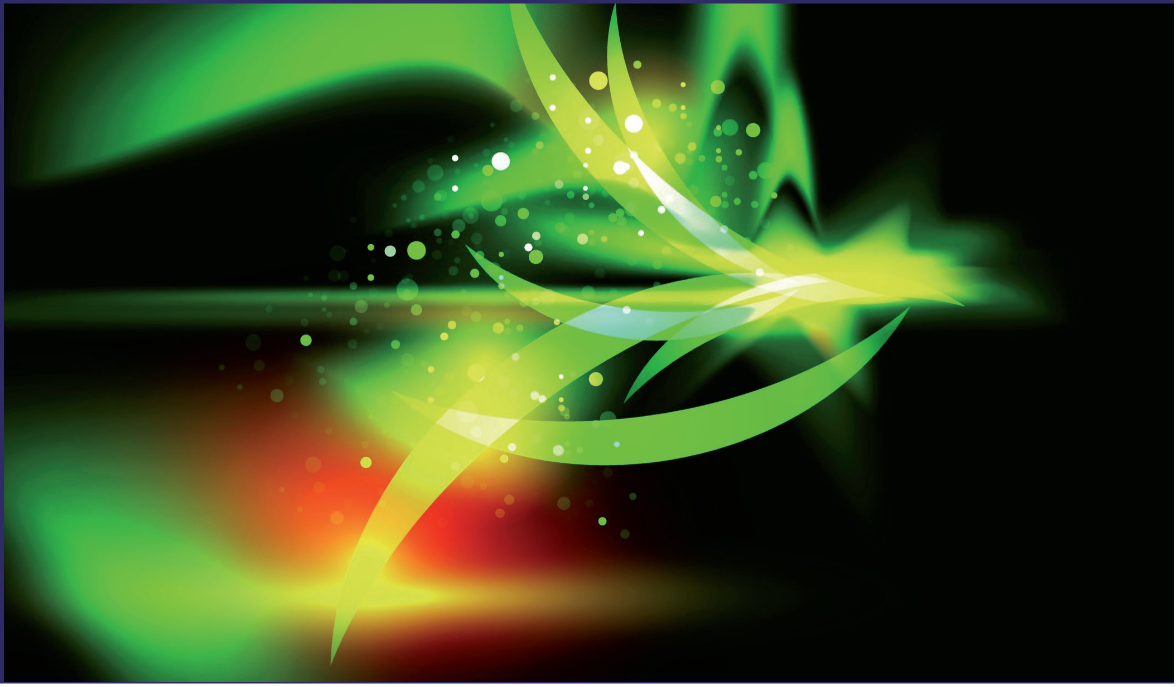
Thirty two new SH-waves in PEM plates

Zakharenko



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Thirty two new SH-waves propagating in PEM plates of class 6 mm

Propagation of thirty two new dispersive shear-horizontal (SH) waves in the piezoelectromagnetic plates of class 6 mm

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PREFACE

This theoretical work demonstrates that as many as thirty two new shear-horizontal (SH) acoustic waves can propagate in the piezoelectromagnetic transversely isotropic (class 6 *mm*) plates. These theoretical investigations relate to the homogeneous boundary conditions when the same set of the mechanical, electrical, and magnetic boundary conditions are applied to the upper and lower free surfaces of the piezoelectromagnetic plate. These new dispersive SH-waves propagating in the piezoelectromagnetic plate can have an infinite number of modes when the phase velocity V_{ph} is larger than the speed V_{tem} of the bulk acoustic SH-wave in the plate. For $V_{ph} < V_{tem}$, the new dispersive SH-waves can have the corresponding fundamental (zero-order) modes. It is apparent that knowledge of plate wave properties can be also beneficial to design of smart devices, biological and chemical sensors, filters, resonators, actuators, etc., and useful for the aerospace industry which calls for innovative smart (composite) materials. Also, it can represent an interest in constitution of piezoelectromagnetic laminate (composite) plates in the microwave technology and nondestructive testing and evaluation.

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COMMENTS BY THE AUTHOR

This activity in the research arena of theoretical physical acoustics was performed for the International Institute of Zakharenko Waves (IIZWs). It is obvious that this work can represent a great interest for researchers and students dealing with the theoretical and experimental researches concerning the acoustic wave propagation in the piezoelectromagnetic plates and various applications in the biological and chemical sensors, filters, actuators, resonators, smart materials and technical devices, labs-on-a-chips, etc. This research field is based on more than 40 years of technological and scientific developments. In the last two decades, the surface generated acoustic wave (SGAW) technology for sensing applications has been attracting the attention of the biochemical scientific community. In fact, some of the SGAW technical devices have demonstrated a high sensitivity in the detection of biorelevant molecules in liquid media. These devices can be based on the following wave phenomena: shear-horizontal surface acoustic wave (SH-SAW), surface transverse wave (STW), Love wave (LW), flexural plate wave (FPW), shear-horizontal acoustic plate mode (SH-APM) and layered guided acoustic plate mode (LG-APM). All these developments have been made with the purpose of reaching a highly sensitive, low cost, small size, multi-channel, portable, reliable, and commercially established SGAW biosensor. A setup with these features can significantly contribute to future developments in the food, health, and environmental industries. It is thought that new two-phase (composite) materials such as piezoelectromagnetics, also called magneto-electro-elastic materials, can significantly contribute because they can be suitable for utilization in smart technical devices.

This theoretical work relates to the shear-horizontal acoustic plate mode (SH-APM) technologies, namely the propagation problems of the new SH-waves in transversely isotropic (class 6 *mm*) piezoelectromagnetic plates. It is well-known that these waves are dispersive and an infinite number of dispersive wave modes can exist. It is apparent that knowledge of plate wave properties can be also beneficial to design of smart devices, biochemisensors, filters, resonators, actuators, etc. Also, it can represent an interest in constitution of piezoelectromagnetic laminate composite plates in the microwave technology and nondestructive testing of the composites. This theoretical research can be also useful for the aerospace industry which calls for innovative smart composite materials. Therefore, it is very important to completely understand wave properties of piezoelectromagnetic plates. This studying subject relates to the disciplines of applied physics and electromagnetic engineering. In physics, ordinary elastic motions in crystals are called acoustic modes. The descriptive term ”acoustic” is used rather than ”elastic”. This is useful because it allows one to distinguish acoustic and optical modes from each other. The optical modes involve internal degrees of freedom within a crystal unit cell. The term ”acoustic” also reflects common terminology among researchers and engineers engaged in developing elastic wave devices for radar and communication systems. This arena of technological development has been strongly influenced by the philosophy, concepts, and techniques of microwave electromagnetics. This is also known as microwave acoustics. Consequently, employment of the term "acoustic" accurately describes the aim and scope of the book.

The International Institute of Zakharenko Waves (IIZWs) was recently created to support researches on different Zakharenko waves, as well as for monitoring the nondispersive Zakharenko type waves in complex systems such as layered and quantum systems. Also, the IIZWs research is focused on treatments of many complex systems in which dispersive waves can propagate. The well-known examples of dispersive waves are Love and Lamb type waves. The Rayleigh and Bleustein-Gulyaev type waves propagating in layered systems can be also dispersive. The International Institute of Zakharenko Waves also has an interest in different applications of the acoustic waves for signal processing (filters, sensors, etc.) and the structural health monitoring. There are currently by about thirty research papers and five books relevant to the IIZWs. These research works also cover some problems of the propagation of the well-known Love, Lamb, Rayleigh, and Bleustein-Gulyaev type waves and discovered new wave phenomena.

It is worth noticing that the IIZWs possessively takes all the planets and smaller natural space bodies in the space outside the Solar System to develop both the IIZWs and the planets concerning economics, ecology, and population. Also, it is thought that this is necessary in order to exclude any sale of the planets and their surfaces by any human or other. This activity of the IIZWs was also created because of some problems to find a spot for the IIZWs on Earth. Note that the single person, namely Mr. Dennis Hope from the United States possesses the planets in the Solar System (but Earth) who sells surfaces of the planets to individuals. It is obvious that the monetary experiment on Earth during thousand years demonstrated a weak power of the financial system to avoid financial problems which cyclically happen. As a result, the following question presents in the air: what is the modern money? It is obvious that monetary systems are coupled only with humans who have given power to each other, but not with any space body such as a planet or star. It is apparent that humans depend on money, but not planets and stars. Indeed, planets and stars are leaving their own lifetimes and their ways of life do not depend on human activities measured in money. Therefore, money can exist only together with the human civilizations. It is not clear that the other civilizations can evaluate their activities in the same way similar to the human civilization does on Earth. Nothing is soundly known about that. It is also noted that only several thousand planets orbiting their own stars can be currently observed in the Star Systems which are situated relatively near the Solar System. This does not mean that only several thousand planets can exist outside the Solar System we can observe. It is expected that in average by about ten planets can orbit each star of enormous number of Star Systems in our Universe. It is thought that our Universe can accumulate more than 10^{999} stars.

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Krasnoyarsk, Russia, 2012

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TABLE OF CONTENTS: **PAGE**

List of Figures: **PAGE**

INTRODUCTION

Elastic wave propagation in isotropic plates (thin elastic films) was first studied as early as in 1917 by H. Lamb [1]. The revealed features of the plate acoustic waves, also known as the Lamb waves, are as follows: (i) the Lamb waves possess an infinite number of both the symmetric and asymmetric dispersive modes, (ii) the Lamb waves have one symmetric fundamental mode and one asymmetric fundamental mode and the fundamental modes are also called the zero-order modes, (iii) the Lamb waves similar to the surface Rayleigh wave [2] have the polarization in the sagittal plane, namely the in-plane polarization. The nondispersive Rayleigh wave discovered in 1885 by Lord Rayleigh (J.W. Strutt [2]) represents the first type of known surface waves guided by the free surface of isotropic materials. However, the surface Rayleigh type waves like the Lamb waves can be dispersive and possess an enormous number of modes, of which the first is also called the fundamental mode. This occurs in layered systems consisting of a layer on a substrate. It is interesting that the fundamental dispersive mode of the surface Rayleigh type waves and the ones of the Lamb type waves can possess the nondispersive Zakharenko waves [3, 4, 5] which divide the modes into dispersive submodes. Also, the reader can find some works which demonstrate possible existence of the other surface-like waves with the in-plane polarization, for instance, see the theoretical work cited in Ref. [6].

The second type of the surface acoustic waves was discovered in 1911 by A.E.H. Love [6] who has treated a two-layer structure consisting of an isotropic layer on an isotropic substrate. The dispersive Love waves have the anti-plane polarization (perpendicular to the sagittal plane) and can also possess an infinite number of dispersive modes with no fundamental mode for the isotropic case. There is the existence condition for the dispersive surface Love wave such that the speed of the shear-horizontal bulk acoustic wave (SH-BAW) for the substrate must be higher than that for the layer. It is worth noting that the phase velocity (V_{ph}) of the Love wave is confined between these two SH-BAWs for the layer and substrate. In addition to the dispersive Love waves, the slow surface Zakharenko waves [8, 9] can propagate in such layered systems. Two different types of the slow surface Zakharenko waves (SSZWs) can exist and they possess only single dispersive mode with the anti-plane polarization. The SSZW phase velocity cannot be larger than the slower SH-BAW speed of two media and becomes equal to zero at nonzero value of kd, where k is the wavenumber in the direction of wave propagation and d is the layer thickness. For one type of the SSZW, the surface wave can exist even when SH-BAW speed for the substrate is smaller than that for the layer and the single dispersive mode can qualitatively look like the asymmetric (*flexural*) fundamental mode of the Lamb waves [8].

It is necessary to state that the anti-plane polarized surface acoustic waves can be nondispersive. To the end of 1960s, two researchers, Bleustein [10] and Gulyaev [11], have theoretically demonstrated independently from each other that a new type of nondispersive surface acoustic waves can propagate in hexagonal (6 *mm*) piezoelectrics, also known as electro-elastic materials. The nondispersive surface Bleustein-Gulyaev (BG) wave can be guided by the free surface of the transversely isotropic material of class 6 *mm* when the propagation direction is perpendicular to both the sixfold symmetry axis and the surface normal. There exist two surface BG-waves which satisfy different electrical boundary conditions: the slower BG-wave for the electrically closed free surface when the electrical potential is vanishing ($\varphi = 0$) and the faster BG-wave for the electrically open surface when the normal component of the electrical displacements is vanishing ($D_3 = 0$). These two surface BG-waves can also exist in the transversely isotropic piezomagnetics, also known as the magneto-elastic materials: the slower BG-wave for the magnetically open surface when the magnetic potential is vanishing ($\psi = 0$) and the faster BG-wave for the magnetically closed surface when the normal component of the magnetic flux is vanishing ($B_3 = 0$).

Also, Gulyaev and Hickernell [12] have stated that the surface BG-waves cannot propagate in the cubic piezoelectrics. This means that they cannot also propagate in the cubic piezomagnetics. However, this is not true. The slower BG-wave can exist in propagation direction <101> of the cubic piezoelectrics and cubic piezomagnetics, but the faster BG-wave cannot propagate, for example, see Refs. [13, 14, 15]. Also, the existence condition for the surface BG-waves such as the propagation direction should be perpendicular to an evenfold symmetry axis (twofold, fourfold, or sixfold) is not true for the cubic materials. For instance, for all the suitable cuts from direction <100> (*Z*-cut) to direction <001> (*X*-cut) in which the acoustic wave can be coupled with the electrical potential in the cubic piezoelectrics or with the magnetic potential in the cubic piezomagnetics, only direction <101> (*XZ*-cut) can support the propagation of the shear-horizontal surface acoustic waves (SH-SAWs). Indeed, direction <101> can brace the propagation of the slower BG-wave and the ultrasonic surface Zakharenko wave (USZW) because the faster BG-wave cannot propagate in the cubic media. Also, Refs. [13, 14, 15] have solidly demonstrated that the value of the corresponding coupling coefficient (CEMC or CMMC) can influence on the USZW speed in such way that the cubic crystals can be even divided into two groups. The first group is for the cubic crystals with the CEMC (CMMC) $< 1/3$ and the second group is for the ones with the CEMC (CMMC) $> 1/3$, where CEMC is the coefficient of the electromechanical coupling and the CMMC is the coefficient of the magnetomechanical coupling. It is noticed that this gradation is absent for the transversely isotropic materials.

This work is concerned with the acoustic wave propagation with the anti-plane polarization in the two-phase transversely isotropic plates which can simultaneously possess the piezoelectric, piezomagnetic, and magnetoelectric effects. These two-phase materials can be composite materials consisting of the piezoelectric phase and the piezomagnetic phase, as well as native materials possessing these two phases. The problems of acoustic wave propagation in the single-phase materials (crystals) such as piezoelectrics and piezomagnetics are studied already for a long while. Indeed, it is useful first of all to be familiar with the wave propagation in the single-phase materials and the reader can find much work on the subject. For instance, some famous books are cited in Refs. [16-43]. The physical properties of crystals, tensor representations, and elasticity theory can be also found in famous classical books cited in Refs. [44-55]. It is natural to treat the suitable high symmetry propagation directions [37, 56] which can be also found in the two-phase transversely isotropic plates (piezoelectromagnetic plates). The suitable propagation direction in the transversely isotropic piezoelectromagnetic (PEM) plate shown in figure 1 is perpendicular to both the surface normal and the sixfold symmetry axis. This is similar to the transversely isotropic piezoelectric plates and the transversely isotropic piezomagnetic plates. In the configuration shown in figure 1, the in-plane polarized plate waves represent purely mechanical Lamb type waves and propagation of the anti-plane polarized shear-horizontal (SH) waves in such plates can be coupled with both the electrical and magnetic potentials. Indeed, it is useful to investigate the wave propagation in the high symmetry directions because in this case only elastic waves with one type of the polarizations mentioned above can be coupled with the electromagnetic waves. Figure 2 shows both the asymmetrical (flexural) and symmetrical modes of the guided Lamb waves.

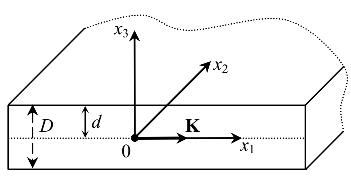

Figure 1. The rectangular coordinate system for the transversely isotropic (6 *mm*) plate of thickness D, where $D = 2d$ and the wavevector **K** is directed along the x_1-axis. The sixfold symmetry axis and the wave polarization are directed along the x_2-axis.

It is obvious that like the single-phase materials such as piezoelectrics and piezomagnetics, the two-phase piezoelectromagnetic (composite) materials can be utilized in various technical devices such as filters, sensors, etc. There is also modern tendency to use suitable two-phase materials to create smart materials and even a laboratory on a single chip in the lab-on-a-chip technology. It is apparent that the application of the piezoelectromagnetic (composite) materials in many technical smart devices can be preferable due to the energy exchange between the electrical and magnetic subsystems via the mechanical subsystem. As a result, a lot of review works cited in Refs. [57-99] can be found in the scientific literature about the various investigations and applications of the two-phase (composite) materials possessing the magnetoelectric effect. Concerning the wave propagation in the piezoelectromagnetics, it is recommended for the reader to read recent books [100, 101, 102] on the subject. Books [100, 101] theoretically investigate the SH-SAW propagation in the piezoelectromagnetics with the cubic symmetry and the hexagonal (6 *mm*) symmetry, respectively. Book [102] published in 2012 has theoretically discovered that twenty two different interfacial SH-waves can be guided by the common interface between two dissimilar transversely isotropic (6 *mm*) piezoelectromagnetics. Also, the reader can find the recent theoretical investigations of the SH-SAWs and interfacial SH-wave propagation in the transversely isotropic cases in Refs. [103-113] when the studied (composite) materials possess the magnetoelectric effect. So, these investigations actually broaden the set of the transversely isotropic materials which can be also used together with piezoelectrics (piezomagnetics) or even instead of them. It is also necessary to mention the pioneer works [114-137] on the foundation and investigations of the magnetoelectric (composite) materials, of which Wood and Austin [136] have discussed the following possible application of such materials, see also work [137]: magnetic-electric energy converting components, solid state nonvolatile memory and solid state memories based on spintronics, multi-state memory which can find application in quantum computing area, electrical/optical polarization components which can find applications in communication, light computing. It is also thought that one of the multi-promising applications of the magnetoelectric materials can be their utilization in biological and chemical sensing technologies. However, the most investigated (therefore, most popular) materials for the sensing applications are piezoelectrics.

Therefore, it is possible to concisely review various acoustic waves' technical devices which can use piezoelectrics for biological and chemical sensing applications.

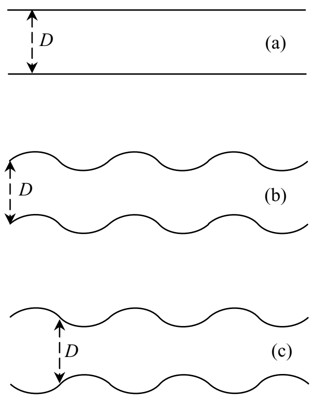

Figure 2. The illustration of (a) the plate of thickness *D* at the rest; (b) asymmetrical (flexural) and (c) symmetrical guided wave modes propagating through the entire thickness of the plate.

Recent reviews [138-142] deal with acoustic wave sensors. Review work [138] is based on more than 40 years of technological and scientific developments of the surface generated acoustic wave (SGAW) technology for biosensing applications. Various technical devices based on the shear-horizontal surface acoustic wave (SH-SAW), surface transverse wave (STW), Love wave (LW), flexural plate wave (FPW), shear-horizontal acoustic plate mode (SH-APM) and layered guided acoustic plate mode (LG-APM) can represent the SGAW devices well-known in the biochemical scientific community. For the last two decades, some of these devices have demonstrated a high sensitivity in the detection of irrelevant molecules in liquid media. During these decades, complementary efforts to improve the sensing films have been done. Future developments of sensor technologies can allow realization of highly sensitive, low cost, small size, multi-channel, portable, reliable and commercially established SGAW biosensors. A created setup with these features can make a significant contribution to future developments in the health, food, and environmental industries. The SGAW biosensors are also likely for the detection of pathogens and this topic is extremely important for the human health. Several commercially available SAW sensors are tabulated in Refs. [141, 142]. Today, these sensors move beyond military and security applications and SAW devices are also moving into the lab-on-a-chip arena [141]. The SAW sensors are competitively priced, and some of them can be passively and wirelessly interrogated.

It is worth mentioning that depending on the method of signal transduction [138], biosensors can be divided into four basic groups such as optical, mass, electrochemical, and thermal. Acoustic wave biosensors represent mass sensors and they operate with mechanical acoustic waves as their transduction mechanism. Depending on the acoustic wave guiding process [143], acoustic wave devices can be also classified into three following groups: BAW, SAW, and APM devices. In BAW devices, the acoustic wave propagates unguided through the volume of the bulk material (substrate). In SAW devices, the acoustic waves can be guided or unguided and propagate along a single surface of the substrate. In APM devices, the acoustic waves are guided by the reflection from multiple surfaces. Figure 3 shows the configuration of the APM device that can be the SH-APM biosensor. The interdigital transducers fabricated on the surface of the material generate the high-amplitude acoustic wave that travels in the plate. Usually, the piezoelectric material is quartz, but GaAs and $LiTaO_3$ substrates have also been used. The piezoelectric material converts any possible changes into a measured frequency change, which can also be described as the acoustic wave velocity. The APM device can be also based on the Lamb waves known as the complex waves traveling through the entire plate. Different families of the Lamb wave modes can be distinguished including symmetrical modes (in-phase displacements of opposite plate surfaces) and asymmetrical or flexural modes (anti-phase displacements of opposite plate surfaces) as shown in figure 2.

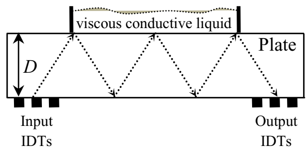

Figure 3. The configuration of the biosensor based on the shear-horizontal acoustic plate mode (SH-APM). The plate SH-waves are generated by the input IDTs and detected by the output IDTs.

The piezoelectric devices actually represent a cost-effective alternative to the other popular transducers for biosensors such as the advanced optical approaches [144]. Among piezoelectric biosensors, QCM-based applications have comprehensively been reviewed [140]. Some approaches to the SGAW biosensors based on STW [145, 146, 147], SH-APM [148-152], and LG-APM [153] devices have been also reported. However, none of these approaches address the detection of pathogens. Ref. [138] also presents that piezoelectric SGAW-based devices such as SH-SAW, LW, and FPW biosensor transducers can be successfully applied to the pathogen detection. The reader can also find some selected investigations of the Lamb waves for different applications cited in Refs. [154-190].

Let's return to the review of investigations of the magnetoelectroelastic materials (piezoelectromagnetics). In the last decade, the reader can find that several research groups have an interest in the investigations of the plate waves in the piezoelectromagnetic materials. Recently published paper [191] used a meshless method based on the local Petrov-Galerkin approach to solve static and dynamic problems of two-layer piezoelectromagnetic composites (piezoelectric layer and piezomagnetic layer) with specific properties. The authors of paper [191] have treated various boundary conditions and geometric parameters to analyze their influence on the value of the electromagnetic parameter and also analyzed the composites under a purely magnetic or combined magneto-mechanical load. They have found that the magnetoelectric effect is dependent on the ratio of the layer thicknesses and also considered functionally graded (FG) material properties of the piezoelectric layer and homogeneous properties of the piezomagnetic layer.

The interest in the problems of the plate wave propagation in the functionally graded two-phase plates [192-197] is growing because the FG materials have a potential to reduce the stress concentration and to increase the fracture toughness. Due to the complexity, plane elasticity problems involving cracks in the FG materials are solved assuming a functional form for variations in material properties, usually a linear or exponential function. It is well-known that the multifunctional materials such as the piezoelectromagnetic composites are extensively used in the modern technical devices, such as sensors, transducers, actuator components, etc. These composites possessing piezoelectric, piezomagnetic, and magnetoelectric properties are obviously sensitive to elastic, electric, and magnetic fields. Some composites can be very brittle and susceptible to fracture that can restrict their employment. Therefore, it is important to understand and to analyze the fracture characteristics of the materials in order to obtain reliable service life predictions for pertinent devices.

Using the state space approach, Ref. [198] numerically studies the bulk wave propagation in laminated piezomagnetic/piezoelectric plates with initial stresses and imperfect interface when either electrically and magnetically open conditions or shorted ones on the top and bottom surfaces are considered. The obtained numerical results indicate that the initial compressive stress can reduce the phase velocity of the wave propagating in a layered multiferroic structure. Also, the imperfect interface can affect the frequency spectra much larger than initial stresses because imperfection can lessen the structural stiffness. The authors of paper [199] have also investigated the dispersion behavior of acoustic waves in multiferroic plates with imperfect interfacial bonding via the method of reverberation-ray matrix, which is directly established from the three-dimensional equations of magnetoelectroelasticity in the form of state space formalism. They have employed a generalized spring-layer model to characterize the interfacial imperfection and numerically calculated the dispersion curves and mode shapes for a typical sandwich piezoelectric/piezomagnetic plate. Also, they have investigated the influence of different interfacial bonding conditions on the dispersion characteristics and corresponding mode shapes and demonstrated that the obtained results are unconditionally stable in comparison with the traditional state space method.

A three-dimensional spectral element method (3D-SEM) was recently developed in Ref. [200] for analysis of propagation of the Lamb waves in composite laminates containing a delamination. Ref. [200] has stated that this method can be more efficient in simulating wave propagation in structures than the conventional finite element method (FEM) because of its unique diagonal form of the mass matrix. Using 3D spectral finite elements, it is possible to simulate three types of composite laminates such as unidirectional-ply laminates, cross-ply laminates, and angle-ply laminates and to evaluate different interactions of the Lamb wave modes with delamination. It has also demonstrated that the symmetric Lamb wave mode may be insensitive to delamination at certain interfaces of laminates, while the asymmetric mode can be suitable for identification of delamination in composite structures. The other recently published paper [201] examines the effect of inclusion shapes, inclusion contents, inclusion elastic constants, and plate thickness on the dispersion relations and modes of wave propagation in inclusion-reinforced composite plates. Ref. [201] has modeled the spheroid-like shapes of inclusions that enable the composite reinforcement geometrical configurations ranging from sphere to short and continuous fiber. This paper also used the Mori–Tanaka mean-field theory to predict the effective elastic moduli of the composite plate which can elucidate the effect of inclusion’s shape, stiffness, and volume fraction on the composite’s anisotropic elastic behavior. Using the resulting moduli in the dynamic stiffness matrix method, the authors of paper [201] can then determine the dispersion relations and the modal patterns of the Lamb waves and also state that the inclusion contents, aspect ratios, and plate thickness affect the propagation velocities, higher-order mode cutoff frequencies, and modal patterns.

Considering the significant nonlinear magnetoelectric (ME) characteristics in laminated composites, the authors of paper [202] have built a numerical model of magnetic-mechanical-electric coupling effect based on the nonlinear magnetostrictive constitutive relation. The change of the ME field coefficients with bias magnetic field predicted by their model can demonstrate a good agreement with the experimental results. This paper also predicts the magnetoelectric conversion characteristics of the composites, calculates and analyzes the influence of the magnetostrictive layer thickness ratio, composite geometrical size, saturation magnetization, and types of piezoelectric materials on the ME conversion coefficient of such composites. Report [203] has discussed a novel frequency multiplier based on the (ME) effect in a simple multiferroic laminate, made up of an amorphous Metglas (FeBSiC) layer bonded onto a $Pb(Zr,Ti)O_3$ plate wrapped with a coil. This report has demonstrated that by applying an input signal with a certain frequency to the coil, an output signal with the doubled value of the frequency can be generated from the PZT plate due to the ME coupling. It also states that this ME laminate-based device can be operated in a broad frequency range and switched by a low bias magnetic field, offering potential opportunities for frequency multipliers in electrical applications. Ref. [204] has investigated elastic wave propagation in two-layer piezoelectric/piezomagnetic plate when the layers are transversely isotropic and perfectly bonded along the interface. In work [204], the upper and lower surfaces of the plate are traction-free but subjected to four types of the electric and magnetic boundary conditions. Utilizing these conditions, the dispersion equations are given in matrix form. This paper has also provided numerical examples for four kinds of the plates composed of piezomagnetic $CoFe_2O_4$ and piezoelectric $BaTiO_3$, PZT-5A, PZT-2, and PZT-4. It also discussed the influences of the boundary conditions, thickness ratio, and piezoelectric material properties on dispersion characteristics and stated that the results can be helpful for the applications of piezoelectric/piezomagnetic composites or structures in acoustic wave and microwave devices.

Based on the 3D linear elastic equations and magnetoelectroelastic (MEE) constitutive relations, propagation of the Lamb waves in an infinite MEE plate is investigated in Ref. [205] for both electrically and magnetically open and shorted cases. The mechanical, electrical, and magnetic responses of the symmetric and antisymmetric fundamental modes are also discussed in Ref. [205] which has obtained results valuable for the analysis and design of broadband ME transducer using composites. Exploiting the finite-element method, the ME coupling effect in a bi-layered magnetostrictive-piezoelectric ($Tb_{0.3}Dy_{0.7}Fe_{1.9}$-$PbZr_{0.52}Ti_{0.48}O_3$) composite structure is numerically simulated in Ref. [206]. This paper has used the numerical algorithm based on a synchronization of the mechanical coupling and resonance between the two layers along the bonding interface. It has also concluded that a significant enhancement of the magnetoelectric effect by optimizing the thicknesses of the two layers is possible.

In paper [207], the anti-plane problem for an interfacial crack between two dissimilar magnetoelectroelastic plates subjected to anti-plane mechanical and in-plane magneto-electrical impact loadings is investigated. Paper [207] adopts four cases of crack surface conditions: magneto-electrically impermeable, magnetically impermeable and electrically permeable, magnetically permeable and electrically impermeable, magneto-electrically permeable. For the first three cases, the effects of loading combination parameters on dynamic energy release rate were demonstrated. Since the magneto-electrically permeable condition is perhaps more physically reasonable for type III crack, the effect of the crack configuration on the dynamic fracture behavior of the crack tips is also studied in Ref. [207] for the fourth case. It is thought that these results can be useful for the design of multilayered magnetoelectroelastic structures and devices. Ref. [208] has developed 3D exact theory for the bending problem of a multiferroic rectangular plate with ME coupling and imperfect interfaces and proposed a generalized spring layer model to characterize the imperfection of the bonding behavior at the interfaces. This theory can particular adopt the linear relation between the electric displacement and the jump of electrical potential, the corresponding one for the magnetic field, and linear relations among different physical fields.

By introducing two displacement functions and two stress functions, the governing equations of the linear theory of magneto-electro-thermo-elasticity with transverse isotropy are simplified in Ref. [209]. On selecting certain physical quantities as the basic unknowns, this paper establishes two new state equations. It also accounts the material inhomogeneity along the sixfold symmetry axis and employs an approximate laminate model to facilitate deriving analytical solutions. Ref. [209] has also treated a functionally graded plate and studied the effect of ME coupling in a $BaTiO_3$-$CoFe_2O_4$ composite predicted from the micromechanics simulation. Buchanan [210] has compared frequencies of vibration for the layered materials versus the multiphase materials as a measure of the accurateness of the derived material constants. Also, the reader can find some works [211, 212] concerning investigations of multilayered piezoelectromagnetic plates.

Finally, the paper cited in Ref. [213] presents some useful discussions on the shear-horizontal (SH) acoustic waves in an inhomogeneous piezoelectromagnetic plate. This paper studies the hexagonal (6 *mm*) material polarized in the SH direction and assumes that the material constants of the plate continuously vary along the thickness direction. Solving the coupled field equations, the authors of work [213] have obtained solutions of the mechanical displacement, electrical and magnetic potentials. They have also considered the influence of the inhomogeneity of the material constants on the phase velocity and stated that their findings are significant in the applications of wave propagation in such structures. The propagation of SH guided waves in a coupled piezoelectric/piezomagnetic plate is also studied in Ref. [214]. Both the layers are transversely isotropic of class 6 *mm* and perfectly bonded along the interface. Ref. [214] has assumed the mechanically free, electrically open, and magnetically closed case for the upper and lower surfaces of the plate. It also obtained the dispersion relations for two different cases: the bulk SH-wave velocity of piezoelectrics is larger or smaller than that of piezomagnetics. The obtained numerical results have also demonstrated that the phase velocity approaches the smaller bulk SH-wave velocity in the two-layer system with the increase in the wavenumber for different modes. It was also found that the thickness ratio and properties of the piezoelectrics have a large effect on the dispersion behaviors.

All the recent works mentioned above have confirmed the fact that various investigations of the magnetoelectric (two-layer) plates can be found in the modern scientific literature because the research community has a growing interest in this matter. However, it is thought that a comprehensive research work on the SH-wave propagation in the piezoelectromagnetic plates does not exist. The author of this work has represented below the theoretical investigations of the SH-wave propagation in the piezoelectromagnetic homogeneous plates which can represent the native magnetoelectric hexagonal (6 *mm*) materials (crystals) and the two-phase composites.

Chapter I

Theory of Wave Propagation in Piezoelectromagnetic Plates

This chapter aims to acquaint the reader with the thermodynamics, constitutive relations, equations of motion, and boundary conditions. For a transversely isotropic piezoelectromagnetics of class 6 *mm*, it is possible to study shear-horizontal (SH) wave propagation in the plate using the rectangular coordinate system shown in figure 1, see in the previous section. Like the previous studies in books [101, 102] of the wave propagation in such materials, the plate SH-waves propagate along the x_1-axis which is perpendicular to both the surface normal directed along the x_3-axis and the sixfold symmetry axis managed along the x_2-axis. It is natural to primarily describe the suitable thermodynamic variables and functions for this case. It is also assumed that the piezoelectromagnetic plate (thin film) possesses the material properties of bulk material.

I.1. Thermodynamic Variables and Functions

It is possible to consider a bulk piezoelectromagnetic solid which simultaneously possesses the piezoelectric, piezomagnetic, and magnetoelectric effects. This complex two-phase system can be thermodynamically described by means of suitable thermodynamic variables and functions. Indeed, it is necessary to choose a thermodynamic potential to properly describe thermoelectromagnetoelastic interactions in a piezoelectromagnetic solid. In general, eight thermodynamic potentials are used and it is thought that in this case the thermodynamic potential called enthalpy H_e can be utilized in order to obtain adiabatic rather than isothermal conditions. It is wellknown that an adiabatic process can be considered as that with the constant entropy S. The entropy S as a thermodynamic variable represents a level of disorder in the system. Treating a linear case, it is possible to account only linear terms in a Taylor series for the enthalpy H_e relative to an equilibrium condition $H_e(S_0)$. It is clear that the constant entropy $S = S_0$ actually gives $\mathrm{d}S = 0$. Therefore, this thermodynamic variable can be excluded from the further consideration, for instance, see in books [100, 101, 102].

