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# Dispersive Rayleigh Type Waves in Layered Systems Consisting of Piezoelectric Crystals Bismuth Silicate and Bismuth Germanate

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**Summary**
The lowest-order modes of the dispersive six-partial waves of Rayleigh type (RTW6) have been numerically obtained for two layered systems consisting of a layer on a substrate in [100] propagation direction of (001)-cut for both cubic crystals of class 23. Dispersion relations are shown for both a layer of $\mathrm{Bi_{12}SiO_{20}}$ on a substrate of $\mathrm{Bi_{12}GeO_{20}}$ and vice versa. Dispersion relations show one mode in each case with clear maximum and minimum, at which it is analytically shown that the group velocity is equal to the phase velocity. It was concluded that in corresponding highly-symmetric cases, the obtained "non- dispersive" six-partial surface waves in the treated layered systems are a new non-dispersive type, termed Rayleigh-Zakharenko type (RZTW6) "non-dispersive" six-partial surface waves. These can exist in layered systems consisting of a layer on a substrate. The possibility of the existence in layered systems of "non-dispersive" waves of both the nine-partial Zakharenko type (ZTW9) for centrosymmetrical crystals, and the thirteen-partial Zakharenko type waves (ZTW13) for non-centrosymmetrical crystals is suggested. In addition, it is shown that dependence of the phase velocity $V_{ph} = V_{ph}(kh_1, kh_2, \ldots, kh_m)$ in a multi-layered system can be reduced to dependence $V_{ph} = V_{ph}(kh)$ for a layered system consisting of a layer on a substrate. Thus, both systems can be studied in the same way.

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## 1. Introduction

Surface acoustic waves (SAW) have very important applications for Acoustoelectronics devices due to their unique properties. Up to the present time, surface Rayleigh waves are widely used in filters, delay lines, etc. Initially, Lord W. Rayleigh [1] analytically discovered that SAW with polarization in the sagittal plane can propagate in an isotropic bulk medium along the surface of the medium, damping with depth of the medium. Later, SAW with the same polarization were numerically studied in isotropic media and in anisotropic, piezoelectric monocrystals in [2]. This reference also shows both the piezoelectric four-partial waves of Rayleigh type (RTW4) and the non-piezoelectric two-partial waves of Rayleigh type (RTW2). These waves are "non-dispersive" waves. The term "non-dispersive" is introduced in order to distinguish these waves, for which the phase velocity $V_{ph}$ is equal to the group velocity $V_g$ ($V_{ph} = V_g \neq 0$), from dispersive waves ($V_g > V_{ph}$ or $V_g < V_{ph}$). For an isotropic medium there is no dependence of $V_{ph}$ on propagation direction.

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With waveguides consisting of a thin film on a substrate, the waves of Rayleigh type are dispersive waves with the phase velocity dependent on the non-dimensional value of $kh$, where $k$ is the wavenumber in the direction of wave propagation, and $h$ is the layer thickness. Figure 1 shows the layered system. It is more complicated to analytically study layered systems, where, in the simplest case, six-partial RTW6-waves propagate [3]. Also, there are the so-called highly-symmetric propagation directions, where the dispersive RTW-waves consist of piezoelectric ten-partial RTW10-waves. Details about the directions of both the dispersive RTW6-waves and the dispersive RTW10-waves in layered systems are given in [4] for thirty classes of crystal symmetry.

Nowadays there is much theoretical and experimental published work concerning the study of dispersive RTW-waves in layered systems consisting of isotropic, anisotropic, piezoelectric materials. See, for example, references [5, 6, 7, 8, 9, 10, 11]. P. Schnitzler [5] has theoretically studied wave propagation in a CdS-layer of class 6mm on a germanium substrate, neglecting the piezoelectric effect. The same case of transversal isotropy, but with the CdS-layer on a sapphire substrate, has been calculated by D. F. Loftus [6]. R. V. Schmidt and F. W. Voltmer [7] have calculated the dispersion relations for a layered sys-


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tem consisting of a CdS-film on a substrate of fused quartz. Some very interesting dependencies of the phase velocity on the layer thickness have been calculated by L. P. Solie [8] for a layer of fused-quartz on a $\mathrm{LiNbO_3}$-substrate, taking into account the piezoeffect. Calculations of dispersive waves for the layered systems consisting of W or Cu layers on a Be-substrate are presented in [4] for applications to filters and delay lines. Here, dispersive RTW6-waves of both the first type and the second type are shown. Also, in [4, 9, 10] one can find complicated formulae for calculating the phase velocity of dispersive RTW6-waves.

Complicated analytical investigations, both for a layered system consisting of a transversal-isotropic "hard" layer on a transversal-isotropic substrate and for a transversal-isotropic "soft" layer on a transversal-isotropic substrate, are presented in [11, 12]. The propagation directions considered here concern the so-called "pure" non-piezoelectric six-partial waves of Rayleigh type (RTW6). The transversal- isotropic media considered were crystals of classes 4, 422, 4 mm, 4/m, 6, 622, 6 mm, 6/m, as well as different textures (for example, class $\infty$ m). Both theoretical and experimental studied of layered systems consisting of a transversal-isotropic thin film of AlN on the substrate of fused-quartz, without taking into account the piezoeffect for the weakly piezoelectric AlN-film, are presented in [13]. In this case there is weak dependence of the phase velocity on the layer thickness, which is convenient for some technical devices. Good agreement was obtained between experimental data and numerical values of the phase velocity for the layered system treated in [13].