These linear terms in a Taylor series for the suitable thermodynamic potential can contain the following thermodynamic variables frequently written in the tensor forms: strain η_{ij}, electrical field E_i, and magnetic field H_i, where the indexes i and j run from 1 to 3. For a piezoelectromagnetics, energetic terms of such complex system described by a thermodynamic potential can be naturally coupled with the following subsystems: elastic subsystem (thermodynamic variable η_{ij}), electric subsystem (variable E_i), magnetic subsystem (variable H_i) and thermal subsystem (entropy S). Therefore, for the likely thermodynamic potential T, one can write the following:

$$T = f(\eta_{kl}, E_k, H_k, S) \tag{I.1}$$

$$\mathrm{d}T = f_0(\mathrm{d}\eta_{kl}, \mathrm{d}E_k, \mathrm{d}H_k, \mathrm{d}S = 0) \tag{I.2}$$

I.2. Constitutive Relations

It is thought that for the problem of acoustic wave propagation in a piezoelectromagnetic solid, it is natural to use the following three thermodynamic functions: stress σ_{ij}, electrical induction D_i, and magnetic induction B_i. The electrical induction D_i is also known as the electrical displacement and the magnetic induction B_i is also called the magnetic displacement or magnetic flux. These thermodynamic functions depend on three independent thermodynamic variables described above: the strain η_{ij}, electrical field E_i, and magnetic field H_i. These three functions are written as follows [100, 101, 102]:

$$\sigma_{ij} = f_1(\eta_{kl}, E_k, H_k) \tag{I.3}$$

$$D_i = f_2(\eta_{kl}, E_k, H_k) \tag{I.4}$$

$$B_i = f_3(\eta_{kl}, E_k, H_k) \tag{I.5}$$

In the linear case, three thermodynamic functions written above depend on three independent thermodynamic mechanical (η_{ij}), electrical (E_i), and magnetic (H_i) variables. For a three-dimensional piezoelectromagnetic solid, the coupled constitutive relations read:

$$\sigma_{ij} = C_{ijkl}\eta_{kl} - e_{kij}E_k - h_{kij}H_k \tag{I.6}$$

$$D_i = e_{ikl}\eta_{kl} + \varepsilon_{ik}E_k + \alpha_{ik}H_k \tag{I.7}$$

$$B_i = h_{ikl}\eta_{kl} + \alpha_{ik}E_k + \mu_{ik}H_k \tag{I.8}$$

In these coupled constitutive relations written above, the used indices i, j, k, and l run from 1 to 3. It is clearly seen in equation (I.6) that the mechanical thermodynamic function such as the stress σ_{ij} also depends on the corresponding factors at the independent thermodynamic mechanical (η_{ij}), electrical (E_i), and magnetic (H_i) variables. These factors represent the corresponding proportionality coefficients for the linear case and are thermodynamically define in the following subsection. They are called the elastic stiffness constants C_{ijkl}, piezoelectric constants e_{kij}, and piezomagnetic coefficients h_{kij}. In equation (I.7), the electrical thermodynamic function such as the electrical displacement D_i also depends on the corresponding factors at the thermodynamic variables, of which the last two are called the dielectric permittivity coefficients ε_{ik} and the electromagnetic constants α_{ik}. In equation (I.8), the magnetic thermodynamic function such as the magnetic displacement B_i also depends on the corresponding factors at the thermodynamic variables, of which the last is called the magnetic permeability coefficients or magnetic constants μ_{ik}. These material constants ε_{ik}, μ_{ik}, and α_{ik} will be also thermodynamically defined in the following subsection.

In equations from (I.6) to (I.8), the first independent thermodynamic variable such as the strain tensor η_{ij} can be defined by the following wellknown relation between the strain and the mechanical displacements for small perturbations:

$$\eta_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{I.9}$$

In relation (I.9), the indices i and j also run from 1 to 3. It is necessary to state that expression (I.9) represents the dependence of the strain tensor components η_{ij} on the corresponding partial first derivatives of the mechanical displacement components U_1, U_2, and U_3 with respect to the real space components x_1, x_2, and x_3.

In equations from (I.6) to (I.8), the second independent thermodynamic variable such as the electrical field E_i can be also defined by the corresponding partial first derivatives. Using the electrical potential φ in the case of the quasi-static (irrotational field) approximation, the components the electrical field (E_i) can be determined as the following partial first derivatives with respect to the real space components such as x_1, x_2, and x_3:

$$E_i = -\frac{\partial \varphi}{\partial x_i} \tag{I.10}$$

Also, the third independent thermodynamic variables such as the magnetic field H_i participating in equations from (I.4) to (I.6) can be also defined by the corresponding partial first derivatives. Utilizing the magnetic potential ψ in the quasi-static approximation, the components of the magnetic field H_i are defined by the following partial first derivatives with respect to the components x_1, x_2, and x_3:

$$H_i = -\frac{\partial \psi}{\partial x_i} \tag{I.11}$$

It is worth mentioning that to exploit the quasi-static approximation when all the derivatives with respect to time t in the corresponding Maxwell equations and definitions (I.10) and (I.11) are omitted is common. This approximation can be used because the speed of the electromagnetic wave is approximately five orders larger than the speed of any elastic wave [29, 37].

I.3. Definitions of Material Constants

In the coupled constitutive relations from (I.6) to (I.8), all the material parameters such as C_{ijkl}, e_{kij}, h_{kij}, ε_{ik}, μ_{ik}, and α_{ik} can be thermodynamically expressed. In equations (I.7) and (I.8), the electromagnetic constants α_{ik} can be defined by the following thermodynamic relations:

$$\alpha_{ik} = \left(\frac{\partial D_i}{\partial H_k}\right)_{\eta,E=\text{const}} = \left(\frac{\partial B_i}{\partial E_k}\right)_{\eta,H=\text{const}} \tag{I.12}$$

Employing equation (I.7), the thermodynamic definition for the dielectric permittivity coefficients ε_{ik} reads:

$$\varepsilon_{ik} = \left(\frac{\partial D_i}{\partial E_k}\right)_{\eta,H=\text{const}} \tag{I.13}$$

It is apparent that the magnetic permeability coefficients μ_{ik} in equation (I.8) can be thermodynamically expressed as follows:

$$\mu_{ik} = \left(\frac{\partial B_i}{\partial H_k}\right)_{\eta,E=\text{const}} \tag{I.14}$$

In the thermodynamic relations from (I.12) to (I.14), the electromagnetic constants α_{ik}, dielectric permittivity coefficients ε_{ik}, and magnetic permeability coefficients μ_{ik} stand for the following symmetric tensors of the second rank (matrices): $\alpha_{ik} = \alpha_{ki}$, $\varepsilon_{ik} = \varepsilon_{ki}$, and $\mu_{ik} = \mu_{ki}$. Indeed, the components of the tensors ε_{ik}, μ_{ik}, and α_{ik} are naturally written as (3×3) symmetric matrices [100, 101, 102].

With equations (I.6) and (I.7), the thermodynamic description of the piezoelectric constants e_{kij} can be given by the following definitions:

$$e_{ijk} = -\left(\frac{\partial \sigma_{ij}}{\partial E_k}\right)_{\eta, H=\text{const}} = e_{ikl} = \left(\frac{\partial D_i}{\partial \eta_{kl}}\right)_{E, H=\text{const}} \tag{I.15}$$

Using relations (I.6) and (I.8), it is also visible that the thermodynamic forms of the piezomagnetic coefficients h_{kij} can be obtained as follows:

$$h_{ijk} = -\left(\frac{\partial \sigma_{ij}}{\partial H_k}\right)_{\eta, E=\text{const}} = h_{ikl} = \left(\frac{\partial B_i}{\partial \eta_{kl}}\right)_{E, H=\text{const}} \tag{I.16}$$

It is necessary to state that the quantities of both the material parameters h_{kij} and e_{kij} can be decreased. The symmetry arguments such as $\sigma_{ij} = \sigma_{ji}$ and $\eta_{ij} = \eta_{ji}$ in equations (I.15) and (I.16) can also demonstrate the corresponding degrees of symmetry for the material tensors e_{kij} and h_{kij}. The symmetry influences allow the existence of the following equalities:

$$e_{kij} = e_{ijk} = e_{kji} = e_{jik} \tag{I.17}$$

$$h_{kij} = h_{ijk} = h_{kji} = h_{jik} \tag{I.18}$$

Utilizing Voigt's notation, the (3×3×3) tensor forms for the piezoelectric constants e_{kij} and piezomagnetic coefficients h_{kij} can be rewritten as the asymmetric (6×3) or (3×6) matrices: $e_{kij} \rightarrow e_{kP}$ or $e_{ijk} \rightarrow e_{Pk}$, $h_{kij} \rightarrow h_{kP}$ or $h_{ijk} \rightarrow h_{Pk}$, where the index P runs from 1 to 6.

Finally, expression (I.6) can be soundly employed for the thermodynamic definition of the elastic stiffness constants C_{ijkl}. These material parameters can be naturally defined as follows:

$$C_{ijkl} = \left(\frac{\partial \sigma_{ij}}{\partial \eta_{kl}} \right)_{E,H=\text{const}} \tag{I.19}$$

Thermodynamic definition (I.19) for the material parameters C_{ijkl} states that they can be determined at constant electrical and magnetic fields. Symmetry arguments allow some simplifications of the quantity of the stiffness constants C_{ijkl} because the stress and strain tensors are symmetric: $\sigma_{ij} = \sigma_{ji}$ and $\eta_{ij} = \eta_{ji}$. Therefore, the stiffness tensor C_{ijkl} must also possess a corresponding degree of symmetry resulting in the following simplifications:

$$C_{ijkl} = C_{klij} = C_{jikl} = C_{klji} = C_{ijlk} = C_{lkij} = C_{jilk} = C_{lkji} \tag{I.20}$$

Using Voigt's notation, the (3×3×3×3) tensor form for the elastic stiffness constants C_{ijkl} defined by expression (I.19) can be compactly written in a form of (6×6) symmetric matrix [29, 37, 44-54]. The transformation procedure of a tensor form into a matrix is wellknown. For this purpose, the following rules are used for the indices: 11 → 1, 22 → 2, 33 → 3, 23 → 4, 13 → 5, 12 → 6. So, the indices are changed as $ijkl \rightarrow PQ$ and $C_{ijkl} \rightarrow C_{PQ}$ where the indices P and Q run from 1 to 6.

It is also necessary to mention for the reader that all the material tensors defined above by the corresponding thermodynamic relations can be transformed from an original coordinate system into a required new one. The original coordinate system usually represents a crystallographic coordinate system. It is necessary to rotate around the x_1-axis, x_2-axis, or x_3-axis in order to obtain a new propagation direction in the new coordinate system called the work coordinate system. The new propagation direction must be directed along the x_1-axis in the work coordinate system. This situation requires a recalculation of all the values of the independent material constants. Therefore, the number of independent material constants and their values must be recalculated. It is obvious that the values of the new material constants are obtained using the values of the old ones. Exploiting the rules for tensor transformations [29, 37, 44], some new values of the material constants with the indexes i, j, k, and l can be obtained by application of the transformation matrices such as a_{im}, a_{jn}, a_{kp}, and a_{lq} to the original values of the material constants with the indexes m, n, p, and q. Therefore, the transformation formulae for all the material tensors introduced above read:

$$C_{ijkl} = a_{im}a_{jn}a_{kp}a_{lq}C_{mnpq} \tag{I.21}$$

$$h_{ijk} = a_{im}a_{jn}a_{kp}h_{mnp} \tag{I.22}$$

$$e_{ijk} = a_{im}a_{jn}a_{kp}e_{mnp} \tag{I.23}$$

$$\alpha_{ij} = a_{im}a_{jn}\alpha_{mn} \tag{I.24}$$

$$\mu_{ij} = a_{im}a_{jn}\mu_{mn} \tag{I.25}$$

$$\varepsilon_{ij} = a_{im}a_{jn}\varepsilon_{mn} \tag{I.26}$$

It is flagrant that after completion of these complicated transformations, the tensors of the material parameters in equations from (I.21) to (I.23) can be also written in their corresponding matrix forms discussed above. It is blatant that these matrix forms are more convenient for the further theoretical considerations.

I.4. *Equations of Motion*

It is wellknown in the physical acoustics that the speed of the electromagnetic waves is approximately five orders larger than that of the acoustic waves. Indeed, the acoustic waves propagating in solids are extremely slow compared with the electromagnetic waves propagating in the same materials. However, propagation of the acoustic waves can be coupled with both the electrical potential φ and the magnetic potential ψ in the quasi-static (irrotational field) approximation. Therefore, the Maxwell four field equations [36] of the electromagnetic theory must be naturally used and also applied to the piezoelectromagnetic solid. Maxwell has creatively formulated the laws of electrostatics, magnetostatics, and electromagnetism. The electrostatic and magnetostatic equilibrium equations can be written using the differential forms of the corresponding Maxwell equations which can be written as follows:

$$\mathrm{div}\mathbf{D} = 0 \text{ and } \mathrm{div}\mathbf{B} = 0 \tag{I.27}$$

The first equality in equations (I.27) with the electrical displacement vector **D** represents Gauss's law without free charge and currents and the second equality represents a divergence of the magnetic flux vector **B**. It is well-known that the mathematical operator such as "div" means a divergence that can convert any vector into a scalar.

Exploiting Maxwell's equations (I.27), the governing electrostatic and magnetostatic equilibrium equations can be respectively expressed as follows:

$$\frac{\partial D_i}{\partial x_i} = 0 \text{ and } \frac{\partial B_i}{\partial x_i} = 0 \tag{I.28}$$

Equations (I.28) represent the partial first derivatives of the electrical displacement components D_i and the magnetic displacement components B_i with respect to the real space components x_i, where the index i runs from 1 to 3.

Besides, the governing mechanical equilibrium equation is also written as the following partial first derivative of the stress tensor components σ_{ij} with respect to the real space components x_j, where the indexes i and j run from 1 to 3:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0 \tag{I.29}$$

With equation (I.29), wave motions of a piezoelectromagnetic material in dependence on time t can be described by the equation of motion written in the following well-known equality [37, 38]:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 U_i}{\partial t^2} \tag{I.30}$$

where ρ is the mass density of the piezoelectromagnetic bulk material. On the right-hand side in equation (I.30), the partial second derivatives of the mechanical displacement components U_i with respect to time t represent corresponding accelerations with the dimension of m/s^2.

In addition to equation of motion (I.30), it is necessary to account the electrostatics and magnetostatics in the quasi-static approximation:

$$\frac{\partial D_i}{\partial x_j} = 0 \text{ and } \frac{\partial B_i}{\partial x_j} = 0 \tag{I.31}$$

It is obvious that equation (I.30) is coupled with equations (I.31) because the mechanical (σ_{ij}), electrical (D_i), and magnetic (B_i) thermodynamic functions respectively defined by equations from (I.6) to (I.8) depend on the mechanical (η_{ij}), electrical (E_i), and magnetic (H_i) thermodynamic variables. The coupled equations of motion, namely equations (I.30) and (I.31) can be readily written in an expended form when the mechanical displacements U_i, electrical potential φ, and magnetic potential ψ are exploited. Employing equations from (I.9) to (I.11) for coupled equations (I.30) and (I.31) written above, the coupled equations of motion are then composed as follows:

$$\rho \frac{\partial^2 U_i}{\partial t^2} = C_{ijkl} \frac{\partial^2 U_l}{\partial x_j \partial x_k} + e_{kij} \frac{\partial^2 \varphi}{\partial x_j \partial x_k} + h_{kij} \frac{\partial^2 \psi}{\partial x_j \partial x_k} \tag{I.32}$$

$$0 = e_{ijk}\frac{\partial^2 U_k}{\partial x_i \partial x_j} - \varepsilon_{ij}\frac{\partial^2 \varphi}{\partial x_i \partial x_j} - \alpha_{ij}\frac{\partial^2 \psi}{\partial x_i \partial x_j} \tag{I.33}$$

$$0 = h_{ijk}\frac{\partial^2 U_k}{\partial x_i \partial x_j} - \alpha_{ij}\frac{\partial^2 \varphi}{\partial x_i \partial x_j} - \mu_{ij}\frac{\partial^2 \psi}{\partial x_i \partial x_j} \tag{I.34}$$

In equations from (I.32) to (I.34), the indexes i, j, k, and l run from 1 to 3. Homogeneous equations from (I.32) to (I.34) represent five partial differential equations of the second order. These coupled equations of motion constitute the wave propagation in a piezoelectromagnetics possessing the piezoelectric, piezomagnetic, and piezoelectromagnetic effects.

I.5. *Tensor Form of Equations of Motion*

It is also possible to write down the wellknown tensor form for the differential form of the coupled equations of motion written in the previous subsection. First of all, it is required to state that these homogeneous partial differential equations of the second order written above must have solutions in the plane wave forms [37, 38]. Therefore, these solutions read:

$$U_I = U_I^0 \exp[\mathrm{j}(k_1 x_1 + k_2 x_2 + k_3 x_3 - \omega t)] \tag{I.35}$$

where the index I runs from 1 to 5 and there is the following: $U_I = U_i$ for $I = i$, $U_4 = \varphi$, and $U_5 = \psi$. Also, U_I^0, $\mathrm{j} = (-1)^{1/2}$, and ω stand for the initial amplitudes, imaginary unity, and angular frequency, respectively. The values of U_1^0, U_2^0, U_3^0, $U_4^0 = \varphi^0$, and $U_5^0 = \psi^0$ should be determined further and the angular frequency is defined by the linear frequency ν, namely $\omega = 2\pi\nu$. In equation (I.35), the parameters such as k_1, k_2, and k_3 are the components of the wavevector **K** directed towards the wave propagation. Also, it is possible to write that the following equality occurs: $(k_1, k_2, k_3) = k(n_1, n_2, n_3)$ where n_1, n_2, and n_3 are the directional cosines. For convenience, they can be defined by $n_1 = 1$, $n_2 = 0$, and $n_3 \equiv n_3$. It is worth noting that the wavenumber k in the direction of wave propagation depends on the wavelength λ as follows: $k = 2\pi/\lambda$.

It is transparent that the utilization of solutions (I.35) and the directional cosines for corresponding substitutions into the differential form of coupled five homogeneous equations from (I.32) to (I.34) can actually lead to five equations written in tensor forms. These five homogeneous equations can be naturally written in the following compact form [100, 101, 102]:

$$\left(GL_{IJ} - \delta_{IJ}\rho V_{ph}\right)U_I^0 = 0 \tag{I.36}$$

where the indices I and J run from 1 to 5 and the phase velocity is defined by

$$V_{ph} = \omega / k \tag{I.37}$$

Also, in the parentheses on the left-hand side in equation (I.36), GL_{IJ} stands for the components of the modified tensor in the well-known Green-Christoffel equation [100, 101, 102] and δ_{IJ} represents the Kronecker delta-function with the following conditions: $\delta_{IJ} = 1$ for $I = J$, $\delta_{IJ} = 0$ for $I \neq J$, and $\delta_{44} = \delta_{55} = 0$. It is also central to state that the modified Green-Christoffel tensor GL_{IJ} is symmetric, i.e. $GL_{IJ} = GL_{JI}$. For that reason, it has only 15 independent tensor components.

Five homogeneous equations written in compact form (I.36) represent the common problem for determination of the eigenvalues and eigenvectors. In this case, the suitable values of n_3 for the corresponding phase velocity represent the eigenvalues and a corresponding eigenvector should exist for each of the suitable eigenvalues. In the common case, the eigenvector can be expressed in the following form:

$$\left(U_1^0, U_2^0, U_3^0, U_4^0 = \varphi^0, U_5^0 = \psi^0\right) \tag{I.38}$$

According to excellent book [37] and classical work [56], it is possible to find high symmetry propagation direction in crystals relating to all classes of symmetry, but the lowest triclinic symmetry. In the high symmetry propagation directions, it is possible that tensor form (I.36) of the coupled equations of motion can consist of two independent sets of homogeneous equations. Indeed, some *GL*-tensor components can become equal to zero when acoustic waves propagate in certain directions on certain cuts. In some certain directions [37, 56] of wave propagation, the in-plane polarized waves can be coupled with both the electrical and magnetic potentials and the anti-plane polarized waves represent purely mechanical waves. Therefore, the corresponding eigenvectors are respectively written as follows:

$$\left(U_1^0, U_3^0, U_4^0, U_5^0\right) \text{ and } \left(U_2^0\right) \tag{I.39}$$

In the other certain directions [37, 56] the in-plane polarized waves represent purely mechanical waves and the anti-plane polarized waves can be coupled with both the electrical and magnetic potentials. As a result, the reader can find that this case corresponds to the following eigenvectors:

$$\left(U_1^0, U_3^0\right) \text{ and } \left(U_2^0, U_4^0, U_5^0\right) \tag{I.40}$$

This work has an interest in the theoretical investigation of the piezoelectromagnetic plate SH-waves coupled with both the electrical and magnetic potentials. Thus, the second eigenvector in expression (I.40) is of the interest. Therefore, the following subsection demonstrates the simplifications for this case.

I.6. *High Symmetry Propagation Directions for SH-Waves*

This subsection acquaints the reader with the suitable high symmetry propagation directions in the transversely isotropic piezoelectromagnetics of class 6 *mm*. There are certain cuts and certain propagation directions in the transversely isotropic piezoelectromagnetic materials [37, 56, 100, 101, 102] in which the propagation of the pure SH-waves with the anti-plane polarization can be coupled with both the electrical and magnetic potentials. In the transversely isotropic (6 *mm*) piezoelectromagnetic plate shown in figure 1, the suitable propagation direction is managed along the x_1-axis in the work coordinate system (x_1, x_2, x_3) in which the sixfold symmetry axis is directed along the x_2-axis. The work coordinate system was obtained from the original coordinate system (x'_1, x'_2, x'_3) in which the sixfold symmetry axis is directed along the surface normal. It is necessary to state that in the obtained work coordinate system (x_1, x_2, x_3), any rotation around the x_2-axis is appropriate and give suitable cuts for the SH-wave propagation. In this case, the SH-wave has the mechanical displacement component directed along the x_2-axis. In the studied propagation direction, the coupled equations of motion written in compact tensor form (I.36) can be actually decomposed [37, 56]. This decomposition allows one to separately write the equations of motion for the in-plane polarized waves and those for the anti-plane polarized waves.

Using equation (I.36) with the second eigenvector in expression (I.40), the SH-wave propagation coupled with both the electrical potential φ and the magnetic potential ψ can be then expressed by the following three homogeneous equations written in the matrix form:

$$\begin{pmatrix} GL_{22}-\rho V_{ph}^2 & GL_{24} & GL_{25} \\ GL_{42} & GL_{44} & GL_{45} \\ GL_{52} & GL_{54} & GL_{55} \end{pmatrix} \begin{pmatrix} U^0 \\ \varphi^0 \\ \psi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{I.41}$$

In equations (I.41), the eigenvector has the following components:

$$\left(U^0, \varphi^0, \psi^0\right) = \left(U_2^0, U_4^0, U_5^0\right) \tag{I.42}$$

The suitable eigenvalues n_3 can be found when the determinant of the coefficient matrix in equations (I.41) equals to zero. Therefore, it is possible to write down the following determinant:

$$\begin{vmatrix} GL_{22}-\rho V_{ph}^2 & GL_{24} & GL_{25} \\ GL_{42} & GL_{44} & GL_{45} \\ GL_{52} & GL_{54} & GL_{55} \end{vmatrix} = 0 \qquad \text{(I.43)}$$

In equations (I.41) and (I.43), the GL-tensor components read:

$$GL_{22} = C(1+n_3^2) \qquad \text{(I.44)}$$

$$GL_{44} = -\varepsilon(1+n_3^2) \qquad \text{(I.45)}$$

$$GL_{55} = -\mu(1+n_3^2) \qquad \text{(I.46)}$$

$$GL_{24} = GL_{42} = e(1+n_3^2) \qquad \text{(I.47)}$$

$$GL_{25} = GL_{52} = h(1+n_3^2) \qquad \text{(I.48)}$$

$$GL_{45} = GL_{54} = -\alpha(1+n_3^2) \qquad \text{(I.49)}$$

In equations from (I.44) to (I.49), the directional cosine n_3 (eigenvalue) is defined by $n_3 = k_3/k$ and the independent material constants for the case are $C = C_{44} = C_{66}$, $e = e_{16} = e_{34}$, $h = h_{16} = h_{34}$, $\varepsilon = \varepsilon_{11} = \varepsilon_{33}$, $\mu = \mu_{11} = \mu_{33}$, and $\alpha = \alpha_{11} = \alpha_{33}$ [101]. Indeed, a corresponding nonzero eigenvector in the form of expression (I.42) should exist for a suitable nonzero eigenvalue n_3. It is also possible that two nonzero eigenvectors can exist for the same eigenvalue. To reveal this significant peculiarity of the two-phase materials is the aim of the following two subsections.

I.7. *Eigenvalues*

First of all, it is thought that it is convenient to operate with the following parameter $m = 1 + n_3^2$ in the GL-tensor components defined by equations from (I.44) to

(I.49). It is also apparent that it is natural to introduce the matrix determinant in equation (I.43) as three factors. Therefore, it is obvious that the determinant in equation (I.43) can be written as follows:

$$\begin{vmatrix} Cm-\rho V_{ph}^{2} & em & hm \\ e & -\varepsilon & -\alpha \\ h & -\alpha & -\mu \end{vmatrix} \times m \times m = 0 \tag{I.50}$$

This form of the determinant written above results from the fact that each component of the second and third rows (or each component of the second and third columns) of determinant (I.43) has the same factor such as m. These two factors are written as the second and third factors in equation (I.50). This peculiarity can significantly simplify further mathematical considerations.

In equation (I.50), it is transparent that m = 0 can soundly satisfy the equality. So, two the same factors such as m can firmly determine four of six normalized eigenvalues n_3. They can be expressed as follows:

$$n_3^{(1,2)} = n_3^{(3,4)} = \pm \mathrm{j} \tag{I.51}$$

Also, the remnant determinant in equation (I.50) represents a number and it is required that it must be equal to zero to reveal the rest two eigenvalues n_3. Expanding this determinant, the following secular equation can be obtained:

$$\left(1+K_{em}^{2}\right)m-\left(V_{ph}/V_{t4}\right)^{2}=0 \tag{I.52}$$

In equation (I.52), V_{t4} and K_{em}^2 stand for the speed of the shear-horizontal bulk acoustic wave (SH-BAW) uncoupled with both the electrical and magnetic potentials and the coefficient of the magnetoelectromechanical coupling (CMEMC), respectively. They are defined by

$$V_{t4} = \sqrt{C/\rho} \text{ and } K_{em}^2 = \frac{\mu e^2 + \varepsilon h^2 - 2\alpha eh}{C(\varepsilon\mu - \alpha^2)} \tag{I.53}$$

Consequently, equation (I.52) provides the following forms of the fifth and sixth eigenvalues:

$$n_3^{(5,6)} = \pm \mathrm{j}\sqrt{1 - (V_{ph}/V_{tem})^2} \text{ or } n_3^{(5,6)} = \pm\sqrt{(V_{ph}/V_{tem})^2 - 1} \tag{I.54}$$

where the first equality is for the case of $V_{ph} < V_{tem}$ and the second is for $V_{ph} > V_{tem}$. In definition (I.54), the velocity denoted by V_{tem} represents the speed of the SH-BAW coupled with both the electrical and magnetic potential. It is defined by the following formula:

$$V_{tem} = V_{t4}\left(1 + K_{em}^2\right)^{1/2} \tag{I.55}$$

Thus, the first problem such as the determination of the explicit forms for all the eigenvalues n_3 is resolved.

I.8. *Eigenvectors*

The second problem is the determination of the eigenvector explicit forms for all six eigenvalues n_3 obtained in the previous subsection. For this purpose, three homogeneous equations (I.41) can be rewritten in the following convenient form:

$$\begin{pmatrix} Cm - \rho V_{ph}^2 & em & hm \\ em & -\varepsilon m & -\alpha m \\ hm & -\alpha m & -\mu m \end{pmatrix}\begin{pmatrix} U^0 \\ \varphi^0 \\ \psi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{I.56}$$

Indeed, the corresponding nonzero eigenvector (U^0,φ^0,ψ^0) for each value of $m=1+n_3^2$ must be found using equation (I.56). All six eigenvalues n_3 are defined by expressions (I.51) and (I.54). Exploiting equation (I.56), it is possible to demonstrate in this subsection below that each of four eigenvalues n_3 in expression (I.51) can have two different eigenvectors and the rest two eigenvalues n_3 in expression (I.54) can have the other two different eigenvectors. However, the corresponding eigenvectors must be coupled.

First of all, it is essential to write the common expressions which can be true for both eigenvalues (I.51) and (I.54). Indeed, it is natural to define the eigenvector component U^0 from the first equation in equations (I.56). Accordingly, U^0 can be readily expressed as the following dependence on both φ^0 and ψ^0:

$$U^0=-\frac{em}{A}\varphi^0-\frac{hm}{A}\psi^0 \text{ with } A=C\left[m-\left(V_{ph}/V_{t4}\right)^2\right] \tag{I.57}$$

Employing definition (I.57) for the second and third equations in equations (I.56), one can obtain the following two homogeneous equations which demonstrate the coupling between the eigenvector components φ^0 and ψ^0:

$$\left(\frac{me^2}{A}+\varepsilon\right)\varphi^0+\left(\frac{meh}{A}+\alpha\right)\psi^0=0 \tag{I.58}$$

$$\left(\frac{meh}{A}+\alpha\right)\varphi^0+\left(\frac{mh^2}{A}+\mu\right)\psi^0=0 \tag{I.59}$$

It is crucial to state that equations from (I.57) to (I.59) can reveal all the eigenvector components such as U^0, φ^0, and ψ^0.

Four eigenvalues (I.51) correspond to m = 0. So, equations (I.56) can be rewritten in the following form to determine the corresponding eigenvector:

$$\begin{pmatrix} 0-\rho V_{ph}^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} U^0 \\ \varphi^0 \\ \psi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{I.60}$$

It is clearly seen in equation (I.60) that the single possibility to have a nonzero eigenvector for nonzero eigenvalues (I.51) is the situation when $U^0 = 0$ for $m = 0$ agrees with expression (I.57) and there are arbitrary nonzero values for both φ^0 and ψ^0. The certain values of both φ^0 and ψ^0 for the case of $m = 0$ can be determined from equations (I.58) and (I.59). Therefore, it is possible to state that four eigenvalues (I.51) actually have the same eigenvector that will be written below. It is also clearly seen in equation (I.60) that the value of the phase velocity V_{ph} is uncertain and cannot be equal to zero to have $U^0 = 0$. It is thought that to use equations (I.58) and (I.59) for determination of the uncertain values of φ^0 and ψ^0 for $m = 0$ is natural because this case couples these two uncertain values with those of the other eigenvector corresponding to the eigenvalues defined by expression (I.54). It is also essential to state that eigenvalues (I.51) and the corresponding eigenvectors do not depend on the phase velocity V_{ph}.

The following useful expressions can be written for eigenvalues (I.54) coupled with the phase velocity V_{ph}:

$$m^{(5,6)} = \left(V_{ph}/V_{tem}\right)^2 \tag{I.61}$$

$$n_3^{(5,6)} = \pm \mathrm{j}\sqrt{1-m^{(5,6)}} = \pm \mathrm{j}b \tag{I.62}$$

$$A^{(5,6)} = -m^{(5,6)} C K_{em}^2 \tag{I.63}$$

Exploiting equation (I.58) and the case of $m \neq 0$ in equation (I.61), one can check that both equalities (I.58) and (I.59) are satisfied when the eigenvector components φ^0 and ψ^0 are expressed as follows:

$$\varphi^0 = \frac{meh}{A} + \alpha \text{ and } \psi^0 = -\frac{me^2}{A} - \varepsilon \tag{I.64}$$

Using definitions (I.61) and (I.63) for the eigenvector components defined by expressions (I.57) and (I.64), the first eigenvector for eigenvalues (I.54) can be formed. It is clearly seen that all the eigenvector components do not depend on the phase velocity V_{ph}.

There is however the second case to satisfy equalities (I.58) and (I.59). Using equation (I.59), it is transparent that the eigenvector components φ^0 and ψ^0 can be also defined by

$$\varphi^0 = \frac{mh^2}{A} + \mu \text{ and } \psi^0 = -\frac{meh}{A} - \alpha \tag{I.65}$$

Thus, the eigenvector components defined by expressions (I.57) and (I.65) can form the second eigenvector for two eigenvalues defined by expression (I.54). It is also obvious that using definition (I.63), all the components of the second eigenvector do not depend on the phase velocity V_{ph}, too. Therefore, all the corresponding eigenvectors for all the eigenvalues do not depend on the phase velocity V_{ph} and only two eigenvalues (I.54) depend on the velocity.

It is indispensable to state that to know two different sets of the eigenvector components is very important because they can lead to two different solutions for the phase velocity V_{ph}. This fact was first revealed in book [101] for the propagation problems of the SH-SAWs guided by the free surface of the transversely isotropic piezoelectromagnetics of class 6 *mm*. This fact can also complicate the theoretical investigations of the plate SH-wave propagation.