Reference [14] is an excellent and classical work dealing with isotropic, anisotropic and piezoelectric layered systems, for example, glass on $\mathrm{LiNbO_3}$, gold on piezo-quartz, gold on nickel, nickel on gold, fused-quartz on sapphire, KCl on nickel, gold or silicon on molybdenum. The study of Rayleigh type waves in different kinds of layered systems has also been an active field of study in recent years. See, for example [15, 16, 17]. In [15, 16, 17] investigations are presented of dispersive RTW10-waves in a layered system consisting of piezoelectric semiconductors. This consists of a transversal-isotropic Z-cut ZnO-layer (class 6mm) on a (001)-cut and [110]-propagation direction in GaAs-substrate (class $\bar{4}$3m), and includes the piezoelectric effect in both the layer and the substrate.

In the present paper, the six-partial dispersive waves of Rayleigh type (RTW6) are studied in layered systems consisting of both a $\mathrm{Bi_{12}SiO_{20}}$-layer on a $\mathrm{Bi_{12}GeO_{20}}$-substrate and a $\mathrm{Bi_{12}GeO_{20}}$-layer on a $\mathrm{Bi_{12}SiO_{20}}$-substrate. These waves can propagate in directions perpendicular to the second order symmetry axis for both materials. In these propagation direction, the waves are non-piezoelectric. With polarization, these waves propagate in the sagittal plane along the $x_1$-axis, as shown in Figure 1. Both the $x_1$-axis and the $x_3$-axis lie in the sagittal plane. The choice to study propagation of the dispersive RTW6-waves in layered systems is taken because of its relative simplicity, and because such layered systems have not been studied before. In addition, there is the possibility of finding a new type of "non-dispersive" wave in layered systems, as is analytically shown below. For the numerical calculations, the elastic constants $C_{ijkl}$ and densities $\rho$ of the treated media have been taken from [18].

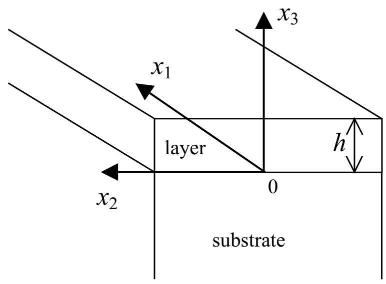


Figure 1. The coordinate system of the layered system, where $h$ is the layer thickness.

## 2. Finding the phase velocity

Finding the phase velocity of the dispersive six-partial Rayleigh type waves (RTW6) in a layered system, consisting of both a layer and a substrate of cubic symmetry, represents the standard procedure for determining the eigenvalues (the corresponding normal components $k_3^N$ of the wavevector $\boldsymbol{k}$) and eigenvectors (the corresponding displacements $U_1^N$ and $U_3^N$ for such polarized waves). The corresponding three components of the Green-Christoffel tensor are $GL_{11}$, $GL_{33}$ and $GL_{13} = GL_{31}$. The determinant for determination of the non-dimensional complex components $n_3 = k_3/k$ can be written as:

$$\begin{vmatrix} C_{44}n_3^2 + C_{11}A_l^2 & (C_{12}+C_{44})n_3 \\ (C_{12}+C_{44})n_3 & C_{11}n_3^2 + C_{44}A_t^2 \end{vmatrix} = 0, \tag{1}$$

where $C_{11} = C_{22} = C_{33}$, $C_{44} = C_{55} = C_{66}$ and $C_{12} = C_{21} = C_{13} = C_{31}$ are the corresponding non-zero components of the stress tensor $C_{ijkl}$, but $A_l^2 = 1 - (V_{ph}/V_l)^2$ and $A_t^2 = 1 - (V_{ph}/V_t)^2$, where $V_{ph} = \omega/k$ is the phase velocity, but $V_l = \sqrt{C_{11}/\rho}$ and $V_t = \sqrt{C_{44}/\rho}$ are the longitudinal and transversal bulk waves, respectively. For example, for $\mathrm{Bi_{12}SiO_{20}}$, there are the following material constants: the material density $\rho$ = 9070 [kg/m$^3$], $C_{11} = 12.962 \cdot 10^{10}$ [N/m$^2$], $C_{44} = 2.451 \cdot 10^{10}$ [N/m$^2$] and $C_{12} = 2.985 \cdot 10^{10}$ [N/m$^2$]. But for $\mathrm{Bi_{12}GeO_{20}}$ there are: $\rho$ = 9200 [kg/m$^3$], $C_{11} = 12.852 \cdot 10^{10}$ [N/m$^2$], $C_{44} = 2.562 \cdot 10^{10}$ [N/m$^2$] and $C_{12} = 2.934 \cdot 10^{10}$ [N/m$^2$]. Expanding the equality (1), the secular equation appears, after straightforward transformations, as:

$$\begin{aligned} &C_{11}C_{44}n_3^4 + \left[C_{11}^2A_l^2 + C_{44}^2A_t^2\right. \\ &\left.-(C_{12}+C_{44})^2\right]n_3^2 + C_{11}C_{44}A_l^2A_t^2 = 0. \end{aligned} \tag{2}$$

Therefore, the corresponding four required non-dimensional values of $n_3$ are:

$$n_3^{1,2,3,4} = \pm\Big[-0.5(A_l^2 + A_t^2 + C^2) \pm 0.5\sqrt{(A_l^2 + A_t^2 + C^2)^2 - 4A_l^2 A_t^2}\Big]^{1/2}, \tag{3}$$

where the anisotropy term $C^2$ is equal to the following:

$$C^2 = \big[(C_{11} - C_{44})^2 - (C_{12} + C_{44})^2\big]/C_{11}C_{44}. \tag{4}$$

It is noted that for isotropic materials there is the well-known condition $C_{11} - C_{44} = C_{12} + C_{44}$, from which it is clearly seen that the anisotropy term $C^2 = 0$ for such materials.

The anisotropy term (4) is positive for both $\mathrm{Bi_{12}SiO_{20}}$ and $\mathrm{Bi_{12}GeO_{20}}$: $C^2(\mathrm{Bi_{12}SiO_{20}}) \sim 2.55 > 0$, but $C^2(\mathrm{Bi_{12}GeO_{20}}) \sim 2.30 > 0$. Already for GaAs there is the negative value: $C^2(\mathrm{GaAs}) \sim -1.32 < 0$ according to [18].

The view of the roots (3) depends on the anisotropy term (4). Let's treat three particular cases. The first case represents the equality $V_{ph} = V_t$, for which there are two zero roots in (3) together with two real(GaAs) / imaginary($\mathrm{Bi_{12}SiO_{20}}$ and $\mathrm{Bi_{12}GeO_{20}}$) roots:

$$n_3^{3,4} = \pm\mathrm{i}\sqrt{A_l^2 + C^2} = \pm\mathrm{i}\sqrt{1 - C_{44}/C_{11} + C^2}, \tag{5}$$

where $A_l^2 = 1 - C_{44}/C_{11} > 0$ for $C_{44} < C_{11}$. There is the relationship between $A_l^2$ and $A_t^2$:

$$C_{11}A_l^2 = C_{44}A_t^2 + (C_{11} - C_{44}), \tag{6}$$

which was used in (5).

The second case represents the equality $V_{ph} = V_l$, for which there are also two zero roots in (3), together with the two following roots, using the relationship (6):

$$n_3^{3,4} = \pm\mathrm{i}\sqrt{A_t^2 + C^2} = \pm\mathrm{i}\sqrt{1 - C_{11}/C_{44} + C^2}, \tag{7}$$

where $A_t^2 = 1 - C_{11}/C_{44} < 0$ for $C_{44} < C_{11}$. The roots (7) can be real. The third particular case in (3) is a very interesting one, which exists for the following equality:

$$A_l^2 + A_t^2 + C^2 = 0. \tag{8}$$

Then there are four roots, which can all be complex:

$$n_3^{1,2,3,4} = \pm\sqrt{\pm 2\mathrm{i}A_l A_t}. \tag{9}$$

The condition (8) is fulfilled for the phase velocity:

$$V'_{ph} = V_l V_t\sqrt{(2 + C^2)/(V_l^2 + V_t^2)}, \tag{10}$$

where the anisotropy term (4) should be $C^2 > -2$ for real phase velocities.

It is clearly seen from (8) and (9) that four complex roots (9) can exist for the phase velocities $V_{ph} < V_t$ in the case $C^2 < 0$. It is well-known from [2, 3] that for some cubic materials there are complex roots, for example, for the suitable phase velocity of the RTW2-waves. But in the case of the positive anisotropy term $C^2 > 0$, the phase velocity (10) can also be greater than the corresponding bulk transversal velocity $V_t$ for two real and two imaginary roots in (9), as well as, probably, even greater than the bulk longitudinal velocity $V_l$ for four complex roots in (9). This could correlate with the interesting numerical results in [19]. Therefore, the boundary conditions determinants could show solutions for such roots (9) for waves in monocrystals, as well as in layered systems, consisting of a solid/liquid layer(s) on a substrate/plate.

Therefore, cubic monocrystals can be divided into two groups, in the first of which there are those with a positive anisotropy term (4), but those with a negative anisotropy term (4) go into the second group. Such different materials can be the subject of further research. The corresponding two eigenvectors $U_1^N$ and $U_3^N$ are:

$$\begin{aligned} U_1^N &= \pm\sqrt{C_{11}(n_3^N)^2 + C_{44}A_t^2}, \\ U_3^N &= \mp\sqrt{C_{44}(n_3^N)^2 + C_{11}A_l^2}. \end{aligned} \tag{11}$$

The boundary conditions should satisfy the following six requirements, which are both at the interface $x_3 = 0$ in Figure 1 and at the free surface $x_3 = h$, for both the corresponding two components of the mechanical displacements and for the corresponding two stress tensor components:

$$\begin{aligned} &\sum_{N=1}^{2} F^{S(N)}U_1^{S(N)} = \sum_{N=1}^{4} F^{L(N)}U_1^{L(N)}, \\ &\sum_{N=1}^{2} F^{S(N)}U_3^{S(N)} = \sum_{N=1}^{4} F^{L(N)}U_3^{L(N)}, \\ &\sum_{N=1}^{2} F^{S(N)}C_{31kl}^{S}\, n_l^{S(N)}U_k^{S(N)} \\ &\qquad = \sum_{N=1}^{4} F^{L(N)}C_{31kl}^{L}\, n_l^{L(N)}U_k^{L(N)}, \\ &\sum_{N=1}^{2} F^{S(N)}C_{33kl}^{S}\, n_l^{S(N)}U_k^{S(N)} \\ &\qquad = \sum_{N=1}^{4} F^{L(N)}C_{33kl}^{L}\, n_l^{L(N)}U_k^{L(N)}, \\ &\sum_{N=1}^{4} F^{L(N)}C_{31kl}^{L}\, n_l^{L(N)}U_k^{L(N)}\exp\big(-\mathrm{i}\,n_3^{L(N)}kh\big] = 0, \\ &\sum_{N=1}^{4} F^{L(N)}C_{33kl}^{L}\, n_l^{L(N)}U_k^{L(N)}\exp\big(-\mathrm{i}\,n_3^{L(N)}kh\big] = 0, \end{aligned} \tag{12}$$

where the index $S$ is for a substrate, and the index $L$ is for a layer.