It is now possible to write all six eigenvectors for six eigenvalues. Using the φ^0 and ψ^0 defined by equations (I.64), the first set of the corresponding eigenvectors for the eigenvalues (I.51) and (I.54) are respectively given as follows:

$$\left(U^{0(1)}, \varphi^{0(1)}, \psi^{0(1)}\right) = \left(U^{0(2)}, \varphi^{0(2)}, \psi^{0(2)}\right) = \left(U^{0(3)}, \varphi^{0(3)}, \psi^{0(3)}\right) = \left(U^{0(4)}, \varphi^{0(4)}, \psi^{0(4)}\right) = (0, \alpha, -\varepsilon) \tag{I.66}$$

$$\left(U^{0(5)},\varphi^{0(5)},\psi^{0(5)}\right)=\left(U^{0(6)},\varphi^{0(6)},\psi^{0(6)}\right)=\left(\frac{e\alpha-h\varepsilon}{CK_{em}^2},-\frac{eh}{CK_{em}^2}+\alpha,\frac{e^2}{CK_{em}^2}-\varepsilon\right) \quad \text{(I.67)}$$

Also, the following equalities exist and couple the corresponding eigenvector components:

$$e\varphi^{0(1)}+h\psi^{0(1)}=e\varphi^{0(5)}+h\psi^{0(5)}=e\alpha-h\varepsilon \quad \text{(I.68)}$$

Using the φ^0 and ψ^0 defined by equations (I.65), it is possible to form the second set of the eigenvector components for the same eigenvalues defined by expressions (I.51) and (I.54). The eigenvectors respectively read:

$$\left(U^{0(1)},\varphi^{0(1)},\psi^{0(1)}\right)=\left(U^{0(2)},\varphi^{0(2)},\psi^{0(2)}\right)=\left(U^{0(3)},\varphi^{0(3)},\psi^{0(3)}\right)=\left(U^{0(4)},\varphi^{0(4)},\psi^{0(4)}\right)=(0,\mu,-\alpha) \quad \text{(I.69)}$$

$$\left(U^{0(5)},\varphi^{0(5)},\psi^{0(5)}\right)=\left(U^{0(6)},\varphi^{0(6)},\psi^{0(6)}\right)=\left(\frac{e\mu-h\alpha}{CK_{em}^2},-\frac{h^2}{CK_{em}^2}+\mu,\frac{eh}{CK_{em}^2}-\alpha\right) \quad \text{(I.70)}$$

For the second set of the eigenvector components, the following useful equalities also exist which can significantly simplify the further analytics:

$$e\varphi^{0(1)}+h\psi^{0(1)}=e\varphi^{0(5)}+h\psi^{0(5)}=e\mu-h\alpha \quad \text{(I.71)}$$

I.9. *Complete Displacements*

Utilizing the eigenvalues and eigenvectors found in the previous two subsections, it is possible to compose the complete mechanical displacement U^{Σ}, complete electrical potential φ^{Σ}, and complete magnetic potential ψ^{Σ}. They can be compactly written in the plane wave forms as follows:

$$U^{\Sigma}=\sum_{p=1,2,3,4,5,6}F^{(p)}U^{0(p)}\exp\left[\mathrm{j}k\left(n_1x_1+n_3^{(p)}x_3-V_{ph}t\right)\right] \quad \text{(I.72)}$$

$$\varphi^{\Sigma} = \sum_{p=1,2,3,4,5,6} F^{(p)} \varphi^{0(p)} \exp\left[\mathrm{j}k\left(n_1 x_1 + n_3^{(p)} x_3 - V_{ph} t\right)\right] \tag{I.73}$$

$$\psi^{\Sigma} = \sum_{p=1,2,3,4,5,6} F^{(p)} \psi^{0(p)} \exp\left[\mathrm{j}k\left(n_1 x_1 + n_3^{(p)} x_3 - V_{ph} t\right)\right] \tag{I.74}$$

In expressions from (I.72) to (I.74), the weight factors $F^{(p)}$ can be determined from equations in which suitable boundary conditions are accounted. The final subsection of this chapter describes the realization of the mechanical, electrical, and magnetic boundary conditions. The complete mechanical displacement U^{Σ}, complete electrical potential φ^{Σ}, and complete magnetic potential ψ^{Σ} can be also written in the following expanded forms:

$$\begin{aligned} U^{\Sigma} = &\left\{F_1 U^{0(1)} \exp(kx_3) + F_2 U^{0(1)} \exp(-kx_3)\right. \\ &\left.+ F_3 U^{0(5)} \exp\left(\mathrm{j}kn_3^{(5)} x_3\right) + F_4 U^{0(5)} \exp\left(-\mathrm{j}kn_3^{(5)} x_3\right)\right\} \times \exp\left[\mathrm{j}k\left(x_1 - V_{ph} t\right)\right] \end{aligned} \tag{I.75}$$

$$\begin{aligned} \varphi^{\Sigma} = &\left\{F_1 \varphi^{0(1)} \exp(kx_3) + F_2 \varphi^{0(1)} \exp(-kx_3)\right. \\ &\left.+ F_3 \varphi^{0(5)} \exp\left(\mathrm{j}kn_3^{(5)} x_3\right) + F_4 \varphi^{0(5)} \exp\left(-\mathrm{j}kn_3^{(5)} x_3\right)\right\} \times \exp\left[\mathrm{j}k\left(x_1 - V_{ph} t\right)\right] \end{aligned} \tag{I.76}$$

$$\begin{aligned} \psi^{\Sigma} = &\left\{F_1 \psi^{0(1)} \exp(kx_3) + F_2 \psi^{0(1)} \exp(-kx_3)\right. \\ &\left.+ F_3 \psi^{0(5)} \exp\left(\mathrm{j}kn_3^{(5)} x_3\right) + F_4 \psi^{0(5)} \exp\left(-\mathrm{j}kn_3^{(5)} x_3\right)\right\} \times \exp\left[\mathrm{j}k\left(x_1 - V_{ph} t\right)\right] \end{aligned} \tag{I.77}$$

where $F_1 = F^{(1)} + F^{(2)}$, $F_2 = F^{(3)} + F^{(4)}$, $F_3 = F^{(5)}$, and $F_4 = F^{(6)}$. Using the wellknown formulae such as $\exp(\pm\Theta) = \cosh(\Theta) \pm \sinh(\Theta)$ and $\exp(\pm \mathrm{j}\Theta) = \cos(\Theta) \pm \mathrm{j}\sin(\Theta)$, it is possible to rewrite the complete parameters for the case of $V_{ph} < V_{tem}$ as follows:

$$\begin{aligned} U^{\Sigma} = &\left\{F_{01} U^{0(1)} \cosh(kx_3) + F_{02} U^{0(1)} \sinh(kx_3) + F_{03} U^{0(5)} \cosh\left(kx_3\sqrt{1-\left(V_{ph}/V_{tem}\right)^2}\right)\right. \\ &\left.+ F_{04} U^{0(5)} \sinh\left(kx_3\sqrt{1-\left(V_{ph}/V_{tem}\right)^2}\right)\right\} \times \exp\left[\mathrm{j}k\left(x_1 - V_{ph} t\right)\right] \end{aligned} \tag{I.78}$$

$$\begin{aligned} \varphi^{\Sigma} = &\left\{F_{01} \varphi^{0(1)} \cosh(kx_3) + F_{02} \varphi^{0(1)} \sinh(kx_3) + F_{03} \varphi^{0(5)} \cosh\left(kx_3\sqrt{1-\left(V_{ph}/V_{tem}\right)^2}\right)\right. \\ &\left.+ F_{04} \varphi^{0(5)} \sinh\left(kx_3\sqrt{1-\left(V_{ph}/V_{tem}\right)^2}\right)\right\} \times \exp\left[\mathrm{j}k\left(x_1 - V_{ph} t\right)\right] \end{aligned} \tag{I.79}$$

$$\psi^{\Sigma} = \left\{ F_{01}\psi^{0(1)}\cosh(kx_3) + F_{02}\psi^{0(1)}\sinh(kx_3) + F_{03}\psi^{0(5)}\cosh\left(kx_3\sqrt{1-(V_{ph}/V_{tem})^2}\right) \right.$$
$$\left. + F_{04}\psi^{0(5)}\sinh\left(kx_3\sqrt{1-(V_{ph}/V_{tem})^2}\right)\right\} \times \exp\left[\mathrm{j}k(x_1 - V_{ph}t)\right] \tag{I.80}$$

where $F_{01} = F_1 + F_2$, $F_{02} = F_1 - F_2$, $F_{03} = F_3 + F_4$, and $F_{04} = F_3 - F_4$. All the formulae written above in this subsection are valid within the plate thickness, namely when $-d \le x_3 \le +d$. |It is also basic to state that the complete mechanical displacement U^{Σ} in expression (I.78) does not depend on the weight factors F_{01} and F_{02} because $U^{0(1)} = 0$.

In the case of $V_{ph} > V_{tem}$, the hyperbolic cosine at the weight factor F_{03} and the hyperbolic sine at the F_{04} are converted as follows:

$$\cosh\left(kx_3\sqrt{1-(V_{ph}/V_{tem})^2}\right) \rightarrow \cos\left(kx_3\sqrt{(V_{ph}/V_{tem})^2 - 1}\right) \tag{I.81}$$

$$\sinh\left(kx_3\sqrt{1-(V_{ph}/V_{tem})^2}\right) \rightarrow \mathrm{j}\sin\left(kx_3\sqrt{(V_{ph}/V_{tem})^2 - 1}\right) \tag{I.82}$$

I.10. *Equations and Parameters for a Vacuum*

In this theoretical investigations there are contacts of the upper ($x_3 = + d$) and lower ($x_3 = – d$) surfaces of the transversely isotropic piezoelectromagnetic plate with the other continuum such as a vacuum, see figure 1 in Introduction. Therefore, it is also necessary to introduce the vacuum material constants and the corresponding expressions for the electrical and magnetic potentials in a vacuum. For a vacuum (free space) the value of the elastic constant C_0 is very small, namely $C_0 = 0.001$ Pa [215]. Indeed, this value of C_0 is thirteen orders smaller than that for a solid such as piezoelectromagnetics. Thus, it is transparent that it is too negligible to account this value in calculations. Also, the vacuum dielectric permittivity constant has the following value: $\varepsilon_0 = 10^{-7}/(4\pi C_L^2) = 8.854187817\times10^{-12}$ [F/m] where $C_L = 2.99782458\times10^{8}$ [m/s] is the speed of light in a vacuum.

For the free space, it is possible to exploit the wellknown Laplace equation of type $\Delta\varphi_f = 0$. This equation can be written as follows:

$$\left(k_1^2 + k_3^2\right)\varphi_{f0} = 0 \tag{I.83}$$

In equation (I.83), the electrical potential above the upper surface (x_3 = + d) and below the lower surface (x_3 = – d) of the piezoelectromagnetics can be respectively written in the following plane wave forms:

$$\varphi_{f0} = F^{(E0)} \exp(-k_1x_3)\exp[\mathrm{j}(k_1x_1 - \omega t)],\ x_3 \geq +d \tag{I.84}$$

$$\varphi_{f0} = F^{(E0)} \exp(k_1x_3)\exp[\mathrm{j}(k_1x_1 - \omega t)],\ x_3 \leq -d \tag{I.85}$$

Also, the free space magnetic permeability constant has the following value: μ_0 = $4\pi\times10^{-7}$ [H/m] = $12.5663706144\times10^{-7}$ [H/m]. For the magnetic potential, Laplace's equation of type $\Delta\psi_f = 0$ can be also written in the following form:

$$\left(k_1^2 + k_3^2\right)\psi_{f0} = 0 \tag{I.86}$$

In equation (I.86), the magnetic potential in a vacuum above the upper surface (x_3 = + d) and below the lower surface (x_3 = – d) of the piezoelectromagnetic plate must be also written in the corresponding plane wave forms. Therefore, it is possible to respectively write the following:

$$\psi_{f0} = F^{(M0)} \exp(-k_1x_3)\exp[\mathrm{j}(k_1x_1 - \omega t)],\ x_3 \geq +d \tag{I.87}$$

$$\psi_{f0} = F^{(M0)} \exp(k_1x_3)\exp[\mathrm{j}(k_1x_1 - \omega t)],\ x_3 \leq -d \tag{I.88}$$

It is clearly seen in equations (I.84), (I.85), (I.87), and (I.88) that both the electrical and magnetic potentials exponentially decrease in a vacuum when $x_3 > + d$ and $x_3 < - d$, see figure 1.

I.11. *Mechanical, Electrical, and Magnetic Boundary Conditions at the Upper and Lower Surfaces*

For the piezoelectromagnetic plate, some certain mechanical, electrical, and magnetic boundary conditions at both the upper ($x_3 = + d$) and lower ($x_3 = - d$) free surfaces (the two interfaces between the solid and a vacuum shown in figure 1 in Introduction) must be used. The mechanically free upper and lower surfaces can be used as the mechanical boundary conditions. Also, the following electrical boundary conditions can be used at both the free surfaces of the plate: the electrically closed surface ($\varphi = 0$), electrically open surface ($D_3 = 0$), and the continuity of both φ and D_3 at the surfaces, i.e. $\varphi = \varphi^f$ and $D_3 = D^f$, where D_3 is the normal component of the electrical displacements and the superscript "*f*" relates to the free space. Besides, the magnetic boundary conditions at both the free surfaces are as follows: the magnetically closed surface ($B_3 = 0$), magnetically open surface ($\psi = 0$), and the continuity of both ψ and B_3 at the surfaces, i.e. $\psi = \psi^f$ and $B_3 = B^f$, where B_3 is the normal component of the magnetic flux. It is worth noticing that the realization of the mechanical, electrical, and magnetic boundary conditions is perfectly described in theoretical work [113] by Al'shits, Darinskii, and Lothe. Also, it is convenient in this subsection to use $x_1 = 0$ and $t = 0$ to cope with $\exp[\mathrm{j}(k_1 x_1 - \omega t)] = 1$.

The mechanically free upper surface of the piezoelectromagnetic plate at $x_3 = + d$ (see figure 1 in Introduction) requires the following condition for the normal component of the stress tensor σ_{32}:

$$\sigma_{32}(x_3 = +d) = 0 \tag{I.89}$$

where

$$\begin{aligned}
&\sigma_{32}(x_3=+d)= \\
&F^{(1)}k_3^{(1)}\left[CU^{0(1)}+e\varphi^{0(1)}+h\psi^{0(1)}\right]\exp\left(jk_3^{(1)}d\right)+F^{(2)}k_3^{(2)}\left[CU^{0(2)}+e\varphi^{0(2)}+h\psi^{0(2)}\right]\exp\left(jk_3^{(2)}d\right) \\
&+F^{(3)}k_3^{(3)}\left[CU^{0(3)}+e\varphi^{0(3)}+h\psi^{0(3)}\right]\exp\left(jk_3^{(3)}d\right)+F^{(4)}k_3^{(4)}\left[CU^{0(4)}+e\varphi^{0(4)}+h\psi^{0(4)}\right]\exp\left(jk_3^{(4)}d\right) \\
&+F^{(5)}k_3^{(5)}\left[CU^{0(5)}+e\varphi^{0(5)}+h\psi^{0(5)}\right]\exp\left(jk_3^{(5)}d\right)+F^{(6)}k_3^{(6)}\left[CU^{0(6)}+e\varphi^{0(6)}+h\psi^{0(6)}\right]\exp\left(jk_3^{(6)}d\right)
\end{aligned} \tag{I.90}$$

The possible electrical boundary condition at the free upper surface of the piezoelectromagnetic plate at $x_3 = + d$ can be written as follows: the normal component of the electrical displacement D_3 must be equal to the electrical displacement D^f for a vacuum, where the superscript "*f*" is used for a vacuum in this subsection below. This condition reads:

$$D_3(x_3=+d)=D^f(x_3=+d) \tag{I.91}$$

where

$$D^f(x_3=+d)=-F_E\varphi_0^f\,jk_1\varepsilon_0\exp(-k_1d) \tag{I.92}$$

$$\begin{aligned}
&D_3(x_3=+d)= \\
&F^{(1)}k_3^{(1)}\left[eU^{0(1)}-\varepsilon\varphi^{0(1)}-\alpha\psi^{0(1)}\right]\exp\left(jk_3^{(1)}d\right)+F^{(2)}k_3^{(2)}\left[eU^{0(2)}-\varepsilon\varphi^{0(2)}-\alpha\psi^{0(2)}\right]\exp\left(jk_3^{(2)}d\right) \\
&+F^{(3)}k_3^{(3)}\left[eU^{0(3)}-\varepsilon\varphi^{0(3)}-\alpha\psi^{0(3)}\right]\exp\left(jk_3^{(3)}d\right)+F^{(4)}k_3^{(4)}\left[eU^{0(4)}-\varepsilon\varphi^{0(4)}-\alpha\psi^{0(4)}\right]\exp\left(jk_3^{(4)}d\right) \\
&+F^{(5)}k_3^{(5)}\left[eU^{0(5)}-\varepsilon\varphi^{0(5)}-\alpha\psi^{0(5)}\right]\exp\left(jk_3^{(5)}d\right)+F^{(6)}k_3^{(6)}\left[eU^{0(6)}-\varepsilon\varphi^{0(6)}-\alpha\psi^{0(6)}\right]\exp\left(jk_3^{(6)}d\right)
\end{aligned} \tag{I.93}$$

The second possible electrical boundary condition is for the electrical potential φ at the free upper surface of the plate at $x_3 = + d$. At this surface, it is possible to require for the electrical potential φ in the plate and the electrical potential φ^f in a vacuum that the following equality must be satisfied:

$$\varphi(x_3=+d)=\varphi^f(x_3=+d) \tag{I.94}$$

where

$$\varphi^f(x_3=+d)=F_E\varphi_0^f\exp(-k_1d) \tag{I.95}$$

$$\begin{aligned}&\varphi(x_3=+d)=F^{(1)}\varphi^{0(1)}\exp\left(\mathrm{j}k_3^{(1)}d\right)+F^{(2)}\varphi^{0(2)}\exp\left(\mathrm{j}k_3^{(2)}d\right)+F^{(3)}\varphi^{0(3)}\exp\left(\mathrm{j}k_3^{(3)}d\right)\\&+F^{(4)}\varphi^{0(4)}\exp\left(\mathrm{j}k_3^{(4)}d\right)+F^{(5)}\varphi^{0(5)}\exp\left(\mathrm{j}k_3^{(5)}d\right)+F^{(6)}\varphi^{0(6)}\exp\left(\mathrm{j}k_3^{(6)}d\right)\end{aligned} \tag{I.96}$$

Besides, the magnetic boundary conditions can be also expressed at the free upper surface of the plate at $x_3 = + d$. First, the magnetic flux normal component B_3 must continue in a vacuum which is characterized by the magnetic flux B^f. Therefore, it is possible to require the following:

$$B_3(x_3=+d)=B^f(x_3=+d) \tag{I.97}$$

where

$$B^f(x_3=+d)=-F_M\psi_0^f\,\mathrm{j}k_1\mu_0\exp(-k_1d) \tag{I.98}$$

$$\begin{aligned}&B_3(x_3=+d)=\\&F^{(1)}k_3^{(1)}\left[hU^{0(1)}-\alpha\varphi^{0(1)}-\mu\psi^{0(1)}\right]\exp\left(\mathrm{j}k_3^{(1)}d\right)+F^{(2)}k_3^{(2)}\left[hU^{0(2)}-\alpha\varphi^{0(2)}-\mu\psi^{0(2)}\right]\exp\left(\mathrm{j}k_3^{(2)}d\right)\\&+F^{(3)}k_3^{(3)}\left[hU^{0(3)}-\alpha\varphi^{0(3)}-\mu\psi^{0(3)}\right]\exp\left(\mathrm{j}k_3^{(3)}d\right)+F^{(4)}k_3^{(4)}\left[hU^{0(4)}-\alpha\varphi^{0(4)}-\mu\psi^{0(4)}\right]\exp\left(\mathrm{j}k_3^{(4)}d\right)\\&+F^{(5)}k_3^{(5)}\left[hU^{0(5)}-\alpha\varphi^{0(5)}-\mu\psi^{0(5)}\right]\exp\left(\mathrm{j}k_3^{(5)}d\right)+F^{(6)}k_3^{(6)}\left[hU^{0(6)}-\alpha\varphi^{0(6)}-\mu\psi^{0(6)}\right]\exp\left(\mathrm{j}k_3^{(6)}d\right)\end{aligned} \tag{I.99}$$

Indeed, the continuity of the magnetic potential ψ at $x_3 = + d$ must be also required. Therefore, the following equality between the magnetic potential ψ in the plate and the one ψ^f in a vacuum must be satisfied:

$$\psi(x_3=+d)=\psi^f(x_3=+d) \tag{I.100}$$

where

$$\psi^f(x_3=+d)=F_M\psi_0^f\exp(-k_1 d) \tag{I.101}$$

$$\begin{aligned}\psi(x_3=+d)=&F^{(1)}\psi^{0(1)}\exp\left(\mathrm{j}k_3^{(1)}d\right)+F^{(2)}\psi^{0(2)}\exp\left(\mathrm{j}k_3^{(2)}d\right)+F^{(3)}\psi^{0(3)}\exp\left(\mathrm{j}k_3^{(3)}d\right)\\&+F^{(4)}\psi^{0(4)}\exp\left(\mathrm{j}k_3^{(4)}d\right)+F^{(5)}\psi^{0(5)}\exp\left(\mathrm{j}k_3^{(5)}d\right)+F^{(6)}\psi^{0(6)}\exp\left(\mathrm{j}k_3^{(6)}d\right)\end{aligned} \tag{I.102}$$

However, the plate possesses the second surface, namely the lower surface at $x_3 = -d$, see figure 1 in Introduction. This study relates to the case of the homogeneous boundary conditions. This means that the same mechanical, electrical, and magnetic boundary conditions must be applied to the lower free surface of the plate. Therefore, it is also necessary to write the mechanical, electrical, and magnetic boundary condition for this free surface. The mechanical condition is the mechanically free surface that means

$$\sigma_{32}(x_3=-d)=0 \tag{I.103}$$

where

$$\begin{aligned}&\sigma_{32}(x_3=-d)=\\&F^{(1)}k_3^{(1)}\left[CU^{0(1)}+e\varphi^{0(1)}+h\psi^{0(1)}\right]\exp\left(-\mathrm{j}k_3^{(1)}d\right)+F^{(2)}k_3^{(2)}\left[CU^{0(2)}+e\varphi^{0(2)}+h\psi^{0(2)}\right]\exp\left(-\mathrm{j}k_3^{(2)}d\right)\\&+F^{(3)}k_3^{(3)}\left[CU^{0(3)}+e\varphi^{0(3)}+h\psi^{0(3)}\right]\exp\left(-\mathrm{j}k_3^{(3)}d\right)+F^{(4)}k_3^{(4)}\left[CU^{0(4)}+e\varphi^{0(4)}+h\psi^{0(4)}\right]\exp\left(-\mathrm{j}k_3^{(4)}d\right)\\&+F^{(5)}k_3^{(5)}\left[CU^{0(5)}+e\varphi^{0(5)}+h\psi^{0(5)}\right]\exp\left(-\mathrm{j}k_3^{(5)}d\right)+F^{(6)}k_3^{(6)}\left[CU^{0(6)}+e\varphi^{0(6)}+h\psi^{0(6)}\right]\exp\left(-\mathrm{j}k_3^{(6)}d\right)\end{aligned} \tag{I.104}$$

The electrical boundary conditions can be written as follows:

$$D_3(x_3=-d)=D^f(x_3=-d) \tag{I.105}$$

$$\varphi(x_3=-d)=\varphi^f(x_3=-d) \tag{I.106}$$

where

$$D^f(x_3=-d)=F_E\varphi_0^f\,\mathrm{j}k_1\varepsilon_0\exp(-k_1 d) \tag{I.107}$$

$$\begin{aligned}
&D_3(x_3=-d)=\\
&F^{(1)}k_3^{(1)}\left[eU^{0(1)}-\varepsilon\varphi^{0(1)}-\alpha\psi^{0(1)}\right]\exp\left(-\mathrm{j}k_3^{(1)}d\right)+F^{(2)}k_3^{(2)}\left[eU^{0(2)}-\varepsilon\varphi^{0(2)}-\alpha\psi^{0(2)}\right]\exp\left(-\mathrm{j}k_3^{(2)}d\right)\\
&+F^{(3)}k_3^{(3)}\left[eU^{0(3)}-\varepsilon\varphi^{0(3)}-\alpha\psi^{0(3)}\right]\exp\left(-\mathrm{j}k_3^{(3)}d\right)+F^{(4)}k_3^{(4)}\left[eU^{0(4)}-\varepsilon\varphi^{0(4)}-\alpha\psi^{0(4)}\right]\exp\left(-\mathrm{j}k_3^{(4)}d\right)\\
&+F^{(5)}k_3^{(5)}\left[eU^{0(5)}-\varepsilon\varphi^{0(5)}-\alpha\psi^{0(5)}\right]\exp\left(-\mathrm{j}k_3^{(5)}d\right)+F^{(6)}k_3^{(6)}\left[eU^{0(6)}-\varepsilon\varphi^{0(6)}-\alpha\psi^{0(6)}\right]\exp\left(-\mathrm{j}k_3^{(6)}d\right)
\end{aligned} \tag{I.108}$$

$$\varphi^f(x_3=-d)=F_E\varphi_0^f\exp(-k_1 d) \tag{I.109}$$

$$\begin{aligned}
&\varphi(x_3=-d)=F^{(1)}\varphi^{0(1)}\exp\left(-\mathrm{j}k_3^{(1)}d\right)+F^{(2)}\varphi^{0(2)}\exp\left(-\mathrm{j}k_3^{(2)}d\right)+F^{(3)}\varphi^{0(3)}\exp\left(-\mathrm{j}k_3^{(3)}d\right)\\
&+F^{(4)}\varphi^{0(4)}\exp\left(-\mathrm{j}k_3^{(4)}d\right)+F^{(5)}\varphi^{0(5)}\exp\left(-\mathrm{j}k_3^{(5)}d\right)+F^{(6)}\varphi^{0(6)}\exp\left(-\mathrm{j}k_3^{(6)}d\right)
\end{aligned} \tag{I.110}$$

Finally, the magnetic boundary conditions at $x_3 = -d$ can be composed as follows:

$$B_3(x_3=-d)=B^f(x_3=-d) \tag{I.111}$$

$$\psi(x_3=-d)=\psi^f(x_3=-d) \tag{I.112}$$

where

$$B^f(x_3=-d)=F_M\psi_0^f\,\mathrm{j}k_1\mu_0\exp(-k_1 d) \tag{I.113}$$

$$\begin{aligned}
&B_3(x_3=-d)=\\
&F^{(1)}k_3^{(1)}\left[hU^{0(1)}-\alpha\varphi^{0(1)}-\mu\psi^{0(1)}\right]\exp\left(-\mathrm{j}k_3^{(1)}d\right)+F^{(2)}k_3^{(2)}\left[hU^{0(2)}-\alpha\varphi^{0(2)}-\mu\psi^{0(2)}\right]\exp\left(-\mathrm{j}k_3^{(2)}d\right)\\
&+F^{(3)}k_3^{(3)}\left[hU^{0(3)}-\alpha\varphi^{0(3)}-\mu\psi^{0(3)}\right]\exp\left(-\mathrm{j}k_3^{(3)}d\right)+F^{(4)}k_3^{(4)}\left[hU^{0(4)}-\alpha\varphi^{0(4)}-\mu\psi^{0(4)}\right]\exp\left(-\mathrm{j}k_3^{(4)}d\right)\\
&+F^{(5)}k_3^{(5)}\left[hU^{0(5)}-\alpha\varphi^{0(5)}-\mu\psi^{0(5)}\right]\exp\left(-\mathrm{j}k_3^{(5)}d\right)+F^{(6)}k_3^{(6)}\left[hU^{0(6)}-\alpha\varphi^{0(6)}-\mu\psi^{0(6)}\right]\exp\left(-\mathrm{j}k_3^{(6)}d\right)
\end{aligned} \tag{I.114}$$

$$\psi^f(x_3=-d)=F_M\psi_0^f\exp(-k_1 d) \tag{I.115}$$

$$\begin{aligned}
&\psi(x_3=-d)=F^{(1)}\psi^{0(1)}\exp\left(-\mathrm{j}k_3^{(1)}d\right)+F^{(2)}\psi^{0(2)}\exp\left(-\mathrm{j}k_3^{(2)}d\right)+F^{(3)}\psi^{0(3)}\exp\left(-\mathrm{j}k_3^{(3)}d\right)\\
&+F^{(4)}\psi^{0(4)}\exp\left(-\mathrm{j}k_3^{(4)}d\right)+F^{(5)}\psi^{0(5)}\exp\left(-\mathrm{j}k_3^{(5)}d\right)+F^{(6)}\psi^{0(6)}\exp\left(-\mathrm{j}k_3^{(6)}d\right)
\end{aligned} \tag{I.116}$$

This short review of the mechanical, electrical, and magnetic boundary conditions introduced in this subsection can allow the reader to better understand the following chapters. The following chapters study the influence of different electrical and magnetic boundary conditions applied at the upper and lower interfaces between the transversely isotropic piezoelectromagnetics and a vacuum. It is thought that the case of the electrically closed surface ($\varphi = 0$) and the magnetically open surface ($\psi = 0$) is a common realization of the boundary conditions to commence the analysis. This is the case of study for the following chapter.

Chapter II

The Electrically Closed and Magnetically Open Boundary Conditions

This chapter studies the SH-wave propagation in the transversely isotropic piezoelectromagnetic plate when homogeneous mechanical, electrical, and magnetic boundary conditions are applied to both the free surfaces of the plate. The homogeneous boundary conditions mean that each of the upper and lower surfaces of the plate, see figure 1 in Introduction, represents the mechanically free, electrically closed (electrical potential φ = 0) and the magnetically open (magnetic potential ψ = 0) surface. In figure 1, the upper free surface is located at $x_3 = + d$ and the lower surface at $x_3 = - d$, where $2d$ is the plate thickness. For these boundary conditions, it is necessary to use equations (I.90), (I.96), and (I.102) from the previous chapter for the upper surface and equations (I.104), (I.110), and (I.116) for the lower surface. For convenience, these six homogeneous equations can be readily transformed using the wellknown formulae such as $\exp(\pm\Theta) = \cosh(\Theta) \pm \sinh(\Theta)$ and $\exp(\pm\,\mathrm{j}\Theta) = \cos(\Theta) \pm \mathrm{j}\sin(\Theta)$ to deal with the following weight factors $F_{01} = F_1 + F_2$, $F_{02} = F_1 - F_2$, $F_{03} = F_3 + F_4$, and $F_{04} = F_3 - F_4$, where $F_1 = F^{(1)} + F^{(2)}$, $F_2 = F^{(3)} + F^{(4)}$, $F_3 = F^{(5)}$, and $F_4 = F^{(6)}$. These weight factors were originally defined in the ninth subsection of the previous chapter.

Therefore, in the case of the upper surface at $x_3 = + d$ the mechanical, electrical, and magnetic boundary conditions are respectively written as follows:

$$\begin{aligned}&\left(e\varphi^{0(1)} + h\psi^{0(1)}\right)\left[F_{01}\sinh(kd) + F_{02}\cosh(kd)\right]\\&+ b\left(CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0\end{aligned} \tag{II.1}$$

$$\left[F_{01}\cosh(kd) + F_{02}\sinh(kd)\right]e\varphi^{0(1)} + \left[F_{03}\cosh(bkd) + F_{04}\sinh(bkd)\right]e\varphi^{0(5)} = 0 \tag{II.2}$$

$$\left[F_{01}\cosh(kd) + F_{02}\sinh(kd)\right]h\psi^{0(1)} + \left[F_{03}\cosh(bkd) + F_{04}\sinh(bkd)\right]h\psi^{0(5)} = 0 \tag{II.3}$$

For the lower free surface at $x_3 = -d$, three homogeneous equations corresponding to three boundary conditions can be transformed as follows:

$$\begin{aligned}&\left(e\varphi^{0(1)} + h\psi^{0(1)}\right)\left[F_{01}\sinh(kd) - F_{02}\cosh(kd)\right]\\&+ b\left(CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) - F_{04}\cosh(bkd)\right] = 0\end{aligned} \tag{II.4}$$

$$\left[F_{01}\cosh(kd) - F_{02}\sinh(kd)\right]e\varphi^{0(1)} + \left[F_{03}\cosh(bkd) - F_{04}\sinh(bkd)\right]e\varphi^{0(5)} = 0 \tag{II.5}$$

$$\left[F_{01}\cosh(kd) - F_{02}\sinh(kd)\right]h\psi^{0(1)} + \left[F_{03}\cosh(bkd) - F_{04}\sinh(bkd)\right]h\psi^{0(5)} = 0 \tag{II.6}$$

In equations from (II.1) to (II.6), $k = k_1$ is the wavenumber in the propagation direction and the parameter b is defined by

$$b = \sqrt{1 - \left(V_{ph}/V_{tem}\right)^2} \tag{II.7}$$

Using six homogeneous equations from (II.1) to (II.6), it is possible to construct a suitable determinant of the boundary conditions. It is clearly seen in these six equations that they have only four unknown weight factors F_{01}, F_{02}, F_{03}, and F_{04} instead of the initial six. To reduce the number of the unknown weight factors is possible because four eigenvalues defined by expression (I.51) consist of the first pair of the equal roots and the second one. So, it is also necessary to reduce the number of homogeneous equations to also cope with the four ones instead of the six. This reduction can be done when one equation is used instead of equations (II.2) and (II.3). It is natural to use an equation that represents a product of a summation of equations (II.2) and (II.3). Also, the same product can be used instead of equations (II.5) and (II.6). Besides, it is necessary to remember that there are two different sets of the eigenvectors: the first set is defined by equations (I.66) and (I.67) and the second set is defined by equations (I.69) and (I.70). Accounting this fact, the fourth-order determinant of the boundary conditions (BCD4) can be represented as follows:

$$\begin{vmatrix} M_1^{0(1)}\sinh(kd) & M_1^{0(1)}\cosh(kd) & bM_1^{0(5)}\sinh(bkd) & bM_1^{0(5)}\cosh(bkd) \\ M_1^{0(1)}\cosh(kd) & M_1^{0(1)}\sinh(kd) & M_1^{0(1)}\cosh(bkd) & M_1^{0(1)}\sinh(bkd) \\ M_2^{0(1)}\sinh(kd) & -M_2^{0(1)}\cosh(kd) & bM_2^{0(5)}\sinh(bkd) & -bM_2^{0(5)}\cosh(bkd) \\ M_2^{0(1)}\cosh(kd) & -M_2^{0(1)}\sinh(kd) & M_2^{0(1)}\cosh(bkd) & -M_2^{0(1)}\sinh(bkd) \end{vmatrix} = 0 \qquad \text{(II.8)}$$

where $M_1^{0(1)}$ and $M_1^{0(5)}$ are used for the upper surface and $M_2^{0(1)}$ and $M_2^{0(5)}$ for the lower surface. Employing equations (I.68) and (I.71), it is essential to state that

$$\frac{M_1^{0(5)}}{M_1^{0(1)}} = \frac{M_2^{0(5)}}{M_2^{0(1)}} = \frac{1+K_{em}^2}{K_{em}^2} \qquad \text{(II.9)}$$

Exploiting equation (I.68), one can get for the upper surface that

$$M_1^{0(1)} = e\varphi^{0(1)} + h\psi^{0(1)} = e\alpha - h\varepsilon\,,\ M_1^{0(5)} = CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)} = (e\alpha - h\varepsilon)\frac{1+K_{em}^2}{K_{em}^2} \qquad \text{(II.10)}$$

With equation (I.71) used for the lower surface, one can also get that

$$M_2^{0(1)} = e\varphi^{0(1)} + h\psi^{0(1)} = e\mu - h\alpha\,,\ M_2^{0(5)} = CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)} = (e\mu - h\alpha)\frac{1+K_{em}^2}{K_{em}^2} \qquad \text{(II.11)}$$

Indeed, one can also use equations (II.11) for the upper surface and equations (II.10) for the lower surface. However, it is apparent that this reverse case does not change anything. Also, it is possible to use $M_1^{0(1)}$ and $M_1^{0(5)}$ for both the upper and lower surfaces or $M_2^{0(1)}$ and $M_2^{0(5)}$ for both the surfaces. It is possible to demonstrate that in all the cases, the same results can be obtained. First of all, in equation (II.8) the first and second rows of the BCD4 must be divided by the parameter $M_1^{0(1)}$ and the rest two rows must be divided by $M_2^{0(1)}$. Utilizing relations (II.9), equation (II.8) can be simplified to the following form:

$$\begin{vmatrix} \sinh(kd) & \cosh(kd) & b\frac{1+K_{em}^2}{K_{em}^2}\sinh(bkd) & b\frac{1+K_{em}^2}{K_{em}^2}\cosh(bkd) \\ \cosh(kd) & \sinh(kd) & \cosh(bkd) & \sinh(bkd) \\ \sinh(kd) & -\cosh(kd) & b\frac{1+K_{em}^2}{K_{em}^2}\sinh(bkd) & -b\frac{1+K_{em}^2}{K_{em}^2}\cosh(bkd) \\ \cosh(kd) & -\sinh(kd) & \cosh(bkd) & -\sinh(bkd) \end{vmatrix} = 0 \qquad \text{(II.12)}$$

It is clearly seen in equation (II.12) that the simplified determinant does not depend on the parameters $M_1^{0(1)}$, $M_1^{0(5)}$, $M_2^{0(1)}$, and $M_2^{0(5)}$. This determinant can be further simplified by means of several mathematical operations applied to the determinant rows. It is natural to add the first and second rows to the third and fourth rows, respectively, and then, to multiply the resulting third and fourth rows by ½ and to subtract them from the first and second rows, respectively. So, equation (II.12) with the transformed BCD4 reads:

$$\begin{vmatrix} 0 & \cosh(kd) & 0 & b\frac{1+K_{em}^2}{K_{em}^2}\cosh(bkd) \\ 0 & \sinh(kd) & 0 & \sinh(bkd) \\ \sinh(kd) & 0 & b\frac{1+K_{em}^2}{K_{em}^2}\sinh(bkd) & 0 \\ \cosh(kd) & 0 & \cosh(bkd) & 0 \end{vmatrix} = 0 \qquad \text{(II.13)}$$

It is transparent that the BCD4 in equation (II.13) consists of two independent determinants of the second order. Exploiting definition (II.7), the first two rows of the BCD4 result in the following dispersion relation for the determination of the phase velocity V_{new1} of the first new plate SH-wave:

$$\sqrt{1-(V_{new1}/V_{tem})^2}\tanh(kd) - \frac{K_{em}^2}{1+K_{em}^2}\tanh\left(kd\sqrt{1-(V_{new1}/V_{tem})^2}\right) = 0 \qquad \text{(II.14)}$$

Dispersion relation (II.14) pertains to the case when the phase velocity is smaller than the SH-BAW speed V_{tem} defined by expression (I.55). It is palpable in equation (II.14) that there exists only single dispersive mode for the new SH-wave in the transversely isotropic piezoelectromagnetic plate because the hyperbolic tangent is changed between zero and unity for $0 \le kd \le +\infty$. Therefore, equation (II.14) is for the fundamental mode, also known as the lowest or zero-order mode.