The boundary conditions determinant from (12) is therefore:

$$\begin{vmatrix} U_1^{S(1)} & U_1^{S(2)} & U_1^{L(1)} & U_1^{L(2)} & U_1^{L(3)} & U_1^{L(4)} \\ U_3^{S(1)} & U_3^{S(2)} & U_3^{L(1)} & U_3^{L(2)} & U_3^{L(3)} & U_3^{L(4)} \\ \begin{matrix} C_{44}^S U_3^{S(1)} \\ +C_{44}^S n_3^{S(1)} U_1^{S(1)} \end{matrix} & \begin{matrix} C_{44}^S U_3^{S(2)} \\ +C_{44}^S n_3^{S(2)} U_1^{S(2)} \end{matrix} & \begin{matrix} C_{44}^L U_3^{L(1)} \\ +C_{44}^L n_3^{L(1)} U_1^{L(1)} \end{matrix} & \begin{matrix} C_{44}^L U_3^{L(2)} \\ +C_{44}^L n_3^{L(2)} U_1^{L(2)} \end{matrix} & \begin{matrix} C_{44}^L U_3^{L(3)} \\ +C_{44}^L n_3^{L(3)} U_1^{L(3)} \end{matrix} & \begin{matrix} C_{44}^L U_3^{L(4)} \\ +C_{44}^L n_3^{L(4)} U_1^{L(4)} \end{matrix} \\ \begin{matrix} C_{12}^S U_1^{S(1)} \\ +C_{11}^S n_3^{S(1)} U_3^{S(1)} \end{matrix} & \begin{matrix} C_{12}^S U_1^{S(2)} \\ +C_{11}^S n_3^{S(2)} U_3^{S(2)} \end{matrix} & \begin{matrix} C_{12}^L U_1^{L(1)} \\ +C_{11}^L n_3^{L(1)} U_3^{L(1)} \end{matrix} & \begin{matrix} C_{12}^L U_1^{L(2)} \\ +C_{11}^L n_3^{L(2)} U_3^{L(2)} \end{matrix} & \begin{matrix} C_{12}^L U_1^{L(3)} \\ +C_{11}^L n_3^{L(3)} U_3^{L(3)} \end{matrix} & \begin{matrix} C_{12}^L U_1^{L(4)} \\ +C_{11}^L n_3^{L(4)} U_3^{L(4)} \end{matrix} \\ 0 & 0 & \begin{matrix} (U_3^{L(1)} \\ +n_3^{L(1)} U_1^{L(1)}) \\ \cdot \mathrm{e}^{-\mathrm{i} n_3^{L(1)} kh} \end{matrix} & \begin{matrix} (U_3^{L(2)} \\ +n_3^{L(2)} U_1^{L(2)}) \\ \cdot \mathrm{e}^{-\mathrm{i} n_3^{L(2)} kh} \end{matrix} & \begin{matrix} (U_3^{L(3)} \\ +n_3^{L(3)} U_1^{L(3)}) \\ \cdot \mathrm{e}^{-\mathrm{i} n_3^{L(3)} kh} \end{matrix} & \begin{matrix} (U_3^{L(4)} \\ +n_3^{L(4)} U_1^{L(4)}) \\ \cdot \mathrm{e}^{-\mathrm{i} n_3^{L(4)} kh} \end{matrix} \\ 0 & 0 & \begin{matrix} (C_{12}^L U_1^{L(1)} \\ +C_{11}^L n_3^{L(1)} U_3^{L(1)}) \\ \cdot \mathrm{e}^{-\mathrm{i} n_3^{L(1)} kh} \end{matrix} & \begin{matrix} (C_{12}^L U_1^{L(2)} \\ +C_{11}^L n_3^{L(2)} U_3^{L(2)}) \\ \cdot \mathrm{e}^{-\mathrm{i} n_3^{L(2)} kh} \end{matrix} & \begin{matrix} (C_{12}^L U_1^{L(3)} \\ +C_{11}^L n_3^{L(3)} U_3^{L(3)}) \\ \cdot \mathrm{e}^{-\mathrm{i} n_3^{L(3)} kh} \end{matrix} & \begin{matrix} (C_{12}^L U_1^{L(4)} \\ +C_{11}^L n_3^{L(4)} U_3^{L(4)}) \\ \cdot \mathrm{e}^{-\mathrm{i} n_3^{L(4)} kh} \end{matrix} \end{vmatrix} = 0. \quad (13)$$

The complete two displacements are:

$$\begin{aligned} U_1 &= \sum_N F^{(N)} U_1^{(N)} \exp\left[\mathrm{i}k(n_1 x_1 + n_3^{(N)} x_3 - V_{ph} t)\right], \\ U_3 &= \sum_N F^{(N)} U_3^{(N)} \exp\left[\mathrm{i}k(n_1 x_1 + n_3^{(N)} x_3 - V_{ph} t)\right]. \end{aligned} \quad (14)$$

The equality (13) must go to zero for given non-dimensional values of $kh$. These depend on the corresponding values of $U_1$, $U_3$ and $n_3$, which represent the functions (3) and (11) of the required phase velocity $V_{ph}$.

## 3. Numerical results and analysis

Figure 2 shows the dependence of the phase velocity of the dispersive six- partial Rayleigh type waves (RTW6) on the layer thickness $kh$. Two curves of the dispersion relation correspond to two different layered systems: the first one is a $\mathrm{Bi_{12}SiO_{20}}$-layer on a $\mathrm{Bi_{12}GeO_{20}}$-substrate, and the second one is a $\mathrm{Bi_{12}GeO_{20}}$-layer on a $\mathrm{Bi_{12}SiO_{20}}$-substrate. It can be clearly seen from Figure 2 that in both cases, when the layer thickness $kh = 0$, the phase velocities of the dispersive RTW6-waves are equal to those of the corresponding two-partial waves of Rayleigh type (RTW2) for the bulk monocrystals. At small $kh < 1$, the phase velocities of the dispersive RTW6-waves are situated out of the phase velocity range between the “non-dispersive” RTW2-waves for bulk $\mathrm{Bi_{12}SiO_{20}}$ and the “non-dispersive” RTW2-waves for bulk $\mathrm{Bi_{12}GeO_{20}}$. At $kh \to \infty$ the phase velocities of the dispersive RTW6-waves approach those of the “non-dispersive” RTW2-waves for the corresponding layer materials. It is possible to find a short abstract about this in [20] in Russian; see also [21].

There is clear maximum or minimum in Figure 2, for the corresponding treated layered systems, in which it is possible to analyze the dependence of the phase velocity on the layer thickness $kh$. This is a very interesting case. First of all, at points of maximum or minimum, it is possible to write:

$$\frac{\mathrm{d}(V_{ph})}{\mathrm{d}(kh)} = 0. \quad (15)$$

The phase velocity can be also written as $V_{ph} = \omega h / kh = \omega / k$, where $\omega = 2\pi\nu$ is the circular frequency and $\nu$ is the frequency. Therefore, $\mathrm{d}(V_{ph})$ can be introduced as

$$\begin{aligned} \mathrm{d}\big(V_{ph}\big) &= \mathrm{d}\Big(\frac{\omega h}{kh}\Big) = \frac{kh d(\omega h) - \omega h d(kh)}{(kh)^2} \\ &= \frac{\mathrm{d}(\omega h)}{kh} - V_{ph} \frac{\mathrm{d}(kh)}{kh}. \end{aligned}$$

Dividing by $\mathrm{d}(kh)$ gives the equality

$$\frac{\mathrm{d}(V_{ph})}{\mathrm{d}(kh)} = \frac{1}{kh}\big(V_g - V_{ph}\big), \quad (16)$$

where $V_g$ is the group velocity, which is equal to

$$V_g = \frac{\mathrm{d}(\omega h)}{\mathrm{d}(kh)} = \frac{\mathrm{d}\omega}{\mathrm{d}k}.$$

And at points of maximum or minimum it is clearly seen from (15) and (16) that

$$V_g = V_{ph}. \quad (17)$$

The formulae (15), (16) and (17) show that at points of maximum and minimum the phase velocity is equal to the group velocity. It means that loading of the layers on the substrates for the treated layered systems at small $kh \approx 0.5$ gives surface RTW6-waves, with the phase velocity equal to the group velocity at both maximum and minimum. This is a special case, because for “non-dispersive” waves (well-known as bulk waves, “non-dispersive” Rayleigh type waves, Bleustein-Gulyaev waves, Maerfeld-Tournois waves, Stoneley waves, leaky waves, bulk-skimming and the exceptional waves in monocrystals) the phase velocity is also equal to the group velocity. Therefore, it is possible to include these surface waves in layered systems as a new type of “non-dispersive” surface wave (namely, “non-dispersive” six-partial Rayleigh-Zakharenko type waves or RZTW6-waves), which can exist in layered systems consisting of a layer on a substrate. Dispersion relations which display a maximum and/or minimum phase velocity when plotted against

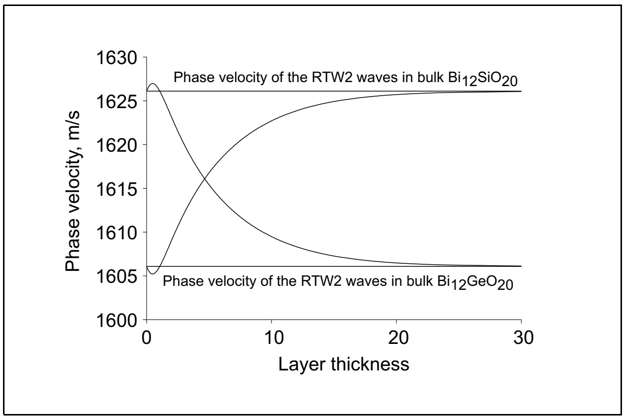


Figure 2. Phase velocity dependencies on layer thickness $kh$ of the first type of RTW6-waves.

the value of $kh$, have been numerically calculated or possible dispersions suggested in many cases; see for example [3, 8, 12]. Unfortunately nobody has analytically shown that corresponding "dispersive" waves are "non-dispersive" at extreme points, and therefore one deals here with a new type of "non-dispersive" wave, which propagates in layered systems. There are many books and papers written in the last decades, in which this phenomenon was observed (it can be readily shown both theoretically and experimentally), which strongly confirms the existence of the phenomenon in many structures with dispersion $\omega(k)$. It is impossible in the present paper to write about all structures in which this phenomenon occurs, but this is not the aim of the paper. Reference [3] presents a theoretical and experimental study of a topographic waveguide consisting of an isotropic medium, where the dependence of the phase velocity on the value of $kd$, with different aspect ratios $h/d$ ($k$ is the wavenumber, $h$ is the height and $d$ is the width), is at a minimum. Therefore, the "non-dispersive" localized Zakharenko waves propagate.