For the case when the phase velocity is larger than the SH-BAW speed V_{tem}, the second hyperbolic tangent in equation (II.14) goes to tangent as a result of the following equality: $\tanh(j\Theta) = j\tan(\Theta)$. Thus, the phase velocity V_{new1} for the higher-order modes of the new SH-waves in the plate is defined by

$$\tanh(kd)\sqrt{(V_{new1}/V_{tem})^2 - 1} - \frac{K_{em}^2}{1+K_{em}^2}\tan\left(kd\sqrt{(V_{new1}/V_{tem})^2 - 1}\right) = 0 \quad \text{(II.15)}$$

Utilizing the third and fourth rows of the BCD4 in equation (II.13) and definition (II.7), the second possible dispersion relation can be obtained. The second dispersion relation defined by equation (II.16) written below can determine the phase velocity V_{new2} for the fundamental mode ($V_{new2} < V_{tem}$) of the second new plate SH-wave. This reads:

$$\tanh\left(kd\sqrt{1-(V_{new2}/V_{tem})^2}\right)\sqrt{1-(V_{new2}/V_{tem})^2} - \frac{K_{em}^2}{1+K_{em}^2}\tanh(kd) = 0 \quad \text{(II.16)}$$

Also, an infinite number of the higher-order modes of the second new plate SH-wave can be found in the case of $V_{new2} > V_{tem}$. For this case, equation (II.16) is transformed into the following form:

$$\tan\left(kd\sqrt{(V_{new2}/V_{tem})^2 - 1}\right)\sqrt{(V_{new2}/V_{tem})^2 - 1} + \frac{K_{em}^2}{1+K_{em}^2}\tanh(kd) = 0 \quad \text{(II.17)}$$

It is clearly seen in dispersion relations (II.14) and (II.16) that for a very large value of the plate thickness *kd*, the phase velocity for both the new plate SH-waves will approach the SH-SAW velocity corresponding to the surface Bleustein-Gulyaev-

Melkumyan (BGM) wave [100, 101, 106, 107]. The surface BGM-wave was first discovered by Melkumyan [107] in 2007. The speed of the nondispersive surface BGM-wave can be calculated with the following explicit formula:

$$V_{BGM} = V_{tem}\left[1-\left(\frac{K_{em}^2}{1+K_{em}^2}\right)^2\right]^{1/2} \qquad \text{(II.18)}$$

Also, it is useful to discuss the explicit forms of the complete mechanical displacement defined by expression (I.78) from the ninth subsection of the previous chapter, complete electrical potential (I.79), and complete magnetic potential (I.80). These complete parameters depend on the unknown weight factors F_{01}, F_{02}, F_{03}, and F_{04}. The weight factors can be explicitly determined from the second row of equation (II.13) for the case of equation (II.14) and from the last row for the cases of equations (II.16) and (II.17). Therefore, they can be respectively written as follows;

$$F_{01} = F_{03} = 0\,,\ F_{02} = \sinh\left(kd\sqrt{1-\left(V_{new1}/V_{tem}\right)^2}\right),\ F_{04} = -\sinh(kd) \quad \text{(II.19)}$$

$$F_{02} = F_{04} = 0\,,\ F_{01} = \cosh\left(kd\sqrt{1-\left(V_{new2}/V_{tem}\right)^2}\right),\ F_{03} = -\cosh(kd) \quad \text{(II.20)}$$

It is worth noting that for the case of equation (II.15), the weight factor F_{02} must have the following forms: $F_{02} = \sin\left(kd\sqrt{\left(V_{new1}/V_{tem}\right)^2-1}\right)$ and $F_{04} = \mathrm{j}\sinh(kd)$. Such forms lead to the real complete parameters defined by expressions (I.78), (I.79), and (I.80).

Figure II.1 shows the dependence of the normalized velocities of the new dispersive SH-waves in the transversely isotropic piezoelectromagnetic plate. For a relatively small value of the coefficient of the magnetoelectroelastic coupling (CMEMC) $K_{em}^2 = 0.3$, the relation between the surface BGM-wave velocity and the SH-BAW velocity is V_{BGM}/V_{tem} ~ 0.973. This is shown in figure II.1 by the dotted line. Below the velocity V_{tem}, the velocity V_{new1} of the first new SH-wave in the plate starts at some value of kd ~ 4.4 and approaches the velocity V_{BGM} when the value of the normalized plate thickness kd goes to infinity. Also, the velocity V_{new2} of the second new SH-wave starts with V_{min}/V_{tem} ~ 0.877 at $kd = 0$ and can reach V_{BGM}/V_{tem} at a large value of the nondimensional parameter kd. This fundamental mode cannot exist below the minimum value of V_{min}/V_{tem} ~ 0.877. It is clearly seen in figure II.1 that only single modes called the fundamental modes can exist below the V_{tem}.

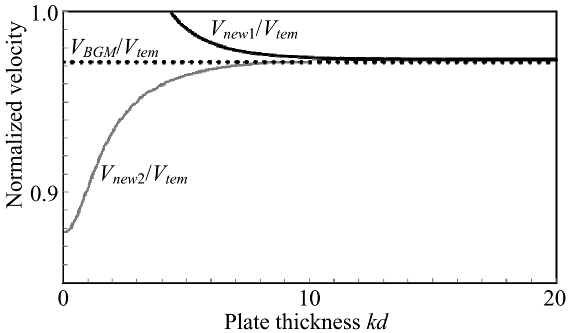

Figure II.1. The fundamental modes' dispersion relations, where the normalized velocities V_{new1}/V_{tem} (black) and V_{new2}/V_{tem} (grey) are defined by equations (II.14) and (II.16), respectively; $K_{em}^2 = 0.3$.

The higher-order modes of the new dispersive SH-waves are shown in figures II.2 and II.3. It is expected that all the higher-order modes start at a very small value of $kd \to 0$ when all the velocities of the first and second new SH-waves go up to some infinitely large values, see figure II.3. Also, it is transparent that the velocities of all the higher-order modes shown in figure II.2 can approach the SH-BAW velocity V_{tem} when the dimensionless parameter kd achieves infinity. However, this is not true for the first higher-order mode shown by the dotted black line in the figure. It is obvious that this first higher-order mode connects with the corresponding fundamental mode about the SH-BAW velocity V_{tem} when the value of the parameter kd is by about 4.4. Thus, it is possible that this first higher-order mode can be also called the fundamental mode due to this connection. Such peculiarity is absent for the other higher-order modes shown by the dotted grey lines in both the figures. This occurs due to the fact that the fundamental mode shown by the grey solid line in figure II.1 cannot reach the value of the SH-BAW velocity V_{tem} and approaches the SH-SAW velocity V_{BGM}.

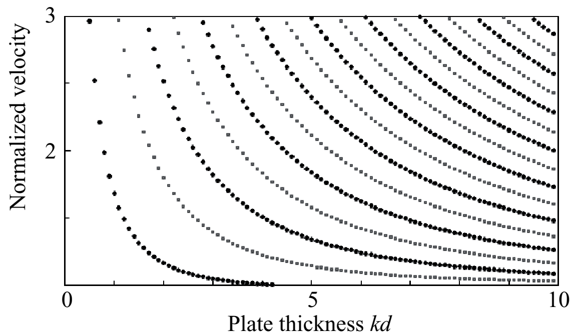

Figure II.2. The higher-order modes' dispersion relations, where the normalized velocities V_{new1}/V_{tem} (black) and V_{new2}/V_{tem} (grey) are defined by equations (II.15) and (II.17), respectively; $K_{em}^2 = 0.3$.

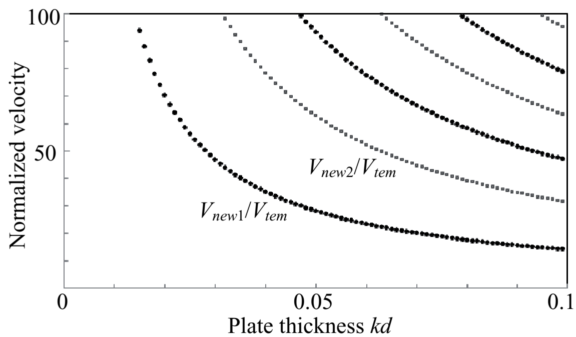

Figure II.3. The beginnings of the first three higher-order modes, where the normalized velocities V_{new1}/V_{tem} (black) and V_{new2}/V_{tem} (grey) are defined by equations (II.15) and (II.17), respectively; $K_{em}^2 = 0.3$.

It is possible to investigate the case of the other electrical and magnetic boundary conditions applied to the free surfaces of the piezoelectromagnetic plate. This work studies the cases of the homogeneous boundary conditions when the same set of the boundary conditions is used for both the upper and lower free surfaces. Therefore, the following chapters acquaint the reader with the other possibilities.

Chapter III

The Electrically Open and Magnetically Closed Boundary Conditions

This chapter studies the second possible case of the electrical and magnetic boundary conditions. In the homogeneous case, the same boundary conditions are utilized at the upper and lower surfaces of the piezoelectromagnetic plate. The mechanical boundary condition is the mechanically free surface, the electrical boundary condition is the electrically open surface (D_3 = 0), and the magnetic boundary condition is the magnetically closed surface (B_3 = 0). It is thought that it is convenient to exploit the weight factors F_{01}, F_{02}, F_{03}, and F_{04}. For the mechanical boundary condition, it is natural to use equation (II.1) from the previous chapter for the upper surface at $x_3 = + d$ and equation (II.4) for the lower surface at $x_3 = - d$. Also, equations (I.93) and (I.108) from the last subsection of Chapter I are likely for the electrical boundary condition and equations (I.99) and (I.114) are suitable for the magnetic one.

For the upper surface at $x_3 = + d$, three homogeneous equations corresponding to the mechanical, electrical, and magnetic boundary conditions read:

$$\begin{aligned}&\left(e\varphi^{0(1)} + h\psi^{0(1)}\right)\left[F_{01}\sinh(kd) + F_{02}\cosh(kd)\right]\\&+ b\left(CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0\end{aligned} \tag{III.1}$$

$$\begin{aligned}&\left(\varepsilon\varphi^{0(1)} + \alpha\psi^{0(1)}\right)\left[F_{01}\sinh(kd) + F_{02}\cosh(kd)\right]\\&- b\left(eU^{0(5)} - \varepsilon\varphi^{0(5)} - \alpha\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0\end{aligned} \tag{III.2}$$

$$\begin{aligned}&\left(\alpha\varphi^{0(1)} + \mu\psi^{0(1)}\right)\left[F_{01}\sinh(kd) + F_{02}\cosh(kd)\right]\\&- b\left(hU^{0(5)} - \alpha\varphi^{0(5)} - \mu\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0\end{aligned} \tag{III.3}$$

These three homogeneous equations for the lower free surface at $x_3 = - d$ can be written in the following forms:

$$\begin{aligned}&\left(e\varphi^{0(1)}+h\psi^{0(1)}\right)\left[F_{01}\sinh(kd)-F_{02}\cosh(kd)\right]\\&+b\left(CU^{0(5)}+e\varphi^{0(5)}+h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]=0\end{aligned}\qquad\text{(III.4)}$$

$$\begin{aligned}&\left(\varepsilon\varphi^{0(1)}+\alpha\psi^{0(1)}\right)\left[F_{01}\sinh(kd)-F_{02}\cosh(kd)\right]\\&-b\left(eU^{0(5)}-\varepsilon\varphi^{0(5)}-\alpha\psi^{0(5)}\right)\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]=0\end{aligned}\qquad\text{(III.5)}$$

$$\begin{aligned}&\left(\alpha\varphi^{0(1)}+\mu\psi^{0(1)}\right)\left[F_{01}\sinh(kd)-F_{02}\cosh(kd)\right]\\&-b\left(hU^{0(5)}-\alpha\varphi^{0(5)}-\mu\psi^{0(5)}\right)\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]=0\end{aligned}\qquad\text{(III.6)}$$

The existence of two different sets of the eigenvectors is the peculiarity that can complicate the theoretical investigations of the SH-wave propagation in the piezoelectromagnetic plate. It is obvious that it is necessary to separately treat each case. Therefore, the following three subsections respectively study the cases when only the first eigenvectors, only the second eigenvectors, and the first and second eigenvectors are used.

III.1. *The first eigenvectors*

Exploiting the first eigenvectors defined by equations (I.66) and (I.67), it is possible to significantly simplify the equations written above. Indeed, it is useful for this case to introduce the following equalities:

$$\varepsilon\varphi^{0(1)}+\alpha\psi^{0(1)}=\varepsilon\alpha-\alpha\varepsilon=0\,,\;eU^{0(5)}-\varepsilon\varphi^{0(5)}-\alpha\psi^{0(5)}=\frac{\alpha e^2-\varepsilon eh}{CK_{em}^2}+\frac{\varepsilon eh}{CK_{em}^2}-\alpha\varepsilon-\frac{\alpha e^2}{CK_{em}^2}+\varepsilon\alpha=0\qquad\text{(III.7)}$$

$$\alpha\varphi^{0(1)}+\mu\psi^{0(1)}=\alpha^2-\varepsilon\mu\,,\;hU^{0(5)}-\alpha\varphi^{0(5)}-\mu\psi^{0(5)}=\frac{\alpha eh-\varepsilon h^2}{CK_{em}^2}+\frac{\alpha eh}{CK_{em}^2}-\alpha^2-\frac{\mu e^2}{CK_{em}^2}+\varepsilon\mu=0\qquad\text{(III.8)}$$

With equalities (III.7) and (III.8), it is flagrant that equations (III.2) and (III.5) can be excluded from the further consideration in this subsection. So, equations

(III.1) and (III.3) for the upper surface and equations (III.4) and (III.6) for the lower surface can be composed in the following forms, using relation (II.9) from the previous chapter:

$$F_{01}\sinh(kd)+F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)+F_{04}\cosh(bkd)\right]=0 \quad \text{(III.9)}$$

$$\left(\varepsilon\mu-\alpha^2\right)\left[F_{01}\sinh(kd)+F_{02}\cosh(kd)\right]=0 \quad \text{(III.10)}$$

$$F_{01}\sinh(kd)-F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]=0 \quad \text{(III.11)}$$

$$\left(\varepsilon\mu-\alpha^2\right)\left[F_{01}\sinh(kd)-F_{02}\cosh(kd)\right]=0 \quad \text{(III.12)}$$

It is apparent that it is convenient to divide both equations (III.10) and (III.12) by ($\varepsilon\mu - \alpha^2$). Then, it is natural to subtract equation (III.10) from equation (III.9) and equation (III.12) from equation (III.11). As a result, equations (III.9) and (III.11) can be written in the following simplified forms:

$$F_{03}\sinh(bkd)+F_{04}\cosh(bkd)=0 \quad \text{(III.13)}$$

$$F_{03}\sinh(bkd)-F_{04}\cosh(bkd)=0 \quad \text{(III.14)}$$

It is blatant that equations (III.13) and (III.14) can give the following equation for the determination of the phase velocity V_{ph} of the plate SH-wave:

$$\sinh\left(kd\sqrt{1-\left(V_{ph}/V_{tem}\right)^2}\right)=0 \quad \text{(III.15)}$$

However, equation (III.15) has only single suitable solution such as $V_{ph} = V_{tem}$ at any value of the dimensionless parameter kd. This is true because the function such as the hyperbolic sine can be equal to zero when the argument equals to zero. This means that the SH-BAW with the speed V_{tem} can propagate in the piezoelectromagnetic plate at any normalized plate thickness kd, but $kd = 0$ because the plate should have nonzero thickness. The solution such as $V_{ph} = V_{tem}$ for equation (III.15) means that any dispersion relation, namely the dependence of the phase velocity V_{ph} on the parameter kd cannot exist below the velocity V_{tem} of the SH-BAW coupled with both the electrical and magnetic potentials.

For this case, it is also possible to discuss the complete parameters defined by expressions (I.78), (I.79), and (I.80) from the ninth subsection of Chapter I. These parameters depend on the unknown weight factors F_{01}, F_{02}, F_{03}, and F_{04}. Utilizing b = 0 and equations (III.13) and (III.14), these weight factors can be found in the following forms:

$$F_{01} = F_{02} = 0\,,\ F_{03} = \cosh(bkd) = 1\,,\ F_{04} = \sinh(bkd) = 0 \qquad \text{(III.16)}$$

On the other hand, equations (III.13) and (III.14) can be rewritten for the case when the phase velocity V_{ph} is larger than the SH-BAW speed V_{tem}. For this case, sine and cosine are used instead of the corresponding hyperbolic functions. Therefore, these equations read:

$$F_{03}\mathrm{j}\sin(b_1 kd) + F_{04}\cos(b_1 kd) = 0 \qquad \text{(III.17)}$$

$$F_{03}\mathrm{j}\sin(b_1 kd) - F_{04}\cos(b_1 kd) = 0 \qquad \text{(III.18)}$$

where

$$b_1 = \sqrt{\left(V_{ph}/V_{tem}\right)^2 - 1} \qquad \text{(III.19)}$$

Consequently, the system of two homogeneous equations (III.17) and (III.18) gives the following dispersion relation for the determination of the phase velocity V_{new3} of the third new SH-wave propagating in the piezoelectromagnetic plate:

$$2\sin(b_1 kd)\cos(b_1 kd) = \sin(2b_1 kd) = 0 \qquad \text{(III.20)}$$

It is clearly seen in equation (III.20) that for this case of $V_{ph} > V_{tem}$, an infinite number of modes of dispersive SH-waves can exist because sine is a periodic function. Therefore, the velocity V_{new3} of the third new SH-wave can be expressed as follows:

$$V_{new3} = V_{tem}\sqrt{1+\left(\frac{n\pi}{2kd}\right)^2},\ n = 1, 2, 3, \ldots \qquad \text{(III.21)}$$

It is worth noting that in definition (III.21) $n = 0$ gives $V_{ph} = V_{tem}$ at any value of kd. So, this is the case of the nondispersive SH-wave that is excluded in definition (III.21).

In this case, the weight factors F_{01}, F_{02}, F_{03}, and F_{04} can have the following likely values to deal with the real complete parameters in equations (I.78), (I.79), and (I.80):

$$F_{01} = F_{02} = 0,\ F_{03} = \cos(b_1 kd),\ F_{04} = \mathrm{j}\sin(b_1 kd) \qquad \text{(III.22)}$$

For $V_{ph} > V_{tem}$, figure III.1 shows several modes of the dispersive new SH-wave propagating in the piezoelectromagnetic plate in the case of the homogeneous boundary conditions at both the upper and lower surfaces of the plate. The values of the normalized velocity V_{new3}/V_{tem} of the third new SH-wave are calculated with formula (II.21) and shown by the black dotted lines in the figure. It is blatant that the normalized velocity V_{new3}/V_{tem} for each mode starts with an infinite value at $kd \to 0$ and approaches unity when $kd \to \infty$, where k is the wavenumber in the propagation direction and d is the half-thickness of the piezoeletromagnetic plate as shown in figure 1 in Introduction.

It is worth noticing that the dispersion relations in figure III.1 are valid for any value of the CMEMC K_{em}^2. It is expected that the nondimensional value of the

CMEMC can be changed from zero to unity. It is also assumed that the CMEMC value can equal to zero for nonzero values of the material constants such as e, h, and α. Indeed, the CMEMC value can have a negative sign for the case when a left-handed material (metamaterial) is studied. Such metamaterials can possess both $\mu < 0$ and $\varepsilon < 0$. This can result in $K_{em}^2 < 0$, see formula (I.53) in Chapter I. However, for the other cases when only either $\mu < 0$ or $\varepsilon < 0$ there is a probability that $K_{em}^2 = 0$ can occur. For zero value of the CMEMC when $V_{tem} = V_{t4}$, it is convenient to use the velocity V_{t4} defined by the first formula in expressions (I.53).

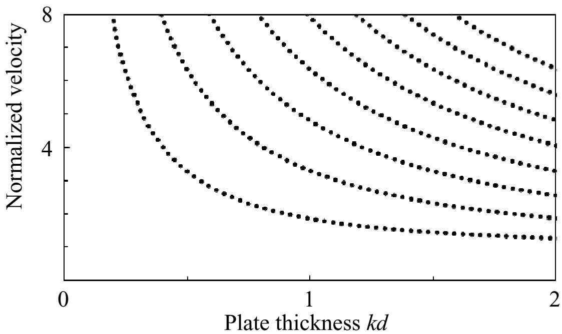

Figure III.1. The higher-order modes, where the normalzed velocity V_{new3}/V_{tem} is defined by equation (III.21).

III.2. *The second eigenvectors*

Employing the second eigenvectors defined by equations (I.69) and (I.70), one can find that

$$\varepsilon\varphi^{0(1)} + \alpha\psi^{0(1)} = \varepsilon\mu - \alpha^2, \; eU^{0(5)} - \varepsilon\varphi^{0(5)} - \alpha\psi^{0(5)} = \frac{\mu e^2 - \alpha e h}{CK_{em}^2} + \frac{\varepsilon h^2}{CK_{em}^2} - \varepsilon\mu - \frac{\alpha e h}{CK_{em}^2} + \alpha^2 = 0 \tag{III.23}$$

$$\alpha\varphi^{0(1)} + \mu\psi^{0(1)} = \alpha\mu - \mu\alpha = 0\,,\; hU^{0(5)} - \alpha\varphi^{0(5)} - \mu\psi^{0(5)} = \frac{e\mu h - \alpha h^2}{CK_{em}^2} + \frac{\alpha h^2}{CK_{em}^2} - \alpha\mu - \frac{e\mu h}{CK_{em}^2} + \alpha\mu = 0 \tag{III.24}$$

These values of the second eigenvectors written above lead to the following four homogeneous equations:

$$F_{01}\sinh(kd) + F_{02}\cosh(kd) + b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0 \tag{III.25}$$

$$\left(\varepsilon\mu - \alpha^2\right)\left[F_{01}\sinh(kd) + F_{02}\cosh(kd)\right] = 0 \tag{III.26}$$

$$F_{01}\sinh(kd) - F_{02}\cosh(kd) + b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd) - F_{04}\cosh(bkd)\right] = 0 \tag{III.27}$$

$$\left(\varepsilon\mu - \alpha^2\right)\left[F_{01}\sinh(kd) - F_{02}\cosh(kd)\right] = 0 \tag{III.28}$$

It is clearly seen that these four equations written above are identical to four equations from (III.9) to (III.12) written in the previous subsection. So, it is flagrant that the second eigenvectors and relations (III.23) and (III.24) actually lead to the same solution defined by expression (III.21) in the previous subsection. The dispersion relations for the case of $V_{ph} > V_{tem}$ are shown in figure III.1 which shows the behavior of the higher-order modes because the fundamental mode does not exist for the case of $V_{ph} < V_{tem}$.

III.3. *The first and second eigenvectors*

This subsection treats the third possible case when first eigenvectors (I.66) and (I.67) are used for the upper free surface of the plate and second eigenvectors (I.69) and (I.70) are utilized for the lower free surface. It is thought that the converse configuration must give the same results for the case of the homogeneous boundary conditions because the upper and lower surfaces are identical. It is obvious that for the upper surface, equations (III.9) and (III.10) must be considered. For the lower surface, equations (III.27) and (III.28) must be chosen. It is flagrant that in this case of the first eigenvectors for the upper surface and the second eigenvectors for the lower surface, the results will be the same to those obtained in the first subsection because equations from (III.9) to (III.12) are identical to equations from (III.25) to (III.28).

Chapter IV

The Electrically Open and Magnetically Open Boundary Conditions

It was demonstrated in Chapters II and III that the exploitation of different electrical and magnetic boundary conditions can lead to different results. The electrical and magnetic boundary conditions used in these two chapters soundly result in the existence of the certain solutions which do not depend on the used eigenvectors. This chapter studies the SH-wave propagation in the transversely isotropic piezoelectromagnetic plate when the following mechanical, electrical, and magnetic boundary conditions are utilized for both the upper and lower free surfaces of the plate: mechanically free ($\sigma_{32} = 0$), electrically open ($D_3 = 0$), and magnetically open ($\psi = 0$) surface. Therefore, it is natural to use suitable equations obtained in the previous two chapters. For the upper free surface at $x_3 = + d$, equations (II.1), (III.2), and (II.3) are the suitable three homogeneous equations corresponding to three boundary conditions. For the lower free surface at $x_3 = - d$, equations (II.4), (III.5), and (II.6) are the suitable ones. These six homogeneous equations are written with the weight factors F_{01}, F_{02}, F_{03}, and F_{04} for convenience and defined below.

At $x_3 = + d$, the mechanical, electrical, and magnetic boundary conditions are respectively written as follows:

$$\begin{aligned}&\left(e\varphi^{0(1)} + h\psi^{0(1)}\right)\left[F_{01}\sinh(kd) + F_{02}\cosh(kd)\right]\\&+ b\left(CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0\end{aligned} \quad \text{(IV.1)}$$

$$\begin{aligned}&\left(\varepsilon\varphi^{0(1)} + \alpha\psi^{0(1)}\right)\left[F_{01}\sinh(kd) + F_{02}\cosh(kd)\right]\\&- b\left(eU^{0(5)} - \varepsilon\varphi^{0(5)} - \alpha\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0\end{aligned} \quad \text{(IV.2)}$$

$$\left[F_{01}\cosh(kd) + F_{02}\sinh(kd)\right]\psi^{0(1)} + \left[F_{03}\cosh(bkd) + F_{04}\sinh(bkd)\right]\psi^{0(5)} = 0 \quad \text{(IV.3)}$$

For the lower free surface at $x_3 = -d$, three homogeneous equations corresponding to three boundary conditions can be transformed as follows:

$$\begin{aligned}&\left(e\varphi^{0(1)} + h\psi^{0(1)}\right)\left[F_{01}\sinh(kd) - F_{02}\cosh(kd)\right]\\&+ b\left(CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) - F_{04}\cosh(bkd)\right] = 0\end{aligned} \tag{IV.4}$$

$$\begin{aligned}&\left(\varepsilon\varphi^{0(1)} + \alpha\psi^{0(1)}\right)\left[F_{01}\sinh(kd) - F_{02}\cosh(kd)\right]\\&- b\left(eU^{0(5)} - \varepsilon\varphi^{0(5)} - \alpha\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) - F_{04}\cosh(bkd)\right] = 0\end{aligned} \tag{IV.5}$$

$$\left[F_{01}\cosh(kd) - F_{02}\sinh(kd)\right]\psi^{0(1)} + \left[F_{03}\cosh(bkd) - F_{04}\sinh(bkd)\right]\psi^{0(5)} = 0 \tag{IV.6}$$

Using the eigenvector components, these six homogeneous equations can be further transformed. Therefore, the following subsections deal with the different cases when the different eigenvectors are utilized.

IV.1. *The first eigenvectors*

Exploitation of the first eigenvectors defined by expressions (I.66) and (I.67) means that relations (II.10) and (III.7) must be accounted for the six homogeneous equations written above. Employing relations (II.9) and (II.10) for equations (IV.1) and (IV.4), the last two equations can be transformed into the following forms:

$$F_{01}\sinh(kd) + F_{02}\cosh(kd) + b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0 \tag{IV.7}$$

$$F_{01}\sinh(kd) - F_{02}\cosh(kd) + b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd) - F_{04}\cosh(bkd)\right] = 0 \tag{IV.8}$$

Using expressions (III.7), equations (IV.2) and (IV.5) can be excluded from the further considerations in this subsection because they are vanishing. Utilizing the first eigenvectors defined by expressions (I.66) and (I.67), equations (IV.3) and (IV.6) can be composed as follows:

$$F_{01}\cosh(kd)+F_{02}\sinh(kd)+\frac{K_{em}^2-K_e^2}{K_{em}^2}\left[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)\right]=0 \quad \text{(IV.9)}$$

$$F_{01}\cosh(kd)-F_{02}\sinh(kd)+\frac{K_{em}^2-K_e^2}{K_{em}^2}\left[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)\right]=0 \quad \text{(IV.10)}$$

where K_e^2 represents the coefficient of the electromechanical coupling (CEMC). Unlike the material characteristic of the two-phase materials such as the CMEMC K_{em}^2, the CEMC K_e^2 is the material characteristic of a single-phase material such as a pure piezoelectrics and can be defined by the following expression [100, 101]:

$$K_e^2=\frac{e^2}{C\varepsilon} \quad \text{(IV.11)}$$

It is natural to further transform the first pair of equations (IV.7) and (IV.8) and the second pair of equations (IV.9) and (IV.10). These transformations can be done in the similar manner completed in Chapter II. First, it is possible to treat equations (IV.7) and (IV.8). It is possible to add equation (IV.8) to equation (IV.7). The resulting equation can be divided by ½ and then subtracted from equation (IV.8). The same simplifications can be carried out with equations (IV.9) and (IV.10). As a result, these four equations can be grouped into two independent pairs, of which the first pair consists of two equations only with the weight factors F_{01} and F_{03}, but the second pair of equations is coupled with the weight factors F_{02} and F_{04}. Thus, the first system of two homogeneous equations can be introduced in the following forms:

$$F_{01}\cosh(kd)+\frac{K_{em}^2-K_e^2}{K_{em}^2}F_{03}\cosh(bkd)=0 \quad \text{(IV.12)}$$

$$F_{01}\sinh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}F_{03}\sinh(bkd)=0 \quad \text{(IV.13)}$$

The second system of two homogeneous equations can be written as follows:

$$F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}F_{04}\cosh(bkd)=0 \tag{IV.14}$$

$$F_{02}\sinh(kd)+\frac{K_{em}^2-K_e^2}{K_{em}^2}F_{04}\sinh(bkd)=0 \tag{IV.15}$$

These four equations are valid for the case when the phase velocity V_{ph} is smaller than the SH-BAW speed V_{tem}. It is evident that the first pair of these four equations written above can reveal the velocity V_{new4} of the fourth new SH-wave propagating in the plate. Indeed, equations (IV.12) and (IV.13) actually lead to the following dispersion relation:

$$\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\tanh(kd)-\sqrt{1-(V_{new4}/V_{tem})^2}\tanh\left(kd\sqrt{1-(V_{new4}/V_{tem})^2}\right)=0 \tag{IV.16}$$

Using equation (IV.12), the weight factors are defined as follows:

$$F_{02}=F_{04}=0\,,\ F_{01}=\frac{K_{em}^2-K_e^2}{K_{em}^2}\cosh(bkd),\ F_{03}=-\cosh(kd) \tag{IV.17}$$

Considering the second pair of equations (IV.14) and (IV.15), the velocity V_{new5} of the fifth new SH-wave propagating in the plate can be revealed. The corresponding dispersion relation reads:

$$\sqrt{1-(V_{new5}/V_{tem})^2}\tanh(kd)-\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\tanh\left(kd\sqrt{1-(V_{new5}/V_{tem})^2}\right)=0 \tag{IV.18}$$

Utilizing equation (IV.14), the weight factors can be determined as follows:

$$F_{01}=F_{03}=0\,,\ F_{02}=b\frac{1+K_{em}^2}{K_{em}^2}\cosh(bkd),\ F_{04}=-\cosh(kd) \tag{IV.19}$$

Following the results obtained in Chapter II, it is possible to mention that the fundamental modes of the first and second new SH-waves shown in figure II.1 are calculated with dispersion relations (II.14) and (II.16), respectively. It was discussed in Chapter II that the velocities of both the new SH-waves approach surface BGM-wave [100, 101, 105, 106, 107] for a large value of the dimensionless plate thickness *kd*. Dispersion relation (IV.18) qualitatively looks like dispersion relation (II.14) with the single significant difference such that the parameter K_e^2 defined by expression (IV.11) is absent in equation (II.14). The same difference can be found when dispersion relations (II.16) and (IV.16) are analyzed for comparison. Therefore, the fundamental modes of the fourth and fifth new SH-waves defined by dispersion relations (IV.16) and (IV.18) can also approach some surface SH-wave at $kd \to \infty$. This surface SH-wave is called the piezomagnetic exchange surface Melkumyan wave or PMESM wave [105]. The surface PMESM-wave was first discovered by Melkumyan [107] in 2007 and also studied in Refs. [100, 101, 105, 106]. The speed of the nondispersive PMESM-wave is explicitly defined by the following formula:

$$V_{PMESM} = V_{tem}\left[1-\left(\frac{K_{em}^2 - K_e^2}{1+K_{em}^2}\right)^2\right]^{1/2} \tag{IV.20}$$

It is also indispensable to study the case when $V_{ph} > V_{tem}$ occurs. Exploiting expression (III.19) from the previous chapter and the wellknown formulae such as $\cosh(bkd) = \cos(b_1 kd)$ and $\sinh(bkd) = \mathrm{j}\sin(b_1 kd)$, equations (IV.12) and (IV.13) can be readily transformed into the following forms:

$$F_{01}\cosh(kd) + \frac{K_{em}^2 - K_e^2}{K_{em}^2} F_{03}\cos(b_1 kd) = 0 \tag{IV.21}$$

$$F_{01}\sinh(kd) - b_1\frac{1+K_{em}^2}{K_{em}^2} F_{03}\sin(b_1 kd) = 0 \tag{IV.22}$$

As a result, dispersion relation (IV.16) for the determination of the velocity V_{new4} of the fourth new SH-wave can be rewritten for the case of $V_{ph} > V_{tem}$ in the following form:

$$\frac{K_{em}^2 - K_e^2}{1 + K_{em}^2}\tanh(kd) + \tan\left(kd\sqrt{(V_{new4}/V_{tem})^2 - 1}\right)\sqrt{(V_{new4}/V_{tem})^2 - 1} = 0 \qquad \text{(IV.23)}$$

Also, the weight factors for the case of equations from (IV.21) to (IV.23) read:

$$F_{02} = F_{04} = 0\,,\ F_{01} = \frac{K_{em}^2 - K_e^2}{K_{em}^2}\cos(b_1 kd),\ F_{03} = -\cosh(kd)` \qquad \text{(IV.24)}$$

where the parameter b_1 is defined by expression (III.19).