Due to the fact that the value of $kh$ should always be positive (see Figure 2), the group velocity can be greater than the phase velocity as determined by equation (16): if the phase velocity increases, the group velocity should be greater than the phase velocity for dispersive waves ($V_g > V_{ph}$), and if the phase velocity decreases, the group velocity should be less than the phase velocity for other types of dispersive wave ($V_g < V_{ph}$). Therefore, it is possible to state that the single mode of the dispersive RTW6-wave of the first type in Figure 2 can be treated as two dispersive sub-modes (two modes) with different dispersion ($V_g > V_{ph}$ or $V_g < V_{ph}$). The first sub-mode is localized between the "non-dispersive" RTW2-wave for the bulk $\mathrm{Bi_{12}SiO_{20}}$ at $kh$ = 0 and the "non-dispersive" RZTW6-wave at $kh \approx 0.5$. The second sub-mode is localized between the "non-dispersive" RZTW6-wave at $kh \approx 0.5$ and the "non-dispersive" RTW2-wave for the bulk $\mathrm{Bi_{12}GeO_{20}}$ at $kh \to \infty$. This choice of treating the propagation directions in a treated layered system is because it is possible to know in advance about the existence of the "non-dispersive" RZTW6-waves. This is because the surface RTW2-wave for the bulk $\mathrm{Bi_{12}GeO_{20}}$ is weaker than that for bulk $\mathrm{Bi_{12}SiO_{20}}$, but the bulk transversal wave for bulk $\mathrm{Bi_{12}GeO_{20}}$ is stronger than that for bulk $\mathrm{Bi_{12}SiO_{20}}$. Perhaps this is why the "non-dispersive" RZTW6-wave exists in treated layered systems. But the same can not be said about the existence of the "non-dispersive" RZTW6-waves in layered systems consisting of either isotropic materials or crystals of another cubic class. Also, nothing can be said at present about why several "non-dispersive" RZTW10-waves can exist in the same propagation direction. This will have to be the subject of further research.

Probably, any mode of dispersive waves can be localized between two different "non-dispersive" waves. However, this is still uncertain, owing to the use of layered systems consisting of isotropic materials or the highly-symmetric propagation directions in layered systems consisting of anisotropic, piezoelectric crystals, in which so-called "pure" waves of both the Rayleigh type and the Love type can propagate.

The author has numerically obtained the dependence of $V_{ph}(kh)$ in the [110]-highly-symmetric propagation direction of (001)-cut for the same layered systems consisting of cubic piezo-electrical crystals of class 23 studied in the present paper, in which the dispersive waves are the piezoelectric ten-partial RTW10-waves. In this propagation direction, several "non-dispersive" ten-partial Rayleigh-Zakharenko type waves (RZTW10) were observed. The questions are: what does the number of "non-dispersive" RZTW10-waves depend on and what is the maximum number of these waves that can exist? For example, in calculations by L. P. Solie [8] there are two "non-dispersive" RZTW10-waves, while in [15] there is only one. However, this is outside the scope of the present paper and may be reported in the future. Also, it is necessary to state that in Figure 2 at $kh \approx 1$, the phase velocity of the corresponding dispersive RTW6-waves in the treated layered systems is equal to that of the corresponding "non-dispersive" RTW2-waves in the bulk monocrystals. This means that it is possible to use a layer with a $kh \approx 1$ on a softer substrate without changing the phase velocity, in order to protect the surface of the substrate.

It can now be noted that equation (16) is correct for any point of the dispersion relations. By further analysis, it can be found that at inflexion points:

$$\frac{\mathrm{d}^2(V_{ph})}{\mathrm{d}(kh)^2} = 0. \tag{18}$$

Therefore, using equation (16)

$$\begin{aligned}\frac{\mathrm{d}^2(V_{ph})}{\mathrm{d}(kh)^2} &= \frac{\mathrm{d}}{\mathrm{d}(kh)}\left[\frac{1}{kh}\left(V_g - V_{ph}\right)\right] \\ &= \frac{1}{kh}\frac{\mathrm{d}(V_g)}{\mathrm{d}(kh)} - \frac{2}{(kh)^2}\left(V_g - V_{ph}\right).\end{aligned}$$

And at inflexion points using (18)

$$\frac{\mathrm{d}(V_g)}{\mathrm{d}(kh)} = 2\frac{\mathrm{d}(V_{ph})}{\mathrm{d}(kh)}. \tag{19}$$

The second derivative of the phase velocity is:

$$\frac{\mathrm{d}^2(V_{ph})}{\mathrm{d}(kh)^2} = \frac{1}{kh}\Big[\frac{\mathrm{d}V_g}{\mathrm{d}kh} - 2\frac{\mathrm{d}V_{ph}}{\mathrm{d}kh}\Big]. \qquad (20)$$

Dividing $\mathrm{d}(V_{ph})$ by $\mathrm{d}(\omega h)$ gives:

$$\frac{\mathrm{d}(V_{ph})}{\mathrm{d}(\omega h)} = \frac{V_{ph}}{\omega h}\Big(1 - \frac{V_{ph}}{V_g}\Big) = \frac{1}{V_g}\frac{\mathrm{d}V_h}{\mathrm{d}(kh)}, \qquad (21)$$

from which it can be clearly seen that the equality (17) is also valid at points of maximum and/or minimum. Therefore, at these extreme points there is no dependence of the phase velocity $V_{ph}$ on both the value of $kh$ and the value of $\omega h$. This means that such waves are non-dispersive. The expression (21) is more convenient than expression (16) for experimentalists, who work with dependence $V_{ph}(\omega h)$, but not with $V_{ph}(kh)$.

## 4. The group velocity

The group velocity and the first and the second derivatives of the group velocity are presented below as functions of phase velocity, $kh$ and phase velocity derivatives:

$$V_g = V_{ph} + kh\frac{\mathrm{d}V_{ph}}{\mathrm{d}kh}, \qquad (22)$$

$$\frac{\mathrm{d}V_g}{\mathrm{d}kh} = 2\frac{\mathrm{d}V_{ph}}{\mathrm{d}kh} + kh\frac{\mathrm{d}^2V_{ph}}{\mathrm{d}(kh)^2}, \qquad (23)$$

$$\frac{\mathrm{d}^2V_g}{\mathrm{d}(kh)^2} = 3\frac{\mathrm{d}^2V_{ph}}{\mathrm{d}(kh)^2} + kh\frac{\mathrm{d}^3V_{ph}}{\mathrm{d}(kh)^3}. \qquad (24)$$

It can be seen that the $n$-th derivative of the group velocity is a function of the $n$-th derivative of the phase velocity and of the $(n+1)$-th derivative of the phase velocity times $kh$:

$$\frac{\mathrm{d}^nV_g}{\mathrm{d}(kh)^n} = (n+1)\frac{\mathrm{d}^nV_{ph}}{\mathrm{d}(kh)^n} + kh\frac{\mathrm{d}^{n+1}V_{ph}}{\mathrm{d}(kh)^{n+1}}, \qquad (25)$$

where $n = 0, 1, 2, \ldots, N$.

The points at which the group velocity is either maximum or minimum can be found from equation (23), namely:

$$-\frac{2}{kh}\frac{\mathrm{d}V_{ph}}{\mathrm{d}kh} = \frac{\mathrm{d}^2V_{ph}}{\mathrm{d}(kh)^2}.$$

This gives the values of $kh$ at these points as:

$$kh_{01}\Big(\tfrac{\mathrm{d}V_g}{\mathrm{d}kh} = 0\Big) = -2\frac{\mathrm{d}V_{ph}}{\mathrm{d}kh}\Big/\frac{\mathrm{d}^2V_{ph}}{\mathrm{d}(kh)^2}. \qquad (26)$$

The second derivative (24) of the group velocity is equal to zero at inflexion points, therefore the values of $kh$ at these points are:

$$kh_{02}\Big(\tfrac{\mathrm{d}^2V_g}{\mathrm{d}(kh)^2} = 0\Big) = -3\frac{\mathrm{d}^2V_{ph}}{\mathrm{d}(kh)^2}\Big/\frac{\mathrm{d}^3V_{ph}}{\mathrm{d}(kh)^3} \qquad (27)$$

and $$-\frac{3}{kh}\frac{\mathrm{d}^2V_{ph}}{\mathrm{d}(kh)^2} = \frac{\mathrm{d}^3V_{ph}}{\mathrm{d}(kh)^3}.$$

The group velocity has a minimum even for layered systems consisting of isotropic materials, therefore, equation (26) gives the layer thickness $kh$ for this minimum. The group velocity has a linear dependence on the value of $kh$ near inflexion points that can be written as $V_g = a + kh_{02}b$, where $kh$ is taken from equation (27) and $a$ and $b$ are taken from equation (22).

The group velocities at points where the derivatives (25) are equal to zero, can be written as:

$$V_g\Big(\tfrac{\mathrm{d}^nV_g}{\mathrm{d}(kh)^n} = 0\Big) = V_{ph}(kh_{on}) + kh_{0n}\frac{\mathrm{d}V_{ph}(kh_{0n})}{\mathrm{d}kh} \qquad (28)$$

and $$kh_{0n} = -(n+1)\frac{\mathrm{d}^nV_{ph}}{\mathrm{d}(kh)^n}\Big/\frac{\mathrm{d}^{n+1}V_{ph}}{\mathrm{d}(kh)^{n+1}},$$

where $n = 1, 2, 3, \ldots, N$.

The group velocity can be obtained in two possible ways: the first is from equation (22) as a function $V_g = V_g(kh, V_{ph}, \mathrm{d}V_{ph}/\mathrm{d}kh)$, where it is suggested that both the value of $kh$ and the $V_{ph}$ are known, but it is necessary to obtain all values of $(\mathrm{d}V_{ph}/\mathrm{d}kh)$ for each corresponding values of $V_{ph}$; and the second is from equation (28), where first of all it is necessary to obtain the first few values of $kh_{0n}$ from equation (28), from which an approximate value of $V_g$ can be obtained.