Concerning the case of $V_{ph} > V_{tem}$, it is also possible to transform the second pair of equations (IV.14) and (IV.15). These equations can be transformed into the following forms:

$$F_{02}\cosh(kd) + \mathrm{j}b_1\frac{1 + K_{em}^2}{K_{em}^2}F_{04}\cos(b_1 kd) = 0 \qquad \text{(IV.25)}$$

$$F_{02}\sinh(kd) + \mathrm{j}\frac{K_{em}^2 - K_e^2}{K_{em}^2}F_{04}\sin(b_1 kd) = 0 \qquad \text{(IV.26)}$$

Consequently, the velocity V_{new5} of the fifth new SH-wave propagating in the plate can be revealed for the case of $V_{ph} > V_{tem}$ by utilizing the following modified form of dispersion relation (IV.18):

$$\tanh(kd)\sqrt{(V_{new5}/V_{tem})^2 - 1} - \frac{K_{em}^2 - K_e^2}{1 + K_{em}^2}\tan\left(kd\sqrt{(V_{new5}/V_{tem})^2 - 1}\right) = 0 \qquad \text{(IV.27)}$$

The corresponding weight factors are defined by

$$F_{01}=F_{03}=0\,,\ F_{02}=b_1\frac{1+K_{em}^2}{K_{em}^2}\cos(b_1kd)\,,\ F_{04}=\mathrm{j}\cosh(kd) \tag{IV.28}$$

The weight factor F_{04} in expressions (IV.28) is chosen imaginary because such choice allow the complete mechanical displacement (I.78), complete electrical potential (I.79), and complete magnetic potential (I.80) to stay real parameters.

IV.2. *The second eigenvectors*

In the case of the first and second eigenvectors, equations (IV.1) and (IV.4) can be transformed into the same forms, see equations from (II.9) to (II.11). For the case of the second eigenvectors defined by expressions (I.69) and (I.70), equations (IV.7) and (IV.8) from the previous subsection can be used and multiplied by $\varepsilon\mu$ for the further transformations. So, they read:

$$\varepsilon\mu[F_{01}\sinh(kd)+F_{02}\cosh(kd)]+\varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}[F_{03}\sinh(bkd)+F_{04}\cosh(bkd)]=0 \tag{IV.29}$$

$$\varepsilon\mu[F_{01}\sinh(kd)-F_{02}\cosh(kd)]+\varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)]=0 \tag{IV.30}$$

Exploiting expressions (III.23), equations (IV.2) and (IV.5) can be written in the following forms for clearance:

$$(\varepsilon\mu-\alpha^2)[F_{01}\sinh(kd)+F_{02}\cosh(kd)]+0\times[F_{03}\sinh(bkd)+F_{04}\cosh(bkd)]=0 \tag{IV.31}$$

$$(\varepsilon\mu-\alpha^2)[F_{01}\sinh(kd)-F_{02}\cosh(kd)]+0\times[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)]=0 \tag{IV.32}$$

Utilizing second eigenvectors (I.69) and (I.70), equations (IV.3) and (IV.6) multiplied by the electromagnetic constant α read:

$$-\alpha^2[F_{01}\cosh(kd)+F_{02}\sinh(kd)]-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)]=0 \qquad \text{(IV.33)}$$

$$-\alpha^2[F_{01}\cosh(kd)-F_{02}\sinh(kd)]-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)]=0 \qquad \text{(IV.34)}$$

In equations (IV.33) and (IV.34) there is the parameter K_α^2 also used in Refs. [100, 101, 102]. It couples together the third term with electromagnetic constants α in the numerator of the CMEMC defined by the second expression in definitions (I.53) and the second term in the denominator. This material characteristic is therefore defined by:

$$K_\alpha^2=\frac{\alpha eh}{C\alpha^2}=\frac{eh}{C\alpha} \qquad \text{(IV.35)}$$

Because of the fact that there are only four unknown weight factors F_{01}, F_{02}, F_{03}, and F_{04}, it is blatant that it is convenient to deal with four homogeneous equations instead of the first six equations written above in this subsection. To reduce the number of equations from six to four, it is natural to properly transform equations (IV.29), (IV.30), (IV.33), and (IV.34). For this purpose, it is possible to treat the first pair of equations (IV.29) and (IV.33). It is obvious that each of these two equations must have the same first term which must be equal to the first term in equation (IV.31). Accounting equation (IV.33) for several simple transformations of equation (IV.29), the last equation can be finally introduced in the following form:

$$\begin{aligned}&(\varepsilon\mu-\alpha^2)[F_{01}\sinh(kd)+F_{02}\cosh(kd)]+\varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}[F_{03}\sinh(bkd)+F_{04}\cosh(bkd)]\\&-\alpha^2[F_{01}(\cosh(kd)-\sinh(kd))+F_{02}(\sinh(kd)-\cosh(kd))]\\&-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)]=0\end{aligned} \qquad \text{(IV.36)}$$

Accounting equation (IV.29) for several simple transformations of equation (IV.33), the last equation can be obtained in the same form written above in equation (IV.36).

It is overt that the same transformations can be done with the second pair of two equations (IV.30) and (IV.34). Indeed, these two equations can be separately transformed into the following form:

$$\begin{aligned}&(\varepsilon\mu-\alpha^2)[F_{01}\sinh(kd)-F_{02}\cosh(kd)]+\varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)]\\&-\alpha^2[F_{01}(\cosh(kd)-\sinh(kd))-F_{02}(\sinh(kd)-\cosh(kd))]\\&-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)]=0\end{aligned}\tag{IV.37}$$

So, the resulting system of four homogeneous equations (IV.31), (IV.32), (IV.36), and (IV.37) with four unknown weight factors F_{01}, F_{02}, F_{03}, and F_{04} must be further treated. It this set of four equations, it is also necessary to account the fact that $F_{01} = F_{02} = 0$. This equality is clearly seen in equations (IV.31) and (IV.32). Therefore, equations (IV.31) and (IV.37) can be rewritten in the following simplified forms:

$$\begin{aligned}&F_{03}\left(\varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}\sinh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\cosh(bkd)\right)\\&+F_{04}\left(\varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}\cosh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\sinh(bkd)\right)=0\end{aligned}\tag{IV.38}$$

$$\begin{aligned}&F_{03}\left(\varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}\sinh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\cosh(bkd)\right)\\&-F_{04}\left(\varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}\cosh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\sinh(bkd)\right)=0\end{aligned}\tag{IV.39}$$

It is clearly seen in equations (IV.38) and (IV.39) that they have only the single difference such as the sign before the weight factor F_{04}. As a result, the following dispersion relation for the determination of the velocity V_{new6} of the sixth new SH-wave propagating in the plate can be obtained by using equation (IV.38) or (IV.39):

$$\sqrt{1-(V_{new6}/V_{tem})^2}\tanh\left(kd\sqrt{1-(V_{new6}/V_{tem})^2}\right)-\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}=0 \qquad \text{(IV.40)}$$

$$F_{01}=F_{02}=F_{04}=0,\ F_{03}=1 \qquad \text{(IV.41)}$$

It is also flagrant that equations (IV.38) and (IV.39) lead to the following second dispersion relation for the determination of the velocity V_{new7} of the seventh new SH-wave that is written with the corresponding weight factors as follows:

$$\sqrt{1-(V_{new7}/V_{tem})^2}-\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\tanh\left(kd\sqrt{1-(V_{new7}/V_{tem})^2}\right)=0 \qquad \text{(IV.42)}$$

$$F_{01}=F_{02}=F_{03}=0,\ F_{04}=1 \qquad \text{(IV.43)}$$

It is worth noting that dispersion relations (IV.40) and (IV.42) are valid for the case of $V_{ph} < V_{tem}$. Thus, these dispersion relations reveal the fundamental modes. These zero-order modes can exist in this case because the corresponding SH-SAW discovered by the author in previous work [101] can propagate with the velocity defined by expression (163) in book [101], see also expression (5) and (6) in paper [105] available on-line for an open access. However, this SH-SAW has a peculiarity such that this SH-SAW speed is slightly smaller than the SH-BAW speed V_{tem}. So, the value of this SH-SAW velocity is situated significantly closer to the value of the SH-BAW velocity V_{tem} in comparison with the other SH-SAWs mentioned in the previous subsection, Chapter II, and the following chapter.

In the case of $V_{ph} > V_{tem}$, dispersion relations (IV.40) and (IV.42) are respectively transformed into the following forms:

$$\tan\left(kd\sqrt{(V_{new6}/V_{tem})^2-1}\right)\sqrt{(V_{new6}/V_{tem})^2-1}+\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}=0 \qquad \text{(IV.44)}$$

$$\sqrt{(V_{new7}/V_{tem})^2-1}-\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\tan\left(kd\sqrt{(V_{new7}/V_{tem})^2-1}\right)=0 \qquad \text{(IV.45)}$$

IV.3. *The first and second eigenvectors*

In this case, first eigenvectors (I.66) and (I.67) are used for the upper free surface of the piezoelectromagnetic plate at $x_3 = + d$, and the second eigenvectors (I.69) and (I.70) are employed for the lower free surface at $x_3 = - d$. Therefore, equations (IV.7) and (IV.9) from the first subsection of this chapter can be borrowed for the upper surface. They read:

$$F_{01}\sinh(kd) + F_{02}\cosh(kd) + b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0 \quad \text{(IV.46)}$$

$$F_{01}\cosh(kd) + F_{02}\sinh(kd) + \frac{K_{em}^2 - K_e^2}{K_{em}^2}\left[F_{03}\cosh(bkd) + F_{04}\sinh(bkd)\right] = 0 \quad \text{(IV.47)}$$

For the lower surface at $x_3 = - d$ with the second eigenvectors (I.69) and (I.70), the corresponding equations are equations (IV.30), (IV.32), and (IV.34) from the previous subsection. These three homogeneous equations can be written as the following two equations:

$$\varepsilon\mu\left[F_{01}\sinh(kd) - F_{02}\cosh(kd)\right] + \varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd) - F_{04}\cosh(bkd)\right] = 0 \quad \text{(IV.48)}$$

$$\left(\varepsilon\mu - \alpha^2\right)\left[F_{01}\sinh(kd) - F_{02}\cosh(kd)\right] + 0\times\left[F_{03}\sinh(bkd) - F_{04}\cosh(bkd)\right] = 0 \quad \text{(IV.49)}$$

$$-\alpha^2\left[F_{01}\cosh(kd) - F_{02}\sinh(kd)\right] - \alpha^2\frac{K_{em}^2 - K_\alpha^2}{K_{em}^2}\left[F_{03}\cosh(bkd) - F_{04}\sinh(bkd)\right] = 0 \quad \text{(IV.50)}$$

These three homogenous equations written above can be reduced to two ones. Indeed, both equations (IV.48) and (IV.50) can be separately transformed into the following form:

$$
\begin{aligned}
&(\varepsilon\mu-\alpha^2)[F_{01}\sinh(kd)-F_{02}\cosh(kd)]+\varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)]\\
&-\alpha^2[F_{01}(\cosh(kd)-\sinh(kd))-F_{02}(\sinh(kd)-\cosh(kd))]\\
&-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)]=0
\end{aligned}
\tag{IV.51}
$$

So, it is natural to use equation (IV.51) instead of equations (IV.48) and (IV.50) to have four homogeneous equations with four unknown weight factors F_{01}, F_{02}, F_{03}, and F_{04}. Using equation (IV.49), it is possible to record the following relationship between the weight factors F_{01} and F_{02}:

$$
F_{01}\sinh(kd)=F_{02}\cosh(kd) \tag{IV.51}
$$

Next, utilization of relation (IV.51) for equations (IV.46), (IV.47), and (IV.51) leads to the following three homogeneous equations for the determination of three weight factors:

$$
F_{01}\sinh(2kd)+b\frac{1+K_{em}^2}{K_{em}^2}\cosh(kd)[F_{03}\sinh(bkd)+F_{04}\cosh(bkd)]=0 \tag{IV.52}
$$

$$
F_{01}\cosh(2kd)+\frac{K_{em}^2-K_e^2}{K_{em}^2}\cosh(kd)[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)]=0 \tag{IV.53}
$$

$$
\begin{aligned}
&-\frac{\alpha^2}{\varepsilon\mu}F_{01}+b\frac{1+K_{em}^2}{K_{em}^2}\cosh(kd)[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)]\\
&-\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\cosh(kd)[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)]=0
\end{aligned}
\tag{IV.54}
$$

It is convenient to use the second of these three equations written above to determine F_{01} as a function of F_{03} and F_{04}. Thus, the following two homogeneous equations for determination of F_{03} and F_{04} can be obtained from equations (IV.52) and (IV.54):

$$F_{03}\left(b\tanh(bkd)-\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\tanh(2kd)\right)$$
$$+F_{04}\left(b-\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\tanh(2kd)\tanh(bkd)\right)=0 \quad \text{(IV.55)}$$

$$F_{03}\left(\frac{\alpha^2}{\varepsilon\mu}\frac{1}{\cosh(2kd)}\frac{K_{em}^2-K_e^2}{1+K_{em}^2}+b\tanh(bkd)-\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)$$
$$+F_{04}\left(\frac{\alpha^2}{\varepsilon\mu}\frac{1}{\cosh(2kd)}\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\tanh(bkd)-b+\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\tanh(bkd)\right)=0 \quad \text{(IV.56)}$$

As a result, the following complicated dispersion relation for the determination of the velocity V_{new8} of the eighth new SH-wave can be written as follows:

$$\left(\sqrt{1-(V_{new8}/V_{tem})^2}-\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\tanh(2kd)\tanh\left(kd\sqrt{1-(V_{new8}/V_{tem})^2}\right)\right)$$
$$\times\left(\frac{\alpha^2}{\varepsilon\mu}\frac{1}{\cosh(2kd)}\frac{K_{em}^2-K_e^2}{1+K_{em}^2}+\sqrt{1-(V_{new8}/V_{tem})^2}\tanh\left(kd\sqrt{1-(V_{new8}/V_{tem})^2}\right)-\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)$$
$$-\left(\sqrt{1-(V_{new8}/V_{tem})^2}\tanh\left(kd\sqrt{1-(V_{new8}/V_{tem})^2}\right)-\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\tanh(2kd)\right)$$
$$\times\left(\frac{\alpha^2}{\varepsilon\mu}\left(\frac{1}{\cosh(2kd)}\frac{K_{em}^2-K_e^2}{1+K_{em}^2}+\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)\tanh\left(kd\sqrt{1-(V_{new8}/V_{tem})^2}\right)-\sqrt{1-(V_{new8}/V_{tem})^2}\right)=0$$

(IV.57)

Note that dispersion relation (IV.57) is for the case of $V_{ph} < V_{tem}$ and it can be readily transformed into the corresponding dispersion relation for the case of $V_{ph} > V_{tem}$. This is like the previous transformations carried out several times in the previous subsections and chapters. So, it is thought that these transformations are already clear and there is no necessity to repeat them because it is necessary to save space for the following chapters. It is obvious that for $V_{ph} < V_{tem}$ there can exist only single fundamental mode and for $V_{ph} > V_{tem}$, an infinite number of the higher-order modes can be found because the hyperbolic tangent goes to the periodic function such as tangent. It is worth noting that obtained dispersion relation (IV.57) is for the case when all four weight factors F_{01}, F_{02}, F_{03}, and F_{04} can have some nonzero values.

This is true because F_{01} and F_{02} are soundly found from relation (IV.51) and the other couple of the weight factors can be found from relation (IV.55) or (IV.56). However, it is possible to demonstrate that an additional dispersion relation can be also found for this mixed case when the first eigenvectors are used for the upper surface of the plate and the second eigenvectors are exploited for the lower surface. Note that this second case is for $F_{01} = F_{02} =0$.

For this purpose, consider five homogeneous equations from (IV.46) to (IV.50) anew. It is possible to separately transform equations (IV.46) and (IV.47) into the same equation. These transformations are similar to those carried out for equations (IV.48) and (IV.50). So, both equation (IV.46) and (IV.47) can be led to the following form:

$$\begin{aligned} & F_{03}\left(b\sinh(bkd)-\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\cosh(bkd)\right) \\ & +F_{04}\left(b\cosh(bkd)-\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\sinh(bkd)\right)=0 \end{aligned} \tag{IV.58}$$

In equation (IV.58), it is already accounted that this is the case when $F_{01} = F_{02}$ $=0$ and the weight factors F_{03} and F_{04} must be found. Consequently, equation (IV.51) reads as follows:

$$\begin{aligned} & F_{03}\left(b\sinh(bkd)-\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\cosh(bkd)\right) \\ & -F_{04}\left(b\cosh(bkd)-\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\sinh(bkd)\right)=0 \end{aligned} \tag{IV.59}$$

For this reason, the following dispersion relation for the determination of the velocity V_{new9} of the ninth new SH-wave is composed as follows:

$$\left(1-\left(V_{new9}/V_{tem}\right)^2+\left(\frac{\alpha^2}{\varepsilon\mu}\right)^2\frac{\left(K_{em}^2-K_e^2\right)\left(K_{em}^2-K_\alpha^2\right)}{\left(1+K_{em}^2\right)^2}\right)\tanh\left(2kd\sqrt{1-\left(V_{new9}/V_{tem}\right)^2}\right)$$
$$-\sqrt{1-\left(V_{new9}/V_{tem}\right)^2}\,\frac{\alpha^2}{\varepsilon\mu}\frac{2K_{em}^2-K_e^2-K_\alpha^2}{1+K_{em}^2}=0 \quad \text{(IV.60)}$$

Obtained dispersion relation (IV.60) is apt for the case of $V_{ph} < V_{tem}$. The weight factors F_{03} and F_{04} can be found from equation (IV.58) or (IV.59). Expression (IV.60) is relatively simple and can be therefore rewritten for the case of $V_{ph} > V_{tem}$ as follows:

$$\left(1-\left(V_{new9}/V_{tem}\right)^2+\left(\frac{\alpha^2}{\varepsilon\mu}\right)^2\frac{\left(K_{em}^2-K_e^2\right)\left(K_{em}^2-K_\alpha^2\right)}{\left(1+K_{em}^2\right)^2}\right)\tan\left(2kd\sqrt{\left(V_{new9}/V_{tem}\right)^2-1}\right)$$
$$-\frac{\alpha^2}{\varepsilon\mu}\frac{2K_{em}^2-K_e^2-K_\alpha^2}{1+K_{em}^2}\sqrt{\left(V_{new9}/V_{tem}\right)^2-1}=0 \quad \text{(IV.61)}$$

It is also possible to treat the reverse case when equations (IV.8) and (IV.10) are used for the upper surface of the piezoelectromagnetic plate and equations (IV.29), (IV.31), and (IV.33) are utilized for the lower surface. For this reverse case, equations (IV.57) and (IV.60) also represent dispersion relations. This means that all possible dispersion relations were found.

Chapter V

The Electrically Closed and Magnetically Closed Boundary Conditions

The previous chapter has provided dispersion relations for propagation of several new SH-waves in the piezoelectromagnetic plate when the upper and lower free surfaces of the plate are mechanically free ($\sigma_{32} = 0$), electrically open ($D_3 = 0$), and magnetically open ($\psi = 0$) surfaces. This chapter studies the other possible case when the mechanical boundary condition remains the same, the electrical one represents the electrically closed surface ($\varphi = 0$), and the magnetic one is the magnetically closed surface ($B_3 = 0$).

Exploiting the weight factors F_{01}, F_{02}, F_{03}, and F_{04}, the mechanical, electrical, and magnetic boundary conditions are respectively written for the case of the upper surface at $x_3 = +d$ in the following forms:

$$\begin{aligned}&\left(e\varphi^{0(1)} + h\psi^{0(1)}\right)\left[F_{01}\sinh(kd) + F_{02}\cosh(kd)\right]\\&+b\left(CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0\end{aligned} \quad \text{(V.1)}$$

$$\left[F_{01}\cosh(kd) + F_{02}\sinh(kd)\right]\varphi^{0(1)} + \left[F_{03}\cosh(bkd) + F_{04}\sinh(bkd)\right]\varphi^{0(5)} = 0 \quad \text{(V.2)}$$

$$\begin{aligned}&\left(\alpha\varphi^{0(1)} + \mu\psi^{0(1)}\right)\left[F_{01}\sinh(kd) + F_{02}\cosh(kd)\right]\\&-b\left(hU^{0(5)} - \alpha\varphi^{0(5)} - \mu\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0\end{aligned} \quad \text{(V.3)}$$

These three homogeneous equations written above represent equations (II.1), (II.2), and (III.3) given in the second and third chapters.

For the lower free surface at $x_3 = -d$, three homogeneous equations corresponding to three boundary conditions can be introduced in the following forms:

$$\begin{aligned}&\left(e\varphi^{0(1)} + h\psi^{0(1)}\right)\left[F_{01}\sinh(kd) - F_{02}\cosh(kd)\right]\\&+b\left(CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) - F_{04}\cosh(bkd)\right] = 0\end{aligned} \quad \text{(V.4)}$$

$$\left[F_{01}\cosh(kd)-F_{02}\sinh(kd)\right]\varphi^{0(1)}+\left[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)\right]\varphi^{0(5)}=0 \quad \text{(V.5)}$$

$$\begin{aligned}&\left(\alpha\varphi^{0(1)}+\mu\psi^{0(1)}\right)\left[F_{01}\sinh(kd)-F_{02}\cosh(kd)\right]\\&-b\left(hU^{0(5)}-\alpha\varphi^{0(5)}-\mu\psi^{0(5)}\right)\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]=0\end{aligned} \quad \text{(V.6)}$$

These three equations represent equations (II.4), (II.5), and (III.6) given in the second and third chapters. So, these six homogeneous equations written above must be further transformed. For this purpose, only the first eigenvectors or only the second eigenvectors, or the first and second eigenvectors can be used.

V.1. *The first eigenvectors*

The first eigenvectors are defined by expressions (I.66) and (I.67). Also, relations (II.10) and (III.7) must be accounted for the six homogeneous equations written above. It is obvious that equations (IV.7) and (IV.8) from the previous chapter are also valid here because the mechanical boundary condition remains the same. Thus, equations (V.1) and (V.4) are written as follows:

$$F_{01}\sinh(kd)+F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)+F_{04}\cosh(bkd)\right]=0 \quad \text{(V.7)}$$

$$F_{01}\sinh(kd)-F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]=0 \quad \text{(V.8)}$$

Equations (V.2) and (V.5) corresponding to the electrical boundary condition can be rewritten in the following forms, utilizing the first eigenvectors defined by expressions (I.66) and (I.67):

$$-\alpha^2\left[F_{01}\cosh(kd)+F_{02}\sinh(kd)\right]-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\left[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)\right]=0 \quad \text{(V.9)}$$

$$-\alpha^2\left[F_{01}\cosh(kd)-F_{02}\sinh(kd)\right]-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\left[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)\right]=0 \quad \text{(V.10)}$$

where both equations were multiplied by the electromagnetic constant α.

Finally, equations (V.3) and (V.6) corresponding to the magnetic boundary condition can be significantly simplified by using relation (III.8) from Chapter III. Therefore, these two equations are rewritten in the following simplified forms:

$$(\alpha^2 - \varepsilon\mu)[F_{01}\sinh(kd) + F_{02}\cosh(kd)] - 0\times[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)] = 0 \quad \text{(V.11)}$$

$$(\alpha^2 - \varepsilon\mu)[F_{01}\sinh(kd) - F_{02}\cosh(kd)] - 0\times[F_{03}\sinh(bkd) - F_{04}\cosh(bkd)] = 0 \quad \text{(V.12)}$$

It is clearly seen that equations (V.9) and (V.10) written for the electrical boundary conditions represent equations (IV.33) and (IV.34) from the second subsection of the previous chapter written for the magnetic boundary condition, respectively. Also, equations (V.11) and (V.12) written for the magnetic boundary condition represent equations (IV.31) and (IV.32) from Chapter IV written for the electrical one, respectively. As a result, it is possible to state that this case of the electrical and magnetic boundary conditions with the first eigenvectors can give the same dispersion relations (IV.40) and (IV.42) obtained in Chapter IV for the second eigenvectors. Dispersion relations (IV.40) and (IV.42) are valid for the case of $V_{ph} < V_{tem}$ when the velocities V_{new6} and V_{new7} of the sixth and seventh new SH-waves (fundamental modes) can be calculated. For the higher-order modes with $V_{ph} > V_{tem}$, these velocities V_{new6} and V_{new7} can be calculated with formulae (IV.44) and (IV.45).

V.2. *The second eigenvectors*

For the second eigenvectors, equations (V.7) and (V.8) corresponding to the mechanical boundary condition are also valid. This is true due to relations from (II.9) to (II.11). Using second eigenvectors (I.69) and (I.70), equations (V.2) and (V.5) corresponding to the electrical boundary condition are written as follows:

$$F_{01}\cosh(kd)+F_{02}\sinh(kd)+\frac{K_{em}^2-K_m^2}{K_{em}^2}\left[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)\right]=0 \quad (V.13)$$

$$F_{01}\cosh(kd)-F_{02}\sinh(kd)+\frac{K_{em}^2-K_m^2}{K_{em}^2}\left[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)\right]=0 \quad (V.14)$$

where K_m^2 represents the coefficient of the magnetomechanical coupling (CMMC). Unlike the material characteristic of the two-phase materials such as the CMEMC K_{em}^2, the CMMC K_m^2 is a material characteristic of a single-phase material such as a pure piezomagnetics. It can be defined by the following expression [100, 101]:

$$K_m^2=\frac{h^2}{C\mu} \quad (V.15)$$

It is apparent that equations (V.3) and (V.6) are vanishing because there are two zero factors defined by expression (III.24) from Chapter III. Also, equations (V.13) and (V.14) look like equations (IV.9) and (IV.10) from the first subsection of the previous chapter. However there is the single significant difference such that equations (V.13) and (V.14) use the CMMC K_m^2, but not the CEMC K_e^2 used in equations (IV.9) and (IV.10). So, it is possible to use K_m^2 instead of K_e^2 in dispersion relations (IV.16) and (IV.18) obtained in the previous chapter. Therefore, it is possible to determine the velocities V_{new10} and V_{new11} of the tenth and eleventh new SH-waves propagating in the transversely isotropic piezoelectromagnetic plate. For the fundamental modes with $V_{ph} < V_{tem}$, these corresponding new dispersion relations respectively read:

$$\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\tanh(kd)-\sqrt{1-(V_{new10}/V_{tem})^2}\tanh\left(kd\sqrt{1-(V_{new10}/V_{tem})^2}\right)=0 \quad (V.16)$$

$$\sqrt{1-(V_{new11}/V_{tem})^2}\tanh(kd)-\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\tanh\left(kd\sqrt{1-(V_{new11}/V_{tem})^2}\right)=0 \quad (V.17)$$

It is transparent that for $kd \to \infty$, both the velocities V_{new10} and V_{new11} of the dispersive SH-waves propagating in the plate will approach some nondispersive SH-SAW velocity recently discovered by Melkumyan [107]. This SH-SAW velocity is called the piezoelectric exchange surface Melkumyan wave or PEESM wave [105] and is defined by the following expression:

$$V_{PEESM} = V_{tem}\left[1 - \left(\frac{K_{em}^2 - K_m^2}{1 + K_{em}^2}\right)^2\right]^{1/2} \tag{V.18}$$

For the case of $V_{ph} > V_{tem}$, dispersion relations (V.16) and (V.17) are transformed into the following forms:

$$\frac{K_{em}^2 - K_m^2}{1 + K_{em}^2}\tanh(kd) - \tanh\left(kd\sqrt{(V_{new10}/V_{tem})^2 - 1}\right)\sqrt{(V_{new10}/V_{tem})^2 - 1} = 0 \tag{V.19}$$

$$\tanh(kd)\sqrt{(V_{new11}/V_{tem})^2 - 1} - \frac{K_{em}^2 - K_m^2}{1 + K_{em}^2}\tanh\left(kd\sqrt{(V_{new11}/V_{tem})^2 - 1}\right) = 0 \tag{V.20}$$

It is worth noticing that dispersion relations (V.19) and (V.20) can reveal an infinite number of the higher-order modes of the new dispersive SH-waves propagating in the transversely isotropic piezoelectromagnetic plate.

V.3. *The first and second eigenvectors*

In this case, the first eigenvectors must be used for the upper surface and the second ones for the lower surface. However, it is also possible to use the reverse case when second eigenvectors are utilized for the upper surface and the first ones for the lower surface. The resulting two dispersion relations must be the same for both the cases. This is similar to the cases treated in the third subsection of the previous chapter. It is also possible even to use two dispersion relations (IV.57) and (IV.60) obtained in Chapter IV and to change them by substituting K_m^2 instead of K_e^2. It is flagrant that the resulting two new dispersion relations will represent the solutions for the case studied in this subsection. So, equation (IV.57) is transformed into the following dispersion relation for the determination of the velocity V_{new12} of the twelfth new SH-wave:

$$\begin{aligned}&\left(\sqrt{1-\left(V_{new12}/V_{tem}\right)^2}-\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\tanh(2kd)\tanh\left(kd\sqrt{1-\left(V_{new12}/V_{tem}\right)^2}\right)\right)\\&\times\left(\frac{\alpha^2}{\varepsilon\mu}\frac{1}{\cosh(2kd)}\frac{K_{em}^2-K_m^2}{1+K_{em}^2}+\sqrt{1-\left(V_{new12}/V_{tem}\right)^2}\tanh\left(kd\sqrt{1-\left(V_{new12}/V_{tem}\right)^2}\right)-\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)\\&-\left(\frac{\alpha^2}{\varepsilon\mu}\left(\frac{1}{\cosh(2kd)}\frac{K_{em}^2-K_m^2}{1+K_{em}^2}+\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)\tanh\left(kd\sqrt{1-\left(V_{new12}/V_{tem}\right)^2}\right)-\sqrt{1-\left(V_{new12}/V_{tem}\right)^2}\right)\\&\times\left(\sqrt{1-\left(V_{new12}/V_{tem}\right)^2}\tanh\left(kd\sqrt{1-\left(V_{new12}/V_{tem}\right)^2}\right)-\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\tanh(2kd)\right)=0\end{aligned}\tag{V.21}$$

Analogically, equation (IV.60) is transformed into the following dispersion relation for the determination of the velocity V_{new13} of the thirteenth new SH-wave:

$$\begin{aligned}&\left(1-\left(V_{new13}/V_{tem}\right)^2+\left(\frac{\alpha^2}{\varepsilon\mu}\right)^2\frac{\left(K_{em}^2-K_m^2\right)\left(K_{em}^2-K_\alpha^2\right)}{\left(1+K_{em}^2\right)^2}\right)\tanh\left(2kd\sqrt{1-\left(V_{new13}/V_{tem}\right)^2}\right)\\&-\sqrt{1-\left(V_{new13}/V_{tem}\right)^2}\,\frac{\alpha^2}{\varepsilon\mu}\frac{2K_{em}^2-K_m^2-K_\alpha^2}{1+K_{em}^2}=0\end{aligned}\tag{V.22}$$

Obtained dispersion relations (V.21) and (V.22) are fitting for the case of the fundamental modes when $V_{ph} < V_{tem}$. For this case, dispersion relation (V.22) reads:

$$\begin{aligned}&\left(1-\left(V_{new13}/V_{tem}\right)^2+\left(\frac{\alpha^2}{\varepsilon\mu}\right)^2\frac{\left(K_{em}^2-K_m^2\right)\left(K_{em}^2-K_\alpha^2\right)}{\left(1+K_{em}^2\right)^2}\right)\tan\left(2kd\sqrt{\left(V_{new13}/V_{tem}\right)^2-1}\right)\\&-\frac{\alpha^2}{\varepsilon\mu}\frac{2K_{em}^2-K_m^2-K_\alpha^2}{1+K_{em}^2}\sqrt{\left(V_{new13}/V_{tem}\right)^2-1}=0\end{aligned}\tag{V.23}$$

Chapter VI

Continuity of φ, D_3, ψ, and B_3

This chapter and four following chapters study the SH-wave propagation in the piezoelectromagnetic plate when the applied electrical and magnetic boundary conditions allow the evaluation of the influence of the material parameters of the free space (vacuum) such as the dielectric permittivity constant ε_0 and the magnetic permeability constant μ_0. The values of these vacuum material parameters are given in the tenth subsection of Chapter I, in which it was also mentioned that the value of the vacuum elastic constant is too negligible to account in the theoretical considerations. It is worth also noticing that the previous four chapters have provided the dispersion relations for several new SH-waves when the used electrical and magnetic boundary conditions allow one to avoid the consideration of the material properties of a vacuum.

So, the mechanical boundary conditions for this case are given by equations (V.1) and (V.4) from the previous chapter for the upper and lower surfaces of the plate, respectively. These two equations are already designed to exploit the weight factors F_{01}, F_{02}, F_{03}, and F_{04}. Also, the electrical and magnetic boundary conditions can be transformed in order to have the weight factors F_{01}, F_{02}, F_{03}, and F_{04}. First of all, it is necessary to write the electrical and magnetic boundary conditions with the weight factors $F^{(1)}$, $F^{(2)}$, $F^{(3)}$, $F^{(4)}$, $F^{(5)}$, and $F^{(6)}$ used in the eleventh subsection of Chapter I. In this subsection of the first chapter, the electrical boundary conditions such as the continuity of both the electrical potential φ and the electrical induction D_3, namely $\varphi = \varphi^f$ and $D_3 = D^f$ are defined by expressions from (I.91) to (I.96) for the upper surface, where the superscript "*f*" relates to the free space (vacuum). It is natural to exclude the weight factor F_E for a vacuum in expressions (I.92) and (I.95).

As a result, the following single equation corresponding to the electrical boundary conditions for the upper surface can be obtained:

$$\begin{aligned}&\left\{F^{(1)}k_3^{(1)}\exp\left(\mathrm{j}k_3^{(1)}d\right)+F^{(2)}k_3^{(2)}\exp\left(\mathrm{j}k_3^{(2)}d\right)+F^{(3)}k_3^{(3)}\exp\left(\mathrm{j}k_3^{(3)}d\right)+F^{(4)}k_3^{(4)}\exp\left(\mathrm{j}k_3^{(4)}d\right)\right\}\\&\times\left(\varepsilon\varphi^{0(1)}+\alpha\psi^{0(1)}\right)-\left\{F^{(5)}k_3^{(5)}\exp\left(\mathrm{j}k_3^{(5)}d\right)+F^{(6)}k_3^{(6)}\exp\left(\mathrm{j}k_3^{(6)}d\right)\right\}\left(eU^{0(5)}-\varepsilon\varphi^{0(5)}-\alpha\psi^{0(5)}\right)\\&-\mathrm{j}k\varepsilon_0\left\{F^{(1)}\varphi^{0(1)}\exp\left(\mathrm{j}k_3^{(1)}d\right)+F^{(2)}\varphi^{0(1)}\exp\left(\mathrm{j}k_3^{(2)}d\right)+F^{(3)}\varphi^{0(1)}\exp\left(\mathrm{j}k_3^{(3)}d\right)\right.\\&\left.+F^{(4)}\varphi^{0(1)}\exp\left(\mathrm{j}k_3^{(4)}d\right)+F^{(5)}\varphi^{0(5)}\exp\left(\mathrm{j}k_3^{(5)}d\right)+F^{(6)}\varphi^{0(5)}\exp\left(\mathrm{j}k_3^{(6)}d\right)\right\}=0\end{aligned}\qquad\text{(VI.1)}$$

Also, the magnetic boundary conditions such as the continuity of both the magnetic potential ψ and the electrical induction B_3, namely $\psi = \psi^f$ and $B_3 = B^f$ are defined by expressions from (I.97) to (I.102) for the upper surface. It is natural to exclude the weight factor F_M for a vacuum in expressions (I.98) and (I.101) and to write down the following single equation corresponding to the magnetic boundary conditions:

$$\begin{aligned}&\left\{F^{(1)}k_3^{(1)}\exp\left(\mathrm{j}k_3^{(1)}d\right)+F^{(2)}k_3^{(2)}\exp\left(\mathrm{j}k_3^{(2)}d\right)+F^{(3)}k_3^{(3)}\exp\left(\mathrm{j}k_3^{(3)}d\right)+F^{(4)}k_3^{(4)}\exp\left(\mathrm{j}k_3^{(4)}d\right)\right\}\\&\times\left(\alpha\varphi^{0(1)}+\mu\psi^{0(1)}\right)-\left\{F^{(5)}k_3^{(5)}\exp\left(\mathrm{j}k_3^{(5)}d\right)+F^{(6)}k_3^{(6)}\exp\left(\mathrm{j}k_3^{(6)}d\right)\right\}\left(hU^{0(5)}-\alpha\varphi^{0(5)}-\mu\psi^{0(5)}\right)\\&-\mathrm{j}k\mu_0\left\{F^{(1)}\psi^{0(1)}\exp\left(\mathrm{j}k_3^{(1)}d\right)+F^{(2)}\psi^{0(1)}\exp\left(\mathrm{j}k_3^{(2)}d\right)+F^{(3)}\psi^{0(1)}\exp\left(\mathrm{j}k_3^{(3)}d\right)\right.\\&\left.+F^{(4)}\psi^{0(1)}\exp\left(\mathrm{j}k_3^{(4)}d\right)+F^{(5)}\psi^{0(5)}\exp\left(\mathrm{j}k_3^{(5)}d\right)+F^{(6)}\psi^{0(5)}\exp\left(\mathrm{j}k_3^{(6)}d\right)\right\}=0\end{aligned}\qquad\text{(VI.2)}$$

For the case of the homogeneous boundary conditions, the electrical and magnetic boundary conditions for the lower surface are defined by expressions from (I.105) to (I.110) and from (I.111) to (I.116), respectively. For the case of $x_3 = -d$, they respectively read:

$$\begin{aligned}&\left\{F^{(1)}k_3^{(1)}\exp\left(-\mathrm{j}k_3^{(1)}d\right)+F^{(2)}k_3^{(2)}\exp\left(-\mathrm{j}k_3^{(2)}d\right)+F^{(3)}k_3^{(3)}\exp\left(-\mathrm{j}k_3^{(3)}d\right)+F^{(4)}k_3^{(4)}\exp\left(-\mathrm{j}k_3^{(4)}d\right)\right\}\\&\times\left(\varepsilon\varphi^{0(1)}+\alpha\psi^{0(1)}\right)-\left\{F^{(5)}k_3^{(5)}\exp\left(-\mathrm{j}k_3^{(5)}d\right)+F^{(6)}k_3^{(6)}\exp\left(-\mathrm{j}k_3^{(6)}d\right)\right\}\left(eU^{0(5)}-\varepsilon\varphi^{0(5)}-\alpha\psi^{0(5)}\right)\\&+\mathrm{j}k\varepsilon_0\left\{F^{(1)}\varphi^{0(1)}\exp\left(-\mathrm{j}k_3^{(1)}d\right)+F^{(2)}\varphi^{0(1)}\exp\left(-\mathrm{j}k_3^{(2)}d\right)+F^{(3)}\varphi^{0(1)}\exp\left(-\mathrm{j}k_3^{(3)}d\right)\right.\\&\left.+F^{(4)}\varphi^{0(1)}\exp\left(-\mathrm{j}k_3^{(4)}d\right)+F^{(5)}\varphi^{0(5)}\exp\left(-\mathrm{j}k_3^{(5)}d\right)+F^{(6)}\varphi^{0(5)}\exp\left(-\mathrm{j}k_3^{(6)}d\right)\right\}=0\end{aligned}\qquad\text{(VI.3)}$$

and

$$
\begin{aligned}
&\left\{F^{(1)}k_3^{(1)}\exp\left(-\mathrm{j}k_3^{(1)}d\right)+F^{(2)}k_3^{(2)}\exp\left(-\mathrm{j}k_3^{(2)}d\right)+F^{(3)}k_3^{(3)}\exp\left(-\mathrm{j}k_3^{(3)}d\right)+F^{(4)}k_3^{(4)}\exp\left(-\mathrm{j}k_3^{(4)}d\right)\right\}\\
&\times\left(\alpha\varphi^{0(1)}+\mu\psi^{0(1)}\right)-\left\{F^{(5)}k_3^{(5)}\exp\left(-\mathrm{j}k_3^{(5)}d\right)+F^{(6)}k_3^{(6)}\exp\left(-\mathrm{j}k_3^{(6)}d\right)\right\}\left(hU^{0(5)}-\alpha\varphi^{0(5)}-\mu\psi^{0(5)}\right)\\
&+\mathrm{j}k\mu_0\left\{F^{(1)}\psi^{0(1)}\exp\left(-\mathrm{j}k_3^{(1)}d\right)+F^{(2)}\psi^{0(1)}\exp\left(-\mathrm{j}k_3^{(2)}d\right)+F^{(3)}\psi^{0(1)}\exp\left(-\mathrm{j}k_3^{(3)}d\right)\right.\\
&\left.+F^{(4)}\psi^{0(1)}\exp\left(-\mathrm{j}k_3^{(4)}d\right)+F^{(5)}\psi^{0(5)}\exp\left(-\mathrm{j}k_3^{(5)}d\right)+F^{(6)}\psi^{0(5)}\exp\left(-\mathrm{j}k_3^{(6)}d\right)\right\}=0
\end{aligned}
\tag{VI.4}
$$

These four homogeneous equations written above with the weight factors $F^{(1)}$, $F^{(2)}$, $F^{(3)}$, $F^{(4)}$, $F^{(5)}$, and $F^{(6)}$ must be transformed into four equations with the weight factors F_{01}, F_{02}, F_{03}, and F_{04}. As a result, three homogeneous equations corresponding to the mechanical, electrical, and magnetic boundary conditions for the upper surface at $x_3 = +\,d$ read:

$$
\begin{aligned}
&\left(e\varphi^{0(1)}+h\psi^{0(1)}\right)\left[F_{01}\sinh(kd)+F_{02}\cosh(kd)\right]\\
&+b\left(CU^{0(5)}+e\varphi^{0(5)}+h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd)+F_{04}\cosh(bkd)\right]=0
\end{aligned}
\tag{VI.5}
$$

$$
\begin{aligned}
&\left(\varepsilon\varphi^{0(1)}+\alpha\psi^{0(1)}\right)\left\{F_{01}\sinh(kd)+F_{02}\cosh(kd)\right\}\\
&-b\left(eU^{0(5)}-\varepsilon\varphi^{0(5)}-\alpha\psi^{0(5)}\right)\left\{F_{03}\sinh(bkd)+F_{04}\cosh(bkd)\right\}\\
&+\varepsilon_0\left\{F_{01}\varphi^{0(1)}\cosh(kd)+F_{02}\varphi^{0(1)}\sinh(kd)+F_{03}\varphi^{0(5)}\cosh(bkd)+F_{04}\varphi^{0(5)}\sinh(bkd)\right\}=0
\end{aligned}
\tag{VI.6}
$$

$$
\begin{aligned}
&\left(\alpha\varphi^{0(1)}+\mu\psi^{0(1)}\right)\left\{F_{01}\sinh(kd)+F_{02}\cosh(kd)\right\}\\
&-b\left(hU^{0(5)}-\alpha\varphi^{0(5)}-\mu\psi^{0(5)}\right)\left\{F_{3}\sinh(bkd)+F_{04}\cosh(bkd)\right\}\\
&+\mu_0\left\{F_{01}\psi^{0(1)}\cosh(kd)+F_{02}\psi^{0(1)}\sinh(kd)+F_{03}\psi^{0(5)}\cosh(bkd)+F_{04}\psi^{0(5)}\sinh(bkd)\right\}=0
\end{aligned}
\tag{VI.7}
$$

For the lower free surface at $x_3 = -\,d$, three homogeneous equations corresponding to the mechanical, electrical, and magnetic boundary conditions can be written in the following forms:

$$
\begin{aligned}
&\left(e\varphi^{0(1)}+h\psi^{0(1)}\right)\left[F_{01}\sinh(kd)-F_{02}\cosh(kd)\right]\\
&+b\left(CU^{0(5)}+e\varphi^{0(5)}+h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]=0
\end{aligned}
\tag{VI.8}
$$

$$
\begin{aligned}
&\left(\varepsilon\varphi^{0(1)}+\alpha\psi^{0(1)}\right)\left\{F_{01}\sinh(kd)-F_{02}\cosh(kd)\right\}\\
&-b\left(eU^{0(5)}-\varepsilon\varphi^{0(5)}-\alpha\psi^{0(5)}\right)\left\{F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right\}\\
&+\varepsilon_0\left\{F_{01}\varphi^{0(1)}\cosh(kd)-F_{02}\varphi^{0(1)}\sinh(kd)+F_{03}\varphi^{0(5)}\cosh(bkd)-F_{04}\varphi^{0(5)}\sinh(bkd)\right\}=0
\end{aligned}
\tag{VI.9}
$$

$$
\begin{aligned}
&\left(\alpha\varphi^{0(1)}+\mu\psi^{0(1)}\right)\{F_{01}\sinh(kd)-F_{02}\cosh(kd)\}\\
&-b\left(hU^{0(5)}-\alpha\varphi^{0(5)}-\mu\psi^{0(5)}\right)\{F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\}\\
&+\mu_0\left\{F_{01}\psi^{0(1)}\cosh(kd)-F_{02}\psi^{0(1)}\sinh(kd)+F_{03}\psi^{0(5)}\cosh(bkd)-F_{04}\psi^{0(5)}\sinh(bkd)\right\}=0
\end{aligned}
\tag{VI.10}
$$

This system of six transformed homogeneous equations from (VI.5) to (VI.10) must be resolved to get dispersion relations for several different cases when only the first eigenvectors, only the second eigenvectors, or the first and second ones are exploited.

VI.1. *The first eigenvectors*

Employing the first eigenvectors defined by expressions (I.66) and (I.67) in the first chapter for the upper surface of the plate, it is obvious that equation (VI.5) can be transformed into equation (V.7). Also, equations (VI.6) and (VI.7) written for the upper surface can be significantly simplified by using expressions (III.7) and (III.8) from the third chapter. Accordingly, transformed equations (VI.5), (VI.6), and (VI.7) read as follows:

$$
\varepsilon(\mu+\mu_0)\left\{F_{01}\sinh(kd)+F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)+F_{04}\cosh(bkd)\right]\right\}=0
\tag{VI.11}
$$

$$
\begin{aligned}
&-\alpha^2\left[F_{01}\sinh(kd)+F_{02}\cosh(kd)\right]\\
&-\alpha^2\left[F_{01}(\cosh(kd)-\sinh(kd))+F_{02}(\sinh(kd)-\cosh(kd))\right]\\
&-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\left[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)\right]=0
\end{aligned}
\tag{VI.12}
$$

$$
\begin{aligned}
&\left(\varepsilon(\mu+\mu_0)-\alpha^2\right)\{F_{01}\sinh(kd)+F_{02}\cosh(kd)\}\\
&+\varepsilon\mu_0\{F_{01}(\cosh(kd)-\sinh(kd))+F_{02}(\sinh(kd)-\cosh(kd))\}\\
&+\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{K_{em}^2}\left[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)\right]=0
\end{aligned}
\tag{VI.13}
$$

For the lower free surface at $x_3 = -d$, the corresponding three homogeneous equations can be written in the following simplified forms:

$$\varepsilon(\mu+\mu_0)\left\{F_{01}\sinh(kd)-F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]\right\}=0 \quad \text{(VI.14)}$$

$$\begin{aligned}&-\alpha^2\left[F_{01}\sinh(kd)-F_{02}\cosh(kd)\right]\\&-\alpha^2\left[F_{01}(\cosh(kd)-\sinh(kd))-F_{02}(\sinh(kd)-\cosh(kd))\right]\\&-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\left[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)\right]=0\end{aligned} \quad \text{(VI.15)}$$

$$\begin{aligned}&\left(\varepsilon(\mu+\mu_0)-\alpha^2\right)\left\{F_{01}\sinh(kd)-F_{02}\cosh(kd)\right\}\\&+\varepsilon\mu_0\left\{F_{01}(\cosh(kd)-\sinh(kd))-F_{02}(\sinh(kd)-\cosh(kd))\right\}\\&+\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{K_{em}^2}\left[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)\right]=0\end{aligned} \quad \text{(VI.16)}$$

It is clearly seen that there are three pairs of equations such as equations (VI.11) and (VI.14), equations (VI.12) and (VI.15), and equations (VI.13) and (VI.16). It is worth noticing that all of these six equations have the same dimension such as s^2/m^2. Following the transformations used in the previous chapters, it is possible to obtain two independent sets, of which each consists of three equations. The first set of three equations is coupled with the weight factors F_{01} and F_{03}. These three homogeneous equations read:

$$\varepsilon(\mu+\mu_0)F_{01}\sinh(kd)+b\varepsilon(\mu+\mu_0)\frac{1+K_{em}^2}{K_{em}^2}F_{03}\sinh(bkd)=0 \quad \text{(VI.17)}$$

$$-\alpha^2F_{01}\sinh(kd)-\alpha^2F_{01}(\cosh(kd)-\sinh(kd))-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}F_{03}\cosh(bkd)=0 \quad \text{(VI.18)}$$

$$\begin{aligned}&\left(\varepsilon(\mu+\mu_0)-\alpha^2\right)F_{01}\sinh(kd)+\varepsilon\mu_0F_{01}(\cosh(kd)-\sinh(kd))\\&+\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{K_{em}^2}F_{03}\cosh(bkd)=0\end{aligned} \quad \text{(VI.19)}$$

It is apparent that each of equations (VI.18) and (VI.19) can be represented as the following equation:

$$F_{01}\left\{\varepsilon(\mu+\mu_0)\sinh(kd)+\left(\varepsilon\mu_0+\alpha^2\right)\left(\cosh(kd)-\sinh(kd)\right)\right\} + F_{03}\left(\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{K_{em}^2}+\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\right)\cosh(bkd)=0 \quad \text{(VI.20)}$$

Equation (VI.20) represent a subtraction of equation (VI.19) from (VI.18) or vice versa. Thus, equations (VI.17) and (VI.20) lead to the following dispersion relation for the determination of the velocity V_{new14} of the fourteenth new SH-wave in the piezoelectromagnetic plate when $V_{ph} < V_{tem}$:

$$\left\{\frac{\varepsilon\mu_0+\alpha^2}{\varepsilon(\mu+\mu_0)}\left(\tanh(kd)-1\right)-\tanh(kd)\right\}\tanh\left(kd\sqrt{1-\left(V_{new14}/V_{tem}\right)^2}\right)\sqrt{1-\left(V_{new14}/V_{tem}\right)^2} + \frac{K_{em}^2-K_e^2+\alpha^2C_L^2(\varepsilon_0/\varepsilon)\left(K_{em}^2-K_\alpha^2\right)}{\left(1+K_{em}^2\right)\left(1+\mu/\mu_0\right)}\tanh(kd)=0 \quad \text{(VI.21)}$$

where C_L is the speed of light in a vacuum.

The second set of three homogeneous linear equations coupled with the weight factors F_{02} and F_{04} can be formed by subtractions of equation (VI.11) from (VI.14), equation (VI.12) from (VI.15), and equation (VI.13) from (VI.16). So, the resulting equations read:

$$\varepsilon(\mu+\mu_0)F_{02}\cosh(kd)+b\varepsilon(\mu+\mu_0)\frac{1+K_{em}^2}{K_{em}^2}F_{04}\cosh(bkd)=0 \quad \text{(VI.22)}$$

$$-\alpha^2F_{02}\cosh(kd)-\alpha^2F_{02}\left(\sinh(kd)-\cosh(kd)\right) -\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}F_{04}\sinh(bkd)=0 \quad \text{(VI.23)}$$

$$\left(\varepsilon(\mu+\mu_0)-\alpha^2\right)F_{02}\cosh(kd)+\varepsilon\mu_0F_{02}\left(\sinh(kd)-\cosh(kd)\right) +\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{K_{em}^2}F_{04}\sinh(bkd)=0 \quad \text{(VI.24)}$$

Similarly, each of equations (VI.23) and (VI.24) can be represented as the following equation:

$$
\begin{aligned}
&F_{02}\left\{\varepsilon(\mu+\mu_0)\cosh(kd)+\left(\varepsilon\mu_0+\alpha^2\right)\left(\sinh(kd)-\cosh(kd)\right)\right\}\\
&+F_{04}\left(\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{K_{em}^2}+\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\right)\sinh(bkd)=0
\end{aligned}
\qquad \text{(VI.25)}
$$

As a result, equations (VI.22) and (VI.25) solidly lead to the following dispersion relation for the determination of the velocity V_{new15} of the fifteenth new SH-wave propagating in the piezoelectromagnetic plate when $V_{ph} < V_{tem}$:

$$
\begin{aligned}
&\frac{K_{em}^2-K_e^2+\alpha^2C_L^2(\varepsilon_0/\varepsilon)\left(K_{em}^2-K_\alpha^2\right)}{\left(1+K_{em}^2\right)\left(1+\mu/\mu_0\right)}\tanh\left(kd\sqrt{1-\left(V_{new15}/V_{tem}\right)^2}\right)-\sqrt{1-\left(V_{new15}/V_{tem}\right)^2}\\
&+\frac{\varepsilon\mu_0+\alpha^2}{\varepsilon(\mu+\mu_0)}\left(1-\tanh(kd)\right)\sqrt{1-\left(V_{new15}/V_{tem}\right)^2}=0
\end{aligned}
\qquad \text{(VI.26)}
$$

It is clearly seen in dispersion relations (VI.21) and (VI.26) that for $kd \to \infty$, the velocities V_{new14} and V_{new15} of the fundamental modes of the new SH-waves approach the corresponding SH-SAW recently discovered in book [101] (see formula (108) in the book) and also investigated in paper [106]. It is also possible to give dispersion relations (VI.21) and (VI.26) for the case of $V_{ph} > V_{tem}$. They respectively read:

$$
\begin{aligned}
&\left\{\tanh(kd)-\frac{\varepsilon\mu_0+\alpha^2}{\varepsilon(\mu+\mu_0)}\left(\tanh(kd)-1\right)\right\}\tan\left(kd\sqrt{\left(V_{new14}/V_{tem}\right)^2-1}\right)\sqrt{\left(V_{new14}/V_{tem}\right)^2-1}\\
&+\frac{K_{em}^2-K_e^2+\alpha^2C_L^2(\varepsilon_0/\varepsilon)\left(K_{em}^2-K_\alpha^2\right)}{\left(1+K_{em}^2\right)\left(1+\mu/\mu_0\right)}\tanh(kd)=0
\end{aligned}
\qquad \text{(VI.27)}
$$

$$
\begin{aligned}
&\frac{K_{em}^2-K_e^2+\alpha^2C_L^2(\varepsilon_0/\varepsilon)\left(K_{em}^2-K_\alpha^2\right)}{\left(1+K_{em}^2\right)\left(1+\mu/\mu_0\right)}\tan\left(kd\sqrt{\left(V_{new15}/V_{tem}\right)^2-1}\right)-\sqrt{\left(V_{new15}/V_{tem}\right)^2-1}\\
&+\frac{\varepsilon\mu_0+\alpha^2}{\varepsilon(\mu+\mu_0)}\left(1-\tanh(kd)\right)\sqrt{\left(V_{new15}/V_{tem}\right)^2-1}=0
\end{aligned}
\qquad \text{(VI.28)}
$$

VI.2. *The second eigenvectors*

The second eigenvectors defined by expressions (I.69) and (I.70) lead to equalities (III.23) and (III.24). Using them for the upper surface of the plate when $x_3 = + d$, equations from (VI.5) to (VI.7) are transformed into the following forms:

$$(\varepsilon+\varepsilon_0)\mu\left\{F_{01}\sinh(kd)+F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)+F_{04}\cosh(bkd)\right]\right\}=0 \quad \text{(VI.29)}$$

$$\begin{aligned}&\left((\varepsilon+\varepsilon_0)\mu-\alpha^2\right)\{F_{01}\sinh(kd)+F_{02}\cosh(kd)\}\\&+\varepsilon_0\mu\left(F_{01}(\cosh(kd)-\sinh(kd))+F_{02}(\sinh(kd)-\cosh(kd))\right)\\&+\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{K_{em}^2}\left[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)\right]=0\end{aligned} \quad \text{(VI.30)}$$

$$\begin{aligned}&-\alpha^2\left[F_{01}\sinh(kd)+F_{02}\cosh(kd)\right]\\&-\alpha^2\left[F_{01}(\cosh(kd)-\sinh(kd))+F_{02}(\sinh(kd)-\cosh(kd))\right]\\&-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\left[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)\right]=0\end{aligned} \quad \text{(VI.31)}$$

For the lower surface at $x_3 = - d$, equations from (VI.8) to (VI.10) can be transformed into the following forms:

$$(\varepsilon+\varepsilon_0)\mu\left\{F_{01}\sinh(kd)-F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]\right\}=0 \quad \text{(VI.32)}$$

$$\begin{aligned}&\left((\varepsilon+\varepsilon_0)\mu-\alpha^2\right)\{F_{01}\sinh(kd)-F_{02}\cosh(kd)\}\\&+\varepsilon_0\mu\left(F_{01}(\cosh(kd)-\sinh(kd))-F_{02}(\sinh(kd)-\cosh(kd))\right)\\&+\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{K_{em}^2}\left[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)\right]=0\end{aligned} \quad \text{(VI.33)}$$

$$\begin{aligned}&-\alpha^2\left[F_{01}\sinh(kd)-F_{02}\cosh(kd)\right]\\&-\alpha^2\left[F_{01}(\cosh(kd)-\sinh(kd))-F_{02}(\sinh(kd)-\cosh(kd))\right]\\&-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\left[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)\right]=0\end{aligned} \quad \text{(VI.34)}$$

These six homogeneous equations from (VI.29) to (VI.34) look like equations from (VI.11) to (VI.16) written in the previous subsection. Therefore, these six equations can be transformed by the same way. Indeed, these six equations can be represented as two independent sets, of which each consists of three equations. The first set is coupled with the weight factors F_{01} and F_{03} and the second set is coupled with the weight factors F_{02} and F_{04}. Then, each set consisting of three equations can be represented as a set of two equations. Consequently, the resulting four equations with the same dimension of s^2/m^2 read:

$$(\varepsilon+\varepsilon_0)\mu F_{01}\sinh(kd)+b(\varepsilon+\varepsilon_0)\mu\frac{1+K_{em}^2}{K_{em}^2}F_{03}\sinh(bkd)=0 \quad \text{(VI.35)}$$

$$\begin{aligned}&F_{01}\left\{(\varepsilon+\varepsilon_0)\mu\sinh(kd)+\left(\varepsilon_0\mu+\alpha^2\right)(\cosh(kd)-\sinh(kd))\right\}\\&+F_{03}\left(\varepsilon_0\mu\frac{K_{em}^2-K_e^2}{K_{em}^2}+\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\right)\cosh(bkd)=0\end{aligned} \quad \text{(VI.36)}$$

$$(\varepsilon+\varepsilon_0)\mu F_{02}\cosh(kd)+b(\varepsilon+\varepsilon_0)\mu\frac{1+K_{em}^2}{K_{em}^2}F_{04}\cosh(bkd)=0 \quad \text{(VI.37)}$$

$$\begin{aligned}&F_{02}\left\{(\varepsilon+\varepsilon_0)\mu\cosh(kd)+\left(\varepsilon_0\mu+\alpha^2\right)(\sinh(kd)-\cosh(kd))\right\}\\&+F_{04}\left(\varepsilon_0\mu\frac{K_{em}^2-K_e^2}{K_{em}^2}+\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\right)\sinh(bkd)=0\end{aligned} \quad \text{(VI.38)}$$

Similar to the results obtained in the previous subsection, two dispersion relations can be obtained: one is for the case of $F_{02}=F_{04}=0$ and the second is for $F_{01}=F_{03}=0$. Therefore, the velocities V_{new16} and V_{new17} of the sixteenth and seventeenth new SH-waves propagating in the plate can be calculated with the following dispersion relations written for $V_{ph}<V_{tem}$:

$$\begin{aligned}&\left\{\frac{\varepsilon_0\mu+\alpha^2}{(\varepsilon+\varepsilon_0)\mu}(\tanh(kd)-1)-\tanh(kd)\right\}\tanh\left(kd\sqrt{1-(V_{new16}/V_{tem})^2}\right)\sqrt{1-(V_{new16}/V_{tem})^2}\\&+\frac{K_{em}^2-K_m^2+\alpha^2C_L^2(\mu_0/\mu)(K_{em}^2-K_\alpha^2)}{(1+K_{em}^2)(1+\varepsilon/\varepsilon_0)}\tanh(kd)=0\end{aligned} \quad \text{(VI.39)}$$

$$\frac{K_{em}^2 - K_m^2 + \alpha^2 C_L^2(\mu_0/\mu)(K_{em}^2 - K_\alpha^2)}{(1+K_{em}^2)(1+\varepsilon/\varepsilon_0)}\tanh\left(kd\sqrt{1-(V_{new17}/V_{tem})^2}\right) - \sqrt{1-(V_{new17}/V_{tem})^2}$$
$$+\frac{\varepsilon_0\mu+\alpha^2}{(\varepsilon+\varepsilon_0)\mu}(1-\tanh(kd))\sqrt{1-(V_{new17}/V_{tem})^2} = 0 \quad \text{(VI.40)}$$

It is clearly seen in dispersion relations (VI.39) and (VI.40) that for $kd \to \infty$, the velocities V_{new16} and V_{new17} of the fundamental modes of the new SH-waves approach the corresponding SH-SAW recently discovered in book [101], see formula (120) in the book. This type of the new SH-SAWs was also investigated in paper [106]. For $V_{ph} > V_{tem}$, these dispersion relations read:

$$\left\{\tanh(kd) - \frac{\varepsilon_0\mu+\alpha^2}{(\varepsilon+\varepsilon_0)\mu}(\tanh(kd)-1)\right\}\tan\left(kd\sqrt{(V_{new16}/V_{tem})^2-1}\right)\sqrt{(V_{new16}/V_{tem})^2-1}$$
$$+\frac{K_{em}^2 - K_m^2 + \alpha^2 C_L^2(\mu_0/\mu)(K_{em}^2 - K_\alpha^2)}{(1+K_{em}^2)(1+\varepsilon/\varepsilon_0)}\tanh(kd) = 0 \quad \text{(VI.41)}$$

$$\frac{K_{em}^2 - K_m^2 + \alpha^2 C_L^2(\mu_0/\mu)(K_{em}^2 - K_\alpha^2)}{(1+K_{em}^2)(1+\varepsilon/\varepsilon_0)}\tan\left(kd\sqrt{(V_{new17}/V_{tem})^2-1}\right) - \sqrt{(V_{new17}/V_{tem})^2-1}$$
$$+\frac{\varepsilon_0\mu+\alpha^2}{(\varepsilon+\varepsilon_0)\mu}(1-\tanh(kd))\sqrt{(V_{new17}/V_{tem})^2-1} = 0 \quad \text{(VI.42)}$$

VI.3. *The first and second eigenvectors*

This subsection investigates the case when the first eigenvectors are used for the upper surface and the second eigenvectors are exploited for the lower surface. Therefore, equations (VI.11), (VI.12), and (VI.13) from the first subsection and equations (VI.32), (VI.33), and (VI.34) from the second subsection must be utilized. However, one can also treat the reverse situation and it is expected that the resulting dispersion relation will be the same. In this reverse case, equations (VI.14), (VI.15), (VI.16), (VI.29), (VI.30), and (VI.31) must be employed. So, the first pair of equations (VI.11) and (VI.32) can be transformed into the following two equations:

$$\varepsilon\mu\left\{F_{01}\sinh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}F_{03}\sinh(bkd)\right\}=0 \quad \text{(VI.43)}$$

$$\varepsilon\mu\left\{F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}F_{04}\cosh(bkd)\right\}=0 \quad \text{(VI.44)}$$

Besides, the second pair of equations (VI.12) and (VI.34) can be also transformed into the following two independent equations:

$$-\alpha^2 F_{01}\cosh(kd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}F_{03}\cosh(bkd)=0 \quad \text{(VI.45)}$$

$$-\alpha^2 F_{02}\sinh(kd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}F_{04}\sinh(bkd)=0 \quad \text{(VI.46)}$$

On the other hand, the last pair of equations (VI.13) and (VI.33) cannot be transformed into two independent equations. Therefore, each of equations (VI.43) and (VI.45) coupled with the weight factors F_{01} and F_{03} can be led to the same equation. Analogically, each of equations (VI.44) and (VI.46) coupled with the weight factors F_{02} and F_{04} can be also led to the same equation. Thus, it is possible finally cope with four equations and four unknown weight factors F_{01}, F_{02}, F_{03}, and F_{04}. This new set of the four homogeneous equations reads:

$$F_{01}\{\varepsilon\mu\sinh(kd)-\alpha^2\cosh(kd)\}+F_{03}\left(\varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}\sinh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\cosh(bkd)\right)=0 \quad \text{(VI.47)}$$

$$F_{02}\{\varepsilon\mu\cosh(kd)-\alpha^2\sinh(kd)\}+F_{04}\left(\varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}\cosh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\sinh(bkd)\right)=0 \quad \text{(VI.48)}$$

$$\begin{aligned}&F_{01}\{(\varepsilon\mu-\alpha^2)\sinh(kd)+\varepsilon\mu_0\cosh(kd)\}+F_{02}\{(\varepsilon\mu-\alpha^2)\cosh(kd)+\varepsilon\mu_0\sinh(kd)\}\\&+\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{K_{em}^2}[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)]=0\end{aligned} \quad \text{(VI.49)}$$

$$\begin{aligned}&F_{01}\{(\varepsilon\mu-\alpha^2)\sinh(kd)+\varepsilon_0\mu\cosh(kd)\}-F_{02}\{(\varepsilon\mu-\alpha^2)\cosh(kd)+\varepsilon_0\mu\sinh(kd)\}\\&+\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{K_{em}^2}[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)]=0\end{aligned} \quad \text{(VI.50)}$$

It is obvious that F_{01} and F_{02} are functions of F_{03} and F_{04} in formulae (VI.47) and (VI.48), respectively. Therefore, equations (VI.49) and (VI.50) can be written only with the weight factors F_{03} and F_{04} as follows:

$$F_{04}\left\{\frac{(\varepsilon\mu-\alpha^2)+\varepsilon\mu_0\tanh(kd)}{\varepsilon\mu-\alpha^2\tanh(kd)}\left(\varepsilon\mu b-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\tanh(bkd)\right)-\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\tanh(bkd)\right\}$$
$$+F_{03}\left\{\frac{(\varepsilon\mu-\alpha^2)\tanh(kd)+\varepsilon\mu_0}{\varepsilon\mu\tanh(kd)-\alpha^2}\left(\varepsilon\mu b\tanh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)-\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\right\}=0 \quad \text{(VI.51)}$$

$$F_{04}\left\{\frac{(\varepsilon\mu-\alpha^2)+\varepsilon_0\mu\tanh(kd)}{\varepsilon\mu-\alpha^2\tanh(kd)}\left(\varepsilon\mu b-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\tanh(bkd)\right)-\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\tanh(bkd)\right\}$$
$$-F_{03}\left\{\frac{(\varepsilon\mu-\alpha^2)\tanh(kd)+\varepsilon_0\mu}{\varepsilon\mu\tanh(kd)-\alpha^2}\left(\varepsilon\mu b\tanh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)-\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\right\}=0 \quad \text{(VI.52)}$$

It is obvious that the following complicated dispersion relation can be obtained for the calculation of the velocity V_{new18} of the eighteenth new SH-wave propagating in the piezoelectromagnetic plate in the case of $V_{ph} < V_{tem}$:

$$\left\{\frac{(\varepsilon\mu-\alpha^2)\tanh(kd)+\varepsilon_0\mu}{\varepsilon\mu\tanh(kd)-\alpha^2}\left(\varepsilon\mu b\tanh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)-\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\right\}$$
$$\times\left\{\frac{(\varepsilon\mu-\alpha^2)+\varepsilon\mu_0\tanh(kd)}{\varepsilon\mu-\alpha^2\tanh(kd)}\left(\varepsilon\mu b-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\tanh(bkd)\right)-\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\tanh(bkd)\right\}$$
$$+\left\{\frac{(\varepsilon\mu-\alpha^2)+\varepsilon_0\mu\tanh(kd)}{\varepsilon\mu-\alpha^2\tanh(kd)}\left(\varepsilon\mu b-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\tanh(bkd)\right)-\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\tanh(bkd)\right\}$$
$$\times\left\{\frac{(\varepsilon\mu-\alpha^2)\tanh(kd)+\varepsilon\mu_0}{\varepsilon\mu\tanh(kd)-\alpha^2}\left(\varepsilon\mu b\tanh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)-\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\right\}=0 \quad \text{(VI.53)}$$

For the case of $kd \to \infty$, this dispersion relation takes the following form:

$$\left\{\frac{(\varepsilon+\varepsilon_0)\mu-\alpha^2}{\varepsilon\mu-\alpha^2}\left(\varepsilon\mu\sqrt{1-(V_{new18}/V_{tem})^2}-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)-\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\right\}$$
$$\times\left\{\frac{\varepsilon(\mu+\mu_0)-\alpha^2}{\varepsilon\mu-\alpha^2}\left(\varepsilon\mu\sqrt{1-(V_{new18}/V_{tem})^2}-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)-\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\right\}=0 \quad \text{(VI.54)}$$

Chapter VII

Magnetically Open Surfaces and Continuity of φ and D_3

This chapter also studies the homogeneous case when the same set of the boundary conditions is exploited for both the upper and lower surfaces of the piezoelectromagnetic plate. The mechanically free surface ($\sigma_{32} = 0$) is the used mechanical boundary condition. Besides, the electrical boundary conditions are the continuity of both φ and D_3 at the surfaces, i.e. $\varphi = \varphi^f$ and $D_3 = D^f$. The magnetic boundary condition is the magnetically open surface ($\psi = 0$). With the weight factors F_{01}, F_{02}, F_{03}, and F_{04}, three homogeneous equations corresponding to the mechanical, electrical, and magnetic boundary conditions for the upper surface at $x_3 = +d$ are composed as follows:

$$\begin{aligned}&\left(e\varphi^{0(1)} + h\psi^{0(1)}\right)\left[F_{01}\sinh(kd) + F_{02}\cosh(kd)\right]\\&+ b\left(CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0\end{aligned} \tag{VII.1}$$

$$\begin{aligned}&\left(\varepsilon\varphi^{0(1)} + \alpha\psi^{0(1)}\right)\left\{F_{01}\sinh(kd) + F_{02}\cosh(kd)\right\}\\&- b\left(eU^{0(5)} - \varepsilon\varphi^{0(5)} - \alpha\psi^{0(5)}\right)\left\{F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right\}\\&+ \varepsilon_0\left\{F_{01}\varphi^{0(1)}\cosh(kd) + F_{02}\varphi^{0(1)}\sinh(kd) + F_{03}\varphi^{0(5)}\cosh(bkd) + F_{04}\varphi^{0(5)}\sinh(bkd)\right\} = 0\end{aligned} \tag{VII.2}$$

$$F_{01}\psi^{0(1)}\cosh(kd) + F_{02}\psi^{0(1)}\sinh(kd) + F_{03}\psi^{0(5)}\cosh(bkd) + F_{04}\psi^{0(5)}\sinh(bkd) = 0 \tag{VII.3}$$

For the case of the lower free surface at $x_3 = -d$, three homogeneous equations corresponding to the mechanical, electrical, and magnetic boundary conditions read:

$$\begin{aligned}&\left(e\varphi^{0(1)} + h\psi^{0(1)}\right)\left[F_{01}\sinh(kd) - F_{02}\cosh(kd)\right]\\&+ b\left(CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) - F_{04}\cosh(bkd)\right] = 0\end{aligned} \tag{VII.4}$$

$$
\begin{aligned}
&\left(\varepsilon\varphi^{0(1)}+\alpha\psi^{0(1)}\right)\{F_{01}\sinh(kd)-F_{02}\cosh(kd)\}\\
&-b\left(eU^{0(5)}-\varepsilon\varphi^{0(5)}-\alpha\psi^{0(5)}\right)\{F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\}\\
&+\varepsilon_0\left\{F_{01}\varphi^{0(1)}\cosh(kd)-F_{02}\varphi^{0(1)}\sinh(kd)+F_{03}\varphi^{0(5)}\cosh(bkd)-F_{04}\varphi^{0(5)}\sinh(bkd)\right\}=0
\end{aligned}
\tag{VII.5}
$$

$$
F_{01}\psi^{0(1)}\cosh(kd)-F_{02}\psi^{0(1)}\sinh(kd)+F_{03}\psi^{0(5)}\cosh(bkd)-F_{04}\psi^{0(5)}\sinh(bkd)=0 \tag{VII.6}
$$

Thus, it is possible now to further transform these six homogeneous equations written above employing the first and second eigenvectors.

VII.1. *The first eigenvectors*

Employing the first eigenvectors defined by expressions (I.66) and (I.67) and equalities (III.7) and (III.8), these six homogeneous equations written above can be transformed into suitable forms. For the upper free surface at $x_3 = + d$, it is possible to write the following forms of transformed equations from (VII.1) to (VII.3):

$$
\left(\varepsilon\mu-\alpha^2\right)\left\{F_{01}\sinh(kd)+F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)+F_{04}\cosh(bkd)\right]\right\}=0 \tag{VII.7}
$$

$$
-\alpha^2\left[F_{01}\cosh(kd)+F_{02}\sinh(kd)\right]-\alpha^2\frac{K_{em}^2-K_{\alpha}^2}{K_{em}^2}\left[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)\right]=0 \tag{VII.8}
$$

$$
\varepsilon\mu\{F_{01}\cosh(kd)+F_{02}\sinh(kd)\}+\varepsilon\mu\frac{K_{em}^2-K_{e}^2}{K_{em}^2}\left[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)\right]=0 \tag{VII.9}
$$

For the lower free surface at $x_3 = - d$, the corresponding three homogeneous equations can be represented as follows:

$$
\left(\varepsilon\mu-\alpha^2\right)\left\{F_{01}\sinh(kd)-F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]\right\}=0 \tag{VII.10}
$$

$$
-\alpha^2\left[F_{01}\cosh(kd)-F_{02}\sinh(kd)\right]-\alpha^2\frac{K_{em}^2-K_{\alpha}^2}{K_{em}^2}\left[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)\right]=0 \tag{VII.11}
$$

$$\varepsilon\mu\{F_{01}\cosh(kd)-F_{02}\sinh(kd)\}+\varepsilon\mu\frac{K_{em}^2-K_e^2}{K_{em}^2}[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)]=0 \quad \text{(VII.12)}$$

To have the same dimension of s^2/m^2, equations (VII.7) and (VII.10) were multiplied by $(\varepsilon\mu - \alpha^2)/(e\alpha - h\varepsilon)$, equations (VII.8) and (VII.11) were multiplied by α/ε_0, and equations (VII.9) and (VII.12) were multiplied by μ. So, it is also possible to use ε_0 instead of ε in the corresponding equations. However, it is thought that to deal with ε is preferable because the value of α^2 must be restricted by the value of $\varepsilon\mu$, namely $\varepsilon\mu > \alpha^2$ [65, 68]. In the case of the utilization of ε_0 instead of ε, the inequality $\varepsilon_0\mu > \alpha^2$ is dim and there is a possibility of $\varepsilon_0\mu < \alpha^2$ for large values of the electromagnetic constant α.

It is also possible to represent these six homogeneous equations with the same dimension which is unequal to s^2/m^2 when equations (VII.7) and (VII.10) have the factor of $(e\alpha - h\varepsilon)$ instead of $(\varepsilon\mu - \alpha^2)$, equations (VII.8) and (VII.11) have the factor of $e\alpha$ instead of α^2, and equations (VII.9) and (VII.12) have the factor of $h\varepsilon$ instead of $\varepsilon\mu$. In this case, these six homogeneous equations lead to the dispersion relations obtained in Chapter II for the other set of the electrical and magnetic boundary conditions such as the electrically closed ($\varphi = 0$) and magnetically open ($\psi = 0$) surface. Therefore, it is expected that the dispersion relations obtained in this subsection below can be also true for the boundary conditions used in Chapter II. However, to use ε_0 instead of ε is unrelated for the case of Chapter II because the vacuum material parameters are absent in the considerations of the second chapter.

Following the transformations exploited in the previous chapters, it is natural to add equation (VII.8) to (VII.9) and equation (VII.11) to (VII.12) to deal with four homogeneous equations with four weight factors F_{01}, F_{02}, F_{03}, and F_{04}. It is also apparent that the resulting four equations can be represented as two independent pairs of the corresponding equations after the usual transformations. Therefore, the transformed equations read:

$$\left(\varepsilon\mu-\alpha^2\right)\left\{F_{01}\sinh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}F_{03}\sinh(bkd)\right\}=0 \quad \text{(VII.13)}$$

$$\left(\varepsilon\mu-\alpha^2\right)F_{01}\cosh(kd)+\left(\varepsilon\mu\frac{K_{em}^2-K_e^2}{K_{em}^2}-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\right)F_{03}\cosh(bkd)=0 \quad \text{(VII.14)}$$

$$\left(\varepsilon\mu-\alpha^2\right)\left\{F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}F_{04}\cosh(bkd)\right\}=0 \quad \text{(VII.15)}$$

$$\left(\varepsilon\mu-\alpha^2\right)F_{02}\sinh(kd)+\left(\varepsilon\mu\frac{K_{em}^2-K_e^2}{K_{em}^2}-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\right)F_{04}\sinh(bkd)=0 \quad \text{(VII.16)}$$

Finally, the first pair of equations (VII.13) and (VII.14) and the second pair of equations (VII.15) and (VII.16) result in two different dispersion relations. These dispersion relations determine the velocities V_{new19} and V_{new20} of the nineteenth and twentieth new SH-waves propagating in the piezoelectromagnetic plate in the case of $V_{ph} < V_{tem}$. So, they are composed for the case of $V_{ph} < V_{tem}$ as follows:

$$\sqrt{1-\left(\frac{V_{new19}}{V_{tem}}\right)^2}\tanh\left(kd\sqrt{1-\left(\frac{V_{new19}}{V_{tem}}\right)^2}\right)-\frac{\varepsilon\mu\left(K_{em}^2-K_e^2\right)-\alpha^2\left(K_{em}^2-K_\alpha^2\right)}{\left(\varepsilon\mu-\alpha^2\right)\left(1+K_{em}^2\right)}\tanh(kd)=0 \quad \text{(VII.17)}$$

$$\sqrt{1-\left(\frac{V_{new20}}{V_{tem}}\right)^2}\tanh(kd)-\frac{\varepsilon\mu\left(K_{em}^2-K_e^2\right)-\alpha^2\left(K_{em}^2-K_\alpha^2\right)}{\left(\varepsilon\mu-\alpha^2\right)\left(1+K_{em}^2\right)}\tanh\left(kd\sqrt{1-\left(\frac{V_{new20}}{V_{tem}}\right)^2}\right)=0 \quad \text{(VII.18)}$$

It is flagrant in dispersion relations (VII.17) and (VII.18) that for $kd \to \infty$, both the velocities V_{new19} and V_{new20} approach the corresponding SH-SAW velocity recently discovered in book [101], see formulae from (148) to (152) in the book. It is also lucid that for the case of $V_{ph} > V_{tem}$ the above written dispersion relations read:

$$\tan\left(kd\sqrt{\left(\frac{V_{new19}}{V_{tem}}\right)^2-1}\right)\sqrt{\left(\frac{V_{new19}}{V_{tem}}\right)^2-1}+\frac{\varepsilon\mu\left(K_{em}^2-K_e^2\right)-\alpha^2\left(K_{em}^2-K_\alpha^2\right)}{\left(\varepsilon\mu-\alpha^2\right)\left(1+K_{em}^2\right)}\tanh(kd)=0 \quad \text{(VII.19)}$$

$$\tanh(kd)\sqrt{\left(\frac{V_{new20}}{V_{tem}}\right)^2-1}-\frac{\varepsilon\mu\left(K_{em}^2-K_e^2\right)-\alpha^2\left(K_{em}^2-K_\alpha^2\right)}{\left(\varepsilon\mu-\alpha^2\right)\left(1+K_{em}^2\right)}\tan\left(kd\sqrt{\left(\frac{V_{new20}}{V_{tem}}\right)^2-1}\right)=0 \quad \text{(VII.20)}$$

VII.2. *The second eigenvectors*

The second eigenvectors defined by expressions (I.69) and (I.70) result in the existence of equalities (III.23) and (III.24). Using them for the upper and lower surfaces of the plate, equations from (VII.1) to (VII.6) can be transformed into equations from (VI.29) to (VI.34) obtained in the second subsection of the previous chapter. So, the resulting dispersion relations will be similar to those defined by expressions (VI.39) and (VI.40).

VII.3. *The first and second eigenvectors*

This case is similar to that treated in the third subsection of the previous chapter. For the upper surface, equations from (VII.7) to (VII.9) must be used and for the lower surface, equations from (VI.32) to (VI.34) defined in the previous chapter are utilized. It is lucid that here there are also two pairs of equations for transformations. Equations (VII.7) and (VI.32) can be transformed into two independent equations (VI.43) and (VI.44) written in the previous chapter. Also, Equations (VII.8) and (VI.34) can be transformed into two independent equations (VI.45) and (VI.46). Next, each of equations (VI.43) and (VI.45) coupled with the weight factors F_{01} and F_{03} is transformed into equation (VI.47) and each of equations (VI.44) and (VI.46) coupled with the weight factors F_{02} and F_{04} is transformed into equation (VI.48). Using equations (VI.47) and (VI.48), the weight factors F_{01} and F_{02} are respectively determined as follows:

$$F_{01}=-F_{03}\frac{\cosh(bkd)}{\varepsilon\mu\sinh(kd)-\alpha^2\cosh(kd)}\left(\varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}\tanh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\right) \quad \text{(VII.21)}$$

$$F_{02} = -F_{04}\frac{\cosh(bkd)}{\varepsilon\mu\cosh(kd)-\alpha^2\sinh(kd)}\left(\varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}-\alpha^2\frac{K_{em}^2-K_{\alpha}^2}{K_{em}^2}\tanh(bkd)\right) \quad \text{(VII.22)}$$

Exploiting expressions (VII.21) and (VII.22), the weight factors F_{01} and F_{02} can be excluded from equations (VII.9) and (VI.33) rewritten below as follows:

$$\varepsilon\mu\{F_{01}\cosh(kd)+F_{02}\sinh(kd)\}+\varepsilon\mu\frac{K_{em}^2-K_e^2}{K_{em}^2}[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)]=0 \quad \text{(VII.23)}$$

$$\begin{aligned}&F_{01}\{(\varepsilon\mu-\alpha^2)\sinh(kd)+\varepsilon_0\mu\cosh(kd)\}-F_{02}\{(\varepsilon\mu-\alpha^2)\cosh(kd)+\varepsilon_0\mu\sinh(kd)\}\\&+\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{K_{em}^2}[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)]=0\end{aligned} \quad \text{(VII.24)}$$

Consequently, these two equations after the transformations to have two equations with two unknown weight factors F_{03} and F_{04} read:

$$\begin{aligned}&F_{04}\left\{\frac{\tanh(kd)}{\varepsilon\mu-\alpha^2\tanh(kd)}\left(\varepsilon\mu b-\alpha^2\frac{K_{em}^2-K_{\alpha}^2}{1+K_{em}^2}\tanh(bkd)\right)-\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\tanh(bkd)\right\}\\&+F_{03}\left\{\frac{1}{\varepsilon\mu\tanh(kd)-\alpha^2}\left(\varepsilon\mu b\tanh(bkd)-\alpha^2\frac{K_{em}^2-K_{\alpha}^2}{1+K_{em}^2}\right)-\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\right\}=0\end{aligned} \quad \text{(VII.25)}$$

$$\begin{aligned}&F_{04}\left\{\frac{(\varepsilon\mu-\alpha^2)+\varepsilon_0\mu\tanh(kd)}{\varepsilon\mu-\alpha^2\tanh(kd)}\left(\varepsilon\mu b-\alpha^2\frac{K_{em}^2-K_{\alpha}^2}{1+K_{em}^2}\tanh(bkd)\right)-\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\tanh(bkd)\right\}\\&-F_{03}\left\{\frac{(\varepsilon\mu-\alpha^2)\tanh(kd)+\varepsilon_0\mu}{\varepsilon\mu\tanh(kd)-\alpha^2}\left(\varepsilon\mu b\tanh(bkd)-\alpha^2\frac{K_{em}^2-K_{\alpha}^2}{1+K_{em}^2}\right)-\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\right\}=0\end{aligned} \quad \text{(VII.26)}$$

Finally, the dispersion relation for the calculation of the velocity V_{new21} of the twenty first new SH-wave propagating in the piezoelectromagnetic plate in the case of $V_{ph} < V_{tem}$ is defined by the following complicated equation:

$$
\begin{aligned}
&\left\{\frac{\left(\varepsilon\mu-\alpha^2\right)+\varepsilon_0\mu\tanh(kd)}{\varepsilon\mu-\alpha^2\tanh(kd)}\left(\varepsilon\mu b-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\tanh(bkd)\right)-\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\tanh(bkd)\right\}\\
&\times\left\{\frac{1}{\varepsilon\mu\tanh(kd)-\alpha^2}\left(\varepsilon\mu b\tanh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)-\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\right\}\\
&+\left\{\frac{\tanh(kd)}{\varepsilon\mu-\alpha^2\tanh(kd)}\left(\varepsilon\mu b-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\tanh(bkd)\right)-\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\tanh(bkd)\right\}\\
&\times\left\{\frac{\left(\varepsilon\mu-\alpha^2\right)\tanh(kd)+\varepsilon_0\mu}{\varepsilon\mu\tanh(kd)-\alpha^2}\left(\varepsilon\mu b\tanh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)-\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\right\}=0
\end{aligned}
\qquad \text{(VII.27)}
$$

Chapter VIII

Magnetically Closed Surfaces and Continuity of φ and D_3

It is also natural to theoretically treat the other possible magnetic boundary condition to compare with the results obtained in the previous chapter. In this chapter, the mechanical and electrical boundary conditions at the upper and lower interfaces between the piezoelectromagnetic plate and a vacuum are similar to those used in the previous chapter: mechanically free surface (σ_{32} = 0) and the continuity of both the electrical potential and electrical induction, namely $\varphi = \varphi^f$ and $D_3 = D^f$, where D_3 is the normal component of the electrical displacements and the superscript "f" relates to the free space. Besides, here the magnetic boundary condition is the magnetically closed surface (B_3 = 0) instead of the magnetically open surface (ψ = 0) utilized in the previous chapter.

Three homogeneous equations corresponding to the mechanical, electrical, and magnetic boundary conditions (upper surface at x_3 = + d) have the following forms:

$$\begin{aligned}&\left(e\varphi^{0(1)} + h\psi^{0(1)}\right)\left[F_{01}\sinh(kd) + F_{02}\cosh(kd)\right]\\&+b\left(CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0\end{aligned} \tag{VIII.1}$$

$$\begin{aligned}&\left(\varepsilon\varphi^{0(1)} + \alpha\psi^{0(1)}\right)\left\{F_{01}\sinh(kd) + F_{02}\cosh(kd)\right\}\\&-b\left(eU^{0(5)} - \varepsilon\varphi^{0(5)} - \alpha\psi^{0(5)}\right)\left\{F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right\}\\&+\varepsilon_0\left\{F_{01}\varphi^{0(1)}\cosh(kd) + F_{02}\varphi^{0(1)}\sinh(kd) + F_{03}\varphi^{0(5)}\cosh(bkd) + F_{04}\varphi^{0(5)}\sinh(bkd)\right\} = 0\end{aligned} \tag{VIII.2}$$

$$\begin{aligned}&\left(\alpha\varphi^{0(1)} + \mu\psi^{0(1)}\right)\left\{F_{01}\sinh(kd) + F_{02}\cosh(kd)\right\}\\&-b\left(hU^{0(5)} - \alpha\varphi^{0(5)} - \mu\psi^{0(5)}\right)\left\{F_{3}\sinh(bkd) + F_{04}\cosh(bkd)\right\} = 0\end{aligned} \tag{VIII.3}$$

For the lower surface of the plate (x_3 = – d) three homogeneous equations corresponding to the mechanical, electrical, and magnetic boundary conditions can be written as follows:

$$\begin{aligned}&\left(e\varphi^{0(1)}+h\psi^{0(1)}\right)\left[F_{01}\sinh(kd)-F_{02}\cosh(kd)\right]\\&+b\left(CU^{0(5)}+e\varphi^{0(5)}+h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]=0\end{aligned}\qquad\text{(VIII.4)}$$

$$\begin{aligned}&\left(\varepsilon\varphi^{0(1)}+\alpha\psi^{0(1)}\right)\left\{F_{01}\sinh(kd)-F_{02}\cosh(kd)\right\}\\&-b\left(eU^{0(5)}-\varepsilon\varphi^{0(5)}-\alpha\psi^{0(5)}\right)\left\{F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right\}\\&+\varepsilon_0\left\{F_{01}\varphi^{0(1)}\cosh(kd)-F_{02}\varphi^{0(1)}\sinh(kd)+F_{03}\varphi^{0(5)}\cosh(bkd)-F_{04}\varphi^{0(5)}\sinh(bkd)\right\}=0\end{aligned}\qquad\text{(VIII.5)}$$

$$\begin{aligned}&\left(\alpha\varphi^{0(1)}+\mu\psi^{0(1)}\right)\left\{F_{01}\sinh(kd)-F_{02}\cosh(kd)\right\}\\&-b\left(hU^{0(5)}-\alpha\varphi^{0(5)}-\mu\psi^{0(5)}\right)\left\{F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right\}=0\end{aligned}\qquad\text{(VIII.6)}$$

So, this set of six homogeneous equations must lead to several dispersion relations obtained by using the first and second eigenvectors.

VIII.1. *The first eigenvectors*

Exploiting the first eigenvectors defined by expressions (I.66) and (I.67) and equalities (III.7) and (III.8), these six homogeneous equations written above can be transformed. For the upper surface ($x_3 = +d$) the corresponding three homogeneous equations read:

$$\varepsilon\mu\left\{F_{01}\sinh(kd)+F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)+F_{04}\cosh(bkd)\right]\right\}=0\qquad\text{(VIII.7)}$$

$$-\alpha^2\left[F_{01}\cosh(kd)+F_{02}\sinh(kd)\right]-\alpha^2\frac{K_{em}^2-K_{\alpha}^2}{K_{em}^2}\left[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)\right]=0\qquad\text{(VIII.8)}$$

$$\left(\varepsilon\mu-\alpha^2\right)\left\{F_{01}\sinh(kd)+F_{02}\cosh(kd)\right\}=0\qquad\text{(VIII.9)}$$

For the lower surface ($x_3 = -d$) the corresponding three homogeneous equations take the following forms:

$$\varepsilon\mu\left\{F_{01}\sinh(kd)-F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]\right\}=0\qquad\text{(VIII.10)}$$

$$-\alpha^2\left[F_{01}\cosh(kd)-F_{02}\sinh(kd)\right]-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\left[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)\right]=0 \quad \text{(VIII.11)}$$

$$\left(\varepsilon\mu-\alpha^2\right)\left\{F_{01}\sinh(kd)-F_{02}\cosh(kd)\right\}=0 \quad \text{(VIII.12)}$$

It is blatant that these six homogeneous equations written above are identical to equations from (IV.29) to (IV.34) obtained in Chapter IV. Therefore, they lead to two dispersion relations given by formulae (IV.40) and (IV.42) for the determination of the velocities V_{new6} and V_{new7} of the sixth and seventh new SH-waves propagating in the piezoelectromagnetic plate. Next, it is possible to consider the case of the second eigenvectors.

VIII.2. *The second eigenvectors*

In this subsection, second eigenvectors defined by expressions (I.69) and (I.70) from the first chapter must be exploited. The second eigenvectors result in equalities (III.23) and (III.24) obtained in Chapter III that must be also utilized for six equations from (VIII.1) to (VIII.6). Using expression (III.24), it is obvious that equations (VIII.3) and (VIII.6) corresponding to the magnetic boundary condition can be excluded from the further theoretical considerations done in this subsection. Thus, this is the case of four equations with four unknown weight factors F_{01}, F_{02}, F_{03}, and F_{04}. For the upper surface at $x_3 = +d$, equations (VI.29) and (VI.30) written in Chapter VI are also suitable here. For the lower surface at $x_3 = -d$, equations (VI.32) and (VI.33) are appropriate. So, four homogeneous equations respectively read:

$$\varepsilon\mu\left\{F_{01}\sinh(kd)+F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)+F_{04}\cosh(bkd)\right]\right\}=0 \quad \text{(VIII.13)}$$

$$\begin{aligned}&F_{01}\left\{\left(\varepsilon\mu-\alpha^2\right)\sinh(kd)+\varepsilon_0\mu\cosh(kd)\right\}+F_{02}\left\{\left(\varepsilon\mu-\alpha^2\right)\cosh(kd)+\varepsilon_0\mu\sinh(kd)\right\}\\&+\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{K_{em}^2}\left[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)\right]=0\end{aligned} \quad \text{(VIII.14)}$$

$$\varepsilon\mu\left\{F_{01}\sinh(kd)-F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]\right\}=0 \quad \text{(VIII.15)}$$

$$F_{01}\left\{\left(\varepsilon\mu-\alpha^2\right)\sinh(kd)+\varepsilon_0\mu\cosh(kd)\right\}-F_{02}\left\{\left(\varepsilon\mu-\alpha^2\right)\cosh(kd)+\varepsilon_0\mu\sinh(kd)\right\}$$
$$+\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{K_{em}^2}\left[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)\right]=0 \quad \text{(VIII.16)}$$

It is obvious that there are two pairs of equations: the first pair is equations (VIII.13) and (VIII.15) and the second pair is equations (VIII.14) and (VIII.16). After the usual transformations used in the previous chapters, these two pairs can be represented as the following two independent pairs of equations:

$$F_{01}\sinh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}F_{03}\sinh(bkd)=0 \quad \text{(VIII.17)}$$

$$F_{01}\left\{\left(\varepsilon\mu-\alpha^2\right)\sinh(kd)+\varepsilon_0\mu\cosh(kd)\right\}+\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{K_{em}^2}F_{03}\cosh(bkd)=0 \quad \text{(VIII.18)}$$

$$F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}F_{04}\cosh(bkd)=0 \quad \text{(VIII.19)}$$

$$F_{02}\left\{\left(\varepsilon\mu-\alpha^2\right)\cosh(kd)+\varepsilon_0\mu\sinh(kd)\right\}+\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{K_{em}^2}F_{04}\sinh(bkd)=0 \quad \text{(VIII.20)}$$

As a result, these two independent pairs of four equations lead to two independent dispersion relations for the case of $V_{ph} < V_{tem}$. Indeed, the velocities V_{new22} and V_{new23} of the twenty second and twenty third new SH-waves propagating in the piezoelectromagnetic plate can be calculated with the following formulae:

$$\sqrt{1-\left(\frac{V_{new22}}{V_{tem}}\right)^2}\tanh\left(kd\sqrt{1-\left(\frac{V_{new22}}{V_{tem}}\right)^2}\right)-\frac{\varepsilon_0\mu\tanh(kd)}{\left(\varepsilon\mu-\alpha^2\right)\tanh(kd)+\varepsilon_0\mu}\frac{K_{em}^2-K_m^2}{1+K_{em}^2}=0 \quad \text{(VIII.21)}$$

$$\sqrt{1-\left(\frac{V_{new23}}{V_{tem}}\right)^2}-\frac{\varepsilon_0\mu}{\varepsilon\mu-\alpha^2+\varepsilon_0\mu\tanh(kd)}\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\tanh\left(kd\sqrt{1-\left(\frac{V_{new23}}{V_{tem}}\right)^2}\right)=0 \quad \text{(VIII.22)}$$

It is also possible to treat the case of $kd \to \infty$. For very large values of kd, it is transparent that the velocities V_{new22} and V_{new23} approach the corresponding SH-SAW velocity recently discovered in book [101], see formulae from (194) to (198) in the book. Finally, dispersion relations (VIII.21) and (VIII.22) can be rewritten for the case of $V_{ph} > V_{tem}$ as follows:

$$\tan\left(kd\sqrt{\left(\frac{V_{new22}}{V_{tem}}\right)^2 - 1}\right)\sqrt{\left(\frac{V_{new22}}{V_{tem}}\right)^2 - 1} + \frac{\varepsilon_0\mu\tanh(kd)}{(\varepsilon\mu - \alpha^2)\tanh(kd) + \varepsilon_0\mu}\frac{K_{em}^2 - K_m^2}{1 + K_{em}^2} = 0 \quad \text{(VIII.23)}$$

$$\sqrt{\left(\frac{V_{new23}}{V_{tem}}\right)^2 - 1} - \frac{\varepsilon_0\mu}{\varepsilon\mu - \alpha^2 + \varepsilon_0\mu\tanh(kd)}\frac{K_{em}^2 - K_m^2}{1 + K_{em}^2}\tan\left(kd\sqrt{\left(\frac{V_{new23}}{V_{tem}}\right)^2 - 1}\right) = 0 \quad \text{(VIII.24)}$$

VIII.3. *The first and second eigenvectors*

For this case of the first eigenvectors for the upper surface of the plate and the second eigenvectors for the lower surface of the plate, it is natural to exploit equations (VIII.7), (VIII.8), and (VIII.9) for the upper surface and equations (VIII.15) and (VIII.16) for the lower surface. Using this system of five homogeneous equations, it is also possible to get two different dispersion relations. First of all, it is possible to add equation (VIII.7) to (VIII.8) to cope with one equation instead of two and to subtract equation (VIII.15) from (VIII.16) to form the second equation. It is natural to have $F_{01} = F_{02} = 0$ due to equation (VIII.9). As a result, the following set of two homogeneous equations which are coupled by the weight factors F_{03} and F_{04} must be treated to obtain the first dispersion relation:

$$F_{03}\left[b\tanh(bkd) - \frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2 - K_\alpha^2}{1 + K_{em}^2}\right] + F_{04}\left[b - \frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2 - K_\alpha^2}{1 + K_{em}^2}\tanh(bkd)\right] = 0 \quad \text{(VIII.25)}$$

$$F_{03}\left[b\tanh(bkd) - \frac{\varepsilon_0\mu}{\varepsilon\mu - \alpha^2}\frac{K_{em}^2 - K_m^2}{1 + K_{em}^2}\right] - F_{04}\left[b - \frac{\varepsilon_0\mu}{\varepsilon\mu - \alpha^2}\frac{K_{em}^2 - K_m^2}{1 + K_{em}^2}\tanh(bkd)\right] = 0 \quad \text{(VIII.26)}$$

Thus, the dispersion relation for the case of $V_{ph} < V_{tem}$ to calculate the velocity V_{new24} of the twenty fourth new SH-wave propagating in the piezoelectromagnetic plate can be composed as follows:

$$\begin{aligned}&\left(\sqrt{1-\left(\frac{V_{new24}}{V_{tem}}\right)^2}-\frac{\varepsilon_0\mu}{\varepsilon\mu-\alpha^2}\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\tanh\left(kd\sqrt{1-\left(\frac{V_{new24}}{V_{tem}}\right)^2}\right)\right)\\&\times\left(\sqrt{1-\left(\frac{V_{new24}}{V_{tem}}\right)^2}\tanh\left(kd\sqrt{1-\left(\frac{V_{new24}}{V_{tem}}\right)^2}\right)-\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)\\&+\left(\sqrt{1-\left(\frac{V_{new24}}{V_{tem}}\right)^2}\tanh\left(kd\sqrt{1-\left(\frac{V_{new24}}{V_{tem}}\right)^2}\right)-\frac{\varepsilon_0\mu}{\varepsilon\mu-\alpha^2}\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\right)\\&\left(\sqrt{1-\left(\frac{V_{new24}}{V_{tem}}\right)^2}-\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\tanh\left(kd\sqrt{1-\left(\frac{V_{new24}}{V_{tem}}\right)^2}\right)\right)=0\end{aligned}\qquad\text{(VIII.27)}$$

To obtain the second possible dispersion relation, it is necessary to add equation (VIII.7) to (VIII.8) to deal with one equation instead of two and to use equation (VIII.9) for determination of F_{02} as a function of F_{01}:

$$F_{02}=-F_{01}\tanh(kd)\qquad\text{(VIII.28)}$$

Utilizing equation (VIII.9), three equations can be written as follows:

$$\begin{aligned}&-\alpha^2F_{01}+F_{03}\cosh(kd)\left(\varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}\sinh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\cosh(bkd)\right)\\&+F_{04}\cosh(kd)\left(\varepsilon\mu b\frac{1+K_{em}^2}{K_{em}^2}\cosh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\sinh(bkd)\right)=0\end{aligned}\qquad\text{(VIII.29)}$$

$$F_{01}=-\frac{b}{2\sinh(kd)}\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]\qquad\text{(VIII.30)}$$

$$F_{01}\left\{2\left(\varepsilon\mu-\alpha^2\right)\sinh(kd)+\varepsilon_0\mu\cosh(kd)+\varepsilon_0\mu\sinh(kd)\tanh(kd)\right\}$$
$$+\varepsilon_0\mu\frac{K_{em}^2-K_m^2}{K_{em}^2}\left[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)\right]=0 \qquad \text{(VIII.31)}$$

Then, it is possible to use equation (VIII.30) to exclude F_{01} in equations (VIII.29) and (VIII.31). So, two equations with the weight factors F_{03} and F_{04} read:

$$F_{03}\left\{\sinh(2kd)\left(\varepsilon\mu b\tanh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)+\alpha^2 b\tanh(bkd)\right\}$$
$$+F_{04}\left\{\sinh(2kd)\left(\varepsilon\mu b-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\tanh(bkd)\right)-\alpha^2 b\right\}=0 \qquad \text{(VIII.32)}$$

$$F_{03}\left\{\frac{\varepsilon_0\mu\sinh(2kd)}{\left(\varepsilon\mu-\alpha^2\right)\sinh(2kd)+\varepsilon_0\mu+2\varepsilon_0\mu\sinh^2(kd)}\frac{K_{em}^2-K_m^2}{1+K_{em}^2}-b\tanh(bkd)\right\}$$
$$-F_{04}\left\{\frac{\varepsilon_0\mu\sinh(2kd)}{\left(\varepsilon\mu-\alpha^2\right)\sinh(2kd)+\varepsilon_0\mu+2\varepsilon_0\mu\sinh^2(kd)}\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\tanh(bkd)-b\right\}=0 \qquad \text{(VIII.33)}$$

Finally, the second dispersion relation to calculate the velocity V_{new25} of the twenty fifth new SH-wave propagating in the piezoelectromagnetic plate can be written for the case of $V_{ph} < V_{tem}$ as follows:

$$\left\{\frac{\varepsilon_0\mu\sinh(2kd)}{\left(\varepsilon\mu-\alpha^2\right)\sinh(2kd)+\varepsilon_0\mu+2\varepsilon_0\mu\sinh^2(kd)}\frac{K_{em}^2-K_m^2}{1+K_{em}^2}\tanh(bkd)-b\right\}$$
$$\times\left\{\sinh(2kd)\left(\varepsilon\mu b\tanh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)+\alpha^2 b\tanh(bkd)\right\}$$
$$+\left\{\frac{\varepsilon_0\mu\sinh(2kd)}{\left(\varepsilon\mu-\alpha^2\right)\sinh(2kd)+\varepsilon_0\mu+2\varepsilon_0\mu\sinh^2(kd)}\frac{K_{em}^2-K_m^2}{1+K_{em}^2}-b\tanh(bkd)\right\}$$
$$\times\left\{\sinh(2kd)\left(\varepsilon\mu b-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\tanh(bkd)\right)-\alpha^2 b\right\}=0 \qquad \text{(VIII.34)}$$

Chapter IX

Electrically Closed Surfaces and Continuity of ψ and B_3

Consider the mechanically free ($\sigma_{32} = 0$) and electrically closed ($\varphi = 0$) case for both the upper and lower surfaces of the piezoelectromagnetic plate of class 6 *mm*. In addition, the magnetic boundary condition is the continuity of both the magnetic potential ψ and the normal component B_3 of the magnetic induction at the surfaces, i.e. $\psi = \psi^f$ and $B_3 = B^f$, where the superscript "*f*" pertains to the free space (vacuum). Thus, three homogeneous equations corresponding to the mechanical, electrical, and magnetic boundary conditions for the upper surface at $x_3 = + d$ must be written as follows:

$$\begin{aligned}&\left(e\varphi^{0(1)} + h\psi^{0(1)}\right)\left[F_{01}\sinh(kd) + F_{02}\cosh(kd)\right]\\&+ b\left(CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0\end{aligned} \tag{IX.1}$$

$$F_{01}\varphi^{0(1)}\cosh(kd) + F_{02}\varphi^{0(1)}\sinh(kd) + F_{03}\varphi^{0(5)}\cosh(bkd) + F_{04}\varphi^{0(5)}\sinh(bkd) = 0 \tag{IX.2}$$

$$\begin{aligned}&\left(\alpha\varphi^{0(1)} + \mu\psi^{0(1)}\right)\left\{F_{01}\sinh(kd) + F_{02}\cosh(kd)\right\}\\&- b\left(hU^{0(5)} - \alpha\varphi^{0(5)} - \mu\psi^{0(5)}\right)\left\{F_{3}\sinh(bkd) + F_{04}\cosh(bkd)\right\}\\&+ \mu_0\left\{F_{01}\psi^{0(1)}\cosh(kd) + F_{02}\psi^{0(1)}\sinh(kd) + F_{03}\psi^{0(5)}\cosh(bkd) + F_{04}\psi^{0(5)}\sinh(bkd)\right\} = 0\end{aligned} \tag{IX.3}$$

For the lower surface at $x_3 = - d$, three homogeneous equations corresponding to the mechanical, electrical, and magnetic boundary conditions can be designed in the following forms:

$$\begin{aligned}&\left(e\varphi^{0(1)} + h\psi^{0(1)}\right)\left[F_{01}\sinh(kd) - F_{02}\cosh(kd)\right]\\&+ b\left(CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) - F_{04}\cosh(bkd)\right] = 0\end{aligned} \tag{IX.4}$$

$$F_{01}\varphi^{0(1)}\cosh(kd) - F_{02}\varphi^{0(1)}\sinh(kd) + F_{03}\varphi^{0(5)}\cosh(bkd) - F_{04}\varphi^{0(5)}\sinh(bkd) = 0 \tag{IX.5}$$

$$\begin{aligned}&\left(\alpha\varphi^{0(1)}+\mu\psi^{0(1)}\right)\left\{F_{01}\sinh(kd)-F_{02}\cosh(kd)\right\}\\&-b\left(hU^{0(5)}-\alpha\varphi^{0(5)}-\mu\psi^{0(5)}\right)\left\{F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right\}\\&+\mu_0\left\{F_{01}\psi^{0(1)}\cosh(kd)-F_{02}\psi^{0(1)}\sinh(kd)+F_{03}\psi^{0(5)}\cosh(bkd)-F_{04}\psi^{0(5)}\sinh(bkd)\right\}=0\end{aligned} \tag{IX.6}$$

This set of six homogeneous equations from (IX.1) to (IX.6) must be resolved to obtain dispersion relations for several different cases when only the first eigenvectors, only the second eigenvectors, or the first and second ones are used. Therefore, this is the main study carried out in the following subsections.

IX.1. *The first eigenvectors*

Employing the first eigenvectors defined by expressions (I.66) and (I.67) in the first chapter and expressions (III.7) and (III.8) from the third chapter for the upper and lower surfaces of the plate, it is possible to find that six equations from (IX.6) to (IX.6) are respectively transformed into equations from (VI.11) to (VI.16) written in the first subsection of the sixth chapter. In view of that, two dispersion relations are defined by equations (VI.21) and (VI.26). They determine the velocities V_{new14} and V_{new15} of the fourteenth and fifteenth new SH-waves propagating in the piezoelectromagnetic plate when $V_{ph} < V_{tem}$.

IX.2. *The second eigenvectors*

Exploiting the second eigenvectors defined by expressions (I.69) and (I.70) and equalities (III.23) and (III.24), these six homogeneous equations written above can be transformed into some appropriate forms. For the upper surface at $x_3 = + d$, equations from (IX.1) to (IX.3) can be transformed into the following forms:

$$\left(\varepsilon\mu-\alpha^2\right)\left\{F_{01}\sinh(kd)+F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)+F_{04}\cosh(bkd)\right]\right\}=0 \tag{IX.7}$$

$$\varepsilon\mu\left(F_{01}\cosh(kd)+F_{02}\sinh(kd)\right)+\varepsilon\mu\frac{K_{em}^2-K_m^2}{K_{em}^2}\left[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)\right]=0 \quad \text{(IX.8)}$$

$$-\alpha^2\left[F_{01}\cosh(kd)+F_{02}\sinh(kd)\right]-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\left[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)\right]=0 \quad \text{(IX.9)}$$

For the lower surface at $x_3 = -d$, equations from (IX.4) to (IX.6) read as follows:

$$\left(\varepsilon\mu-\alpha^2\right)\left\{F_{01}\sinh(kd)-F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]\right\}=0 \quad \text{(IX.10)}$$

$$\varepsilon\mu\left(F_{01}\cosh(kd)-F_{02}\sinh(kd)\right)+\varepsilon\mu\frac{K_{em}^2-K_m^2}{K_{em}^2}\left[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)\right]=0 \quad \text{(IX.11)}$$

$$-\alpha^2\left[F_{01}\cosh(kd)-F_{02}\sinh(kd)\right]-\alpha^2\frac{K_{em}^2-K_\alpha^2}{K_{em}^2}\left[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)\right]=0 \quad \text{(IX.12)}$$

It is lucid that the set of equations from (IX.7) to (IX.12) look like the other set of equations from (VII.7) to (VII.12) obtained in the first subsection of Chapter VII. However, one very important difference occurs such that K_m^2 is used in equations (IX.8) and (IX.11) instead of K_e^2 utilized in equations (VII.9) and (VII.12). Therefore, this significant difference results in two new dispersion relations which must look like dispersion relations (VII.17) and (VII.18) with K_m^2 instead of K_e^2. So, the velocities V_{new26} and V_{new27} of the twenty sixth and twenty seventh new SH-waves propagating in the piezoelectromagnetic plate in the case of $V_{ph} < V_{tem}$ can be calculated with the following formulae:

$$\sqrt{1-\left(\frac{V_{new26}}{V_{tem}}\right)^2}\tanh\left(kd\sqrt{1-\left(\frac{V_{new26}}{V_{tem}}\right)^2}\right)-\frac{\varepsilon\mu\left(K_{em}^2-K_m^2\right)-\alpha^2\left(K_{em}^2-K_\alpha^2\right)}{\left(\varepsilon\mu-\alpha^2\right)\left(1+K_{em}^2\right)}\tanh(kd)=0 \quad \text{(IX.13)}$$

$$\sqrt{1-\left(\frac{V_{new27}}{V_{tem}}\right)^2}\tanh(kd)-\frac{\varepsilon\mu\left(K_{em}^2-K_m^2\right)-\alpha^2\left(K_{em}^2-K_\alpha^2\right)}{\left(\varepsilon\mu-\alpha^2\right)\left(1+K_{em}^2\right)}\tanh\left(kd\sqrt{1-\left(\frac{V_{new27}}{V_{tem}}\right)^2}\right)=0 \quad \text{(IX.14)}$$

It is transparently seen in formulae (IX.13) and (IX.14) that for $kd \to \infty$, both V_{new26} and V_{new27} approach the corresponding SH-SAW velocity discovered in book [101], see formulae from (133) to (140) in the book. It is also evident that for $V_{ph} > V_{tem}$ the above written dispersion relations are transformed into the following forms:

$$\tan\left(kd\sqrt{\left(\frac{V_{new26}}{V_{tem}}\right)^2 - 1}\right)\sqrt{\left(\frac{V_{new26}}{V_{tem}}\right)^2 - 1} + \frac{\varepsilon\mu\left(K_{em}^2 - K_m^2\right) - \alpha^2\left(K_{em}^2 - K_\alpha^2\right)}{\left(\varepsilon\mu - \alpha^2\right)\left(1 + K_{em}^2\right)}\tanh(kd) = 0 \quad \text{(IX.15)}$$

$$\tanh(kd)\sqrt{\left(\frac{V_{new27}}{V_{tem}}\right)^2 - 1} - \frac{\varepsilon\mu\left(K_{em}^2 - K_m^2\right) - \alpha^2\left(K_{em}^2 - K_\alpha^2\right)}{\left(\varepsilon\mu - \alpha^2\right)\left(1 + K_{em}^2\right)}\tan\left(kd\sqrt{\left(\frac{V_{new27}}{V_{tem}}\right)^2 - 1}\right) = 0 \quad \text{(IX.16)}$$

IX.3. *The first and second eigenvectors*

For this case, equations from (VI.11) to (VI.13) can be used for the upper surface at $x_3 = + d$. These equations can be written in some suitable forms. For the lower surface at $x_3 = - d$, equations from (IX.10) to (IX.12) must be also used in the corresponding suitable forms. These six homogeneous equations can be transformed by the same way carried out in the third subsection of Chapter VII. So, the resulting dispersion relation will be similar to that given in equation (VII.27) with the following differences: $K_e^2 \to K_m^2$, $K_m^2 \to K_e^2$, and $\varepsilon_0\mu \to \varepsilon\mu_0$. Therefore, the dispersion relation for the calculation of the velocity V_{new28} of the twenty eighth new SH-wave propagating in the plate when $V_{ph} < V_{tem}$ is defined by

$$\begin{aligned}
&\left\{\frac{\left(\varepsilon\mu - \alpha^2\right) + \varepsilon\mu_0\tanh(kd)}{\varepsilon\mu - \alpha^2\tanh(kd)}\left(\varepsilon\mu b - \alpha^2\frac{K_{em}^2 - K_\alpha^2}{1 + K_{em}^2}\tanh(bkd)\right) - \varepsilon\mu_0\frac{K_{em}^2 - K_e^2}{1 + K_{em}^2}\tanh(bkd)\right\} \\
&\times\left\{\frac{1}{\varepsilon\mu\tanh(kd) - \alpha^2}\left(\varepsilon\mu b\tanh(bkd) - \alpha^2\frac{K_{em}^2 - K_\alpha^2}{1 + K_{em}^2}\right) - \frac{K_{em}^2 - K_m^2}{1 + K_{em}^2}\right\} \\
&+\left\{\frac{\tanh(kd)}{\varepsilon\mu - \alpha^2\tanh(kd)}\left(\varepsilon\mu b - \alpha^2\frac{K_{em}^2 - K_\alpha^2}{1 + K_{em}^2}\tanh(bkd)\right) - \frac{K_{em}^2 - K_m^2}{1 + K_{em}^2}\tanh(bkd)\right\} \\
&\times\left\{\frac{\left(\varepsilon\mu - \alpha^2\right)\tanh(kd) + \varepsilon\mu_0}{\varepsilon\mu\tanh(kd) - \alpha^2}\left(\varepsilon\mu b\tanh(bkd) - \alpha^2\frac{K_{em}^2 - K_\alpha^2}{1 + K_{em}^2}\right) - \varepsilon\mu_0\frac{K_{em}^2 - K_e^2}{1 + K_{em}^2}\right\} = 0
\end{aligned} \quad \text{(IX.17)}$$

Chapter X

Electrically Open Surfaces and Continuity of ψ and B_3

In this chapter, the following mechanical, electrical, and magnetic boundary conditions are exploited for both the upper and lower surface surfaces of the piezoelectromagnetic plate:

1) mechanically free surface ($\sigma_{32} = 0$);
2) electrically open surface ($D_3 = 0$);
3) continuity of both the magnetic potential ψ and the normal component B_3 of the magnetic induction at the surfaces, i.e. $\psi = \psi^f$ and $B_3 = B^f$, where the superscript "*f*" relates to the free space.

As a consequence, three homogeneous equations corresponding to the mechanical, electrical, and magnetic boundary conditions for the upper surface at $x_3 = + d$ read:

$$\begin{aligned}&\left(e\varphi^{0(1)} + h\psi^{0(1)}\right)\left[F_{01}\sinh(kd) + F_{02}\cosh(kd)\right]\\&+ b\left(CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0\end{aligned} \tag{X.1}$$

$$\begin{aligned}&\left(\varepsilon\varphi^{0(1)} + \alpha\psi^{0(1)}\right)\left[F_{01}\sinh(kd) + F_{02}\cosh(kd)\right]\\&- b\left(eU^{0(5)} - \varepsilon\varphi^{0(5)} - \alpha\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) + F_{04}\cosh(bkd)\right] = 0\end{aligned} \tag{X.2}$$

$$\begin{aligned}&\left(\alpha\varphi^{0(1)} + \mu\psi^{0(1)}\right)\left\{F_{01}\sinh(kd) + F_{02}\cosh(kd)\right\}\\&- b\left(hU^{0(5)} - \alpha\varphi^{0(5)} - \mu\psi^{0(5)}\right)\left\{F_{3}\sinh(bkd) + F_{04}\cosh(bkd)\right\}\\&+ \mu_0\left\{F_{01}\psi^{0(1)}\cosh(kd) + F_{02}\psi^{0(1)}\sinh(kd) + F_{03}\psi^{0(5)}\cosh(bkd) + F_{04}\psi^{0(5)}\sinh(bkd)\right\} = 0\end{aligned} \tag{X.3}$$

For the lower surface at $x_3 = - d$, corresponding three homogeneous equations read as follows:

$$\begin{aligned}&\left(e\varphi^{0(1)} + h\psi^{0(1)}\right)\left[F_{01}\sinh(kd) - F_{02}\cosh(kd)\right]\\&+ b\left(CU^{0(5)} + e\varphi^{0(5)} + h\psi^{0(5)}\right)\left[F_{03}\sinh(bkd) - F_{04}\cosh(bkd)\right] = 0\end{aligned} \tag{X.4}$$

$$\begin{aligned}&\left(\varepsilon\varphi^{0(1)}+\alpha\psi^{0(1)}\right)\left[F_{01}\sinh(kd)-F_{02}\cosh(kd)\right]\\&-b\left(eU^{0(5)}-\varepsilon\varphi^{0(5)}-\alpha\psi^{0(5)}\right)\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]=0\end{aligned}\qquad(\text{X.5})$$

$$\begin{aligned}&\left(\alpha\varphi^{0(1)}+\mu\psi^{0(1)}\right)\left\{F_{01}\sinh(kd)-F_{02}\cosh(kd)\right\}\\&-b\left(hU^{0(5)}-\alpha\varphi^{0(5)}-\mu\psi^{0(5)}\right)\left\{F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right\}\\&+\mu_0\left\{F_{01}\psi^{0(1)}\cosh(kd)-F_{02}\psi^{0(1)}\sinh(kd)+F_{03}\psi^{0(5)}\cosh(bkd)-F_{04}\psi^{0(5)}\sinh(bkd)\right\}=0\end{aligned}\qquad(\text{X.6})$$

So, this set of six homogeneous equations must be properly transformed into suitable forms to get dispersion relations with which it is possible to calculate the dispersive wave phase velocity. This is the main purpose of the following subsections.

X.1. *The first eigenvectors*

The first eigenvectors are defined by expressions (I.66) and (I.67) in the first chapter. Using them, expressions (III.7) and (III.8) were obtained in the third chapter and they lead to the situation when equations (X.2) and (X.5) can be excluded from the further consideration in this subsection. For the upper surface of the plate, it is possible to find that equations (X.1) and (X.3) can be written as follows:

$$F_{01}\sinh(kd)+F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)+F_{04}\cosh(bkd)\right]=0\qquad(\text{X.7})$$

$$\begin{aligned}&\left(\varepsilon\mu-\alpha^2\right)\left\{F_{01}\sinh(kd)+F_{02}\cosh(kd)\right\}+\varepsilon\mu_0\left\{F_{01}\cosh(kd)+F_{02}\sinh(kd)\right\}\\&+\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{K_{em}^2}\left[F_{03}\cosh(bkd)+F_{04}\sinh(bkd)\right]=0\end{aligned}\qquad(\text{X.8})$$

For the lower surface at $x_3 = -d$ (see figure 1 in Introduction) the corresponding three homogeneous equations can be written in the following simplified forms:

$$F_{01}\sinh(kd)-F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}\left[F_{03}\sinh(bkd)-F_{04}\cosh(bkd)\right]=0\qquad(\text{X.9})$$

$$
\begin{aligned}
&\left(\varepsilon\mu-\alpha^2\right)\left\{F_{01}\sinh(kd)-F_{02}\cosh(kd)\right\}+\varepsilon\mu_0\left\{F_{01}\cosh(kd)-F_{02}\sinh(kd)\right\}\\
&+\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{K_{em}^2}\left[F_{03}\cosh(bkd)-F_{04}\sinh(bkd)\right]=0
\end{aligned}
\qquad \text{(X.10)}
$$

It is natural that these four linear homogeneous equations in four unknowns F_{01}, F_{02}, F_{03}, and F_{04} can be transformed into two independent sets, of which the first set represents two equations in two unknowns F_{01} and F_{03}. The second set represents two equations in two unknowns F_{02} and F_{04}. These four equations then read:

$$F_{01}\sinh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}F_{03}\sinh(bkd)=0 \quad \text{(X.11)}$$

$$F_{01}\left\{\left(\varepsilon\mu-\alpha^2\right)\sinh(kd)+\varepsilon\mu_0\cosh(kd)\right\}+\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{K_{em}^2}F_{03}\cosh(bkd)=0 \quad \text{(X.12)}$$

$$F_{02}\cosh(kd)+b\frac{1+K_{em}^2}{K_{em}^2}F_{04}\cosh(bkd)=0 \quad \text{(X.13)}$$

$$F_{02}\left\{\left(\varepsilon\mu-\alpha^2\right)\cosh(kd)+\varepsilon\mu_0\sinh(kd)\right\}+\varepsilon\mu_0\frac{K_{em}^2-K_e^2}{K_{em}^2}F_{04}\sinh(bkd)=0 \quad \text{(X.14)}$$

Therefore, it is possible to obtain two dispersion relations. The velocities V_{new29} and V_{new30} of the twenty ninth and thirtieth new SH-waves propagating in the piezoelectromagnetic plate in the case of $V_{ph} < V_{tem}$ can be respectively calculated with the following formulae:

$$\sqrt{1-\left(\frac{V_{new29}}{V_{tem}}\right)^2}\tanh\left(kd\sqrt{1-\left(\frac{V_{new29}}{V_{tem}}\right)^2}\right)-\frac{\varepsilon\mu_0\tanh(kd)}{\left(\varepsilon\mu-\alpha^2\right)\tanh(kd)+\varepsilon\mu_0}\frac{K_{em}^2-K_e^2}{1+K_{em}^2}=0 \quad \text{(X.15)}$$

$$\sqrt{1-\left(\frac{V_{new30}}{V_{tem}}\right)^2}-\frac{\varepsilon\mu_0}{\left(\varepsilon\mu-\alpha^2\right)+\varepsilon\mu_0\tanh(kd)}\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\tanh\left(kd\sqrt{1-\left(\frac{V_{new30}}{V_{tem}}\right)^2}\right)=0 \quad \text{(X.16)}$$

It is also possible to analyze dispersion relations (X.15) and (X.16). It is clearly seen in the dispersion relations that for $kd \to \infty$, both the velocities V_{new29} and V_{new30}

approach the corresponding SH-SAW velocity recently discovered in book [101], see formulae from (180) to (184) in the book.

For the case of $V_{ph} > V_{tem}$, dispersion relations (X.15) and (X.16) can be also rewritten as follows:

$$\tan\left(kd\sqrt{\left(\frac{V_{new29}}{V_{tem}}\right)^2-1}\right)\sqrt{\left(\frac{V_{new29}}{V_{tem}}\right)^2-1}+\frac{\varepsilon\mu_0\tanh(kd)}{(\varepsilon\mu-\alpha^2)\tanh(kd)+\varepsilon\mu_0}\frac{K_{em}^2-K_e^2}{1+K_{em}^2}=0 \quad (X.17)$$

$$\sqrt{\left(\frac{V_{new30}}{V_{tem}}\right)^2-1}-\frac{\varepsilon\mu_0}{(\varepsilon\mu-\alpha^2)+\varepsilon\mu_0\tanh(kd)}\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\tan\left(kd\sqrt{\left(\frac{V_{new30}}{V_{tem}}\right)^2-1}\right)=0 \quad (X.18)$$

X.2. *The second eigenvectors*

The second eigenvectors are defined by expressions (I.69) and (I.70). Thus, equalities (III.23) and (III.24) obtained in Chapter III must be used here. For the upper surface at $x_3 = + d$ and the lower surface at $x_3 = - d$, six linear homogeneous equations from (X.1) to (X.6) can be transformed into the forms given by equations from (VIII.7) to (VIII.12) which are identical to equations from (IV.29) to (IV.34) obtained in Chapter IV. Therefore, they obviously result in two dispersion relations given by formulae (IV.40) and (IV.42) for the computation of the velocities V_{new6} and V_{new7} of the sixth and seventh new SH-waves propagating in the piezoelectromagnetic plate.

X.3. *The first and second eigenvectors*

For the upper surface ($x_3 = + d$) of the plate, it is natural to use equations (X.7) and (X.8) written in the first subsection of this chapter. For the lower surface ($x_3 = - d$) the corresponding three homogeneous equations are equations from (VIII.10) to (VIII.12) given in Chapter VIII. Therefore, it is possible to state that in this subsection two new dispersion relations can be also obtained. This is similar to the results obtained in Chapter VIII. It is natural to use equation (VIII.27) with the following substitutions: $K_m^2 \to K_e^2$ and $\varepsilon_0\mu \to \varepsilon\mu_0$. Thus, the first dispersion relation for the case of $V_{ph} < V_{tem}$ to calculate the velocity V_{new31} of the thirty first new SH-wave propagating in the piezoelectromagnetic plate can be composed as follows:

$$\begin{aligned}&\left(\sqrt{1-\left(\frac{V_{new31}}{V_{tem}}\right)^2}-\frac{\varepsilon\mu_0}{\varepsilon\mu-\alpha^2}\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\tanh\left(kd\sqrt{1-\left(\frac{V_{new31}}{V_{tem}}\right)^2}\right)\right)\\&\times\left(\sqrt{1-\left(\frac{V_{new31}}{V_{tem}}\right)^2}\tanh\left(kd\sqrt{1-\left(\frac{V_{new31}}{V_{tem}}\right)^2}\right)-\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)\\&+\left(\sqrt{1-\left(\frac{V_{new31}}{V_{tem}}\right)^2}\tanh\left(kd\sqrt{1-\left(\frac{V_{new31}}{V_{tem}}\right)^2}\right)-\frac{\varepsilon\mu_0}{\varepsilon\mu-\alpha^2}\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\right)\\&\left(\sqrt{1-\left(\frac{V_{new31}}{V_{tem}}\right)^2}-\frac{\alpha^2}{\varepsilon\mu}\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\tanh\left(kd\sqrt{1-\left(\frac{V_{new31}}{V_{tem}}\right)^2}\right)\right)=0\end{aligned} \tag{X.18}$$

The second possible dispersion relation can be obtained by the same way used in the third subsection in Chapter VIII. It is also possible to use equation (VIII.34) with the following substitutions: $K_m^2 \to K_e^2$ and $\varepsilon_0\mu \to \varepsilon\mu_0$. Finally, the second dispersion relation to calculate the velocity V_{new32} of the thirty second new SH-wave propagating in the piezoelectromagnetic plate can be written for the case of $V_{ph} < V_{tem}$ as follows:

$$\begin{aligned}&\left\{\frac{\varepsilon\mu_0\sinh(2kd)}{(\varepsilon\mu-\alpha^2)\sinh(2kd)+\varepsilon\mu_0+2\varepsilon\mu_0\sinh^2(kd)}\frac{K_{em}^2-K_e^2}{1+K_{em}^2}\tanh(bkd)-b\right\}\\&\times\left\{\sinh(2kd)\left(\varepsilon\mu b\tanh(bkd)-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\right)+\alpha^2 b\tanh(bkd)\right\}\\&+\left\{\frac{\varepsilon\mu_0\sinh(2kd)}{(\varepsilon\mu-\alpha^2)\sinh(2kd)+\varepsilon\mu_0+2\varepsilon\mu_0\sinh^2(kd)}\frac{K_{em}^2-K_e^2}{1+K_{em}^2}-b\tanh(bkd)\right\}\\&\times\left\{\sinh(2kd)\left(\varepsilon\mu b-\alpha^2\frac{K_{em}^2-K_\alpha^2}{1+K_{em}^2}\tanh(bkd)\right)-\alpha^2 b\right\}=0\end{aligned} \tag{X.19}$$

Chapter XI

Conclusive Discussion

This theoretical work has demonstrated that investigations of the transversely isotropic piezoelectromagnetic plate of class 6 *mm* concerning the shear-horizontal (SH) acoustic wave propagation can reveal existence of as many as thirty two new SH-waves. Dispersion relations for determination of the phase velocities of new SH-waves in the plate can be obtained in relatively simple explicit forms. The obtained results correspond to different sets of the electrical and magnetic boundary conditions when the mechanical boundary condition such as the mechanically free surface ($\sigma_{32} = 0$) is set the same throughout the investigations carried out in this work. It is worth noting that homogeneous boundary conditions were used for the upper and lower surfaces of the plate when the same set of the boundary conditions is applied to either surface. However, the utilization of different sets of the eigenvectors resulted in three different cases occurred for each set of the mechanical, electrical, and magnetic boundary conditions. Possible electrical boundary conditions exploited in this work include the electrically closed surface ($\varphi = 0$), electrically open surface ($D_3 = 0$), and the continuity of both φ and D_3 at the surfaces, i.e. $\varphi = \varphi^f$ and $D_3 = D^f$, where D_3 is the normal component of the electrical induction. Also, the superscript "*f*" relates to the free space (vacuum). Also, the used magnetic boundary conditions are the magnetically closed surface ($B_3 = 0$), magnetically open surface ($\psi = 0$), and the continuity of both ψ and B_3 at the surfaces, i.e. $\psi = \psi^f$ and $B_3 = B^f$.

This work also demonstrates that for all the treated suitable cases, the obtained new dispersion relations for calculation of the velocities of the new dispersive SH-waves propagating in the plates can have fundamental modes. These fundamental mode velocities are situated below the SH-BAW speed denoted by V_{tem}. It is mentioned that the propagation of this SH-BAW is coupled with both the electrical and magnetic potentials. It is also necessary to state that the velocities of the fundamental mode dispersive waves can approach the corresponding SH-SAW velocities as soon as the dimensionless parameter kd reaches very large values, namely $kd \to \infty$ where k and d are the wavenumber in the propagation direction and the plate half-thickness, respectively, see figure 1 in Introduction. It is natural that it was found that the velocities of the new dispersive SH-waves can approach to ten known SH-SAWs recently discovered by Melkumyan [107] and by the author of the book cited in Ref. [101]. Three SH-SAWs discovered by Melkumyan in his theoretical work [107] are apt for this theoretical work. They are called the surface Bleustein-Gulyaev-Melkumyan wave characterized by the velocity V_{BGM}, the piezomagnetic exchange surface Melkumyan wave or PMESM wave (V_{PMESM}), and the piezoelectric exchange surface Melkumyan wave or PEESM wave (V_{PEESM}). The last two Melkumyan waves were also studied in paper [105].

To study the transversely isotropic piezoelectromagnetics concerning the SH-wave propagation is problematic because many suitable solutions can be found due to the existence of two different sets of the eigenvectors. Two different sets of the eigenvectors also exist for piezoelectromagnetics possessing the cubic symmetry. This fact was demonstrated in book [100]. However, the existence of two different eigenvectors for the cubic piezoelectromagnetics does not lead to two different solutions. In fact, two different eigenvectors give the same SH-SAW velocity for the cubic piezoelectromagnetics. Therefore, it is thought that it can be more preferable for experimentalists to cope with the cubic piezoelectromagnetics. However, explicit forms of the SH-SAW velocities of non-dispersive SH-waves propagating in the cubic piezoelectromagnetics cannot be obtained and it is necessary to perform numerical calculations with the corresponding formulae [100] which can be quite complicated. It is thought that it is also useful to carry out theoretical investigations of problems of dispersive SH-wave propagation in the cubic piezoelectromagnetic plates because the cubic piezoelectromagnetics can also possess a strong magnetoelectric effect.

It is transparent that the obtained results in this work can be useful for further theoretical and experimental investigations of the SH-wave propagation in the piezoelectromagnetic (composite) plates consisting of dissimilar materials. It is thought that the simplest example of such laminated plate is a bi-layered plate consisting of two dissimilar piezoelectromagnetic thin films possessing the common interface. Indeed, the results can be also suitable for some particular cases when a single-phase material such as a piezoelectrics or piezomagnetics is used instead of one of two-phase materials such as piezoelectromagnetics. It is obvious that such problems of SH-wave propagation in the plates can be analytically resolved even in the case of the two-layer plate consisting of two dissimilar piezoelectromagnetics. Today there are attempts to numerically resolve similar problems. For instance, paper [111] numerically studies static and dynamic problems of two-layer magnetoelectroelastic composites with specific properties, where one layer is a pure piezoelectrics and the second layer is a pure piezomagnetics. The authors of paper [111] have used a meshless method based on the local Petrov-Galerkin approach. It is obvious that an analytical study is more preferable in comparison with a numerical investigation. Indeed, an analytical study can lead to illumination of common features for all suitable bi-layer plates, while a numerical treatment can give only results concerning an individual bi-layer plate. It is obvious that when a system has a lot of parameters similar to piezoelectromagnetic bi-layer plates, it is hard to numerically reveal common features. For this purpose, it is necessary to calculate a lot of such materials. Therefore, it is necessary to continue analytical investigations of such relatively simple systems such as the homogeneous plates and bi-layer plates. This can be also useful for more complicated problems that can be resolved only numerically.

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