The first method is more complicated than the second one. Two boundary points for the group velocity can be found: one at $kh = 0$, where $V_g = V_{ph}$ (the case without a layer for "non-dispersive" waves) and one at $kh \to \infty$, where $V_g \to V_{ph}$ (or it is possible to write $V_g = V_{ph}$).

## 5. Multi-layered systems

For a multi-layered system, the dependence of the phase velocity $V_{ph}$ on many layers can be expressed as:

$$V_{ph} = V_{ph}(k_{l1}h_{l1}, k_{l2}h_{l2}, \ldots, k_{lm}h_{lm}); \qquad (29)$$
$$k = k_{l1} = k_{l2} = \ldots = k_{lm},$$

where $k$ is the wavenumber in the direction of wave propagation. Thus the following equation can be written:

$$V_{ph} = V_{ph}\Big(kh_l\frac{h_{l1}}{h_l}, kh_l\frac{h_{l2}}{h_l}, \ldots, kh_l\frac{h_{lm}}{h_l}\Big). \qquad (30)$$

Therefore, it follows that the phase velocity $V_{ph}$ depends on the non- dimensional value of $kh_l$, which can be taken to be equal to a real layer thickness, and on the non-dimentional values of $h_{l1}/h_l$, $h_{l2}/h_l$, …, $h_{lm}/h_l$:

$$V_{ph} = V_{ph}\Big(kh_l, \frac{h_{l1}}{h_l}, \frac{h_{l2}}{h_l}, \ldots, \frac{h_{lm}}{h_l}\Big). \qquad (31)$$

Finally, since in experiments the values of $h_{l1}/hl$, $h_{l2}/h_l$, …, $h_{lm}/h_l$ in equation (31) are constant, it is possible to deal only with the following:

$$V_{ph} = V_{ph}(kh_l). \qquad (32)$$

Expresion (32) for a multi-layered system is similar to the one for a layered system consisting of a layer on a subsrate. Therefore, non-dispersive Zakharenko waves in multi-layered systems can be dealt with in a similar way to the treatment of systems consisting of a layer on a substrate.

## 6. Conclusions

The obtained dispersion relations of the phase velocity as a function of the non-dimensional value of $kh$ have shown that for a layered system consisting of a $\mathrm{Bi_{12}GeO_{20}}$-layer on a $\mathrm{Bi_{12}SiO_{20}}$-substrate, there is one clear maximum. However, for a layered system consisting of a $\mathrm{Bi_{12}SiO_{20}}$-layer on a $\mathrm{Bi_{12}GeO_{20}}$-substrate, there is one clear minimum. The maximum and minimum indicate that at these points the phase velocity of the dispersive waves of Rayleigh type (RTW6) is equal to the corresponding group velocity of the dispersive RTW6-waves. In general, for treated layered systems, the phase velocity of the surface RTW6-waves is localized in the narrow phase velocity range (see Figure 2) between the "non- dispersive" two-partial Rayleigh type waves (RTW2) for bulk $\mathrm{Bi_{12}SiO_{20}}$ and for bulk $\mathrm{Bi_{12}GeO_{20}}$. However, at small values of $kh$, the phase velocities of the dispersive RTW6-waves for both cases lie out of this phase velocity range. Also, there are extreme points at which the phase velocities of the RTW6-waves are equal to the group velocities analytically shown in the present paper (see formulae (15), (16) and (17)). This phenomenon is referred to as the "non-dispersive" Zakharenko waves. The analytically obtained universal formulae (16), (20) and (25) are valid in arbitrary propagation directions of arbitrary cuts for any layered systems (for both Lamb type waves and Love type waves).

Probably, the "non-dispersive" Zakharenko type waves (ZTW) can propagate for longer distances than dispersive waves in a corresponding mode. A layered system, in which the "non-dispersive" ZTW-waves can exist, could be treated as a "monocrystal" at corresponding layer thickness $kh(V_{ph} = V_g)$. Therefore, a layered system at such a value of $kh$ could be used instead of a true monocrystal. Thus it is possible to excite several "non-dispersive" ZTW-waves in the same propagation direction in a layered system. This raises the possibility of using one layered structure instead of several true monocrystals for further microminiaturization. Also, it is possible to use the unique dispersive properties of both $V_g > V_{ph}$ and $V_g < V_{ph}$ that can occur for the same mode. In addition there are now researchers who have begun to think about the possibilities of using the dependencies of the derivatives (25) with respect to $kh$ in some technical devices. Both the phase velocity and the group velocity of SAW and their dispersions can be readily measured; for example, by an original interferometric procedure described in [22], which is based on simultaneous optical probing of SAW at two different points and uses the new possibilities offered by the use of a dual-beam optical interferometer.

For non-highly-symmetric propagation directions (the so-called low-symmetric cases, in which waves are neither Love type nor Rayleigh type), both the "non-dispersive" nine-partial Zakharenko type waves (ZTW9) for centrosymmetrical crystals and the "non-dispersive" thirteen-partial Zakharenko type waves (ZTW13) for non-centrosymmetrical crystals will be useful for numerical investigations of the group velocity in arbitrary cuts and arbitrary directions of waves propagation. In anisotropic cases, many waves of different types can exist in layered systems consisting of a layer on a substrate. The difficulties involved in understanding these will be helped by the relations written above and by the presence of the "non-dispersive" ZTW- waves. In the same cases a layer on a substrate will result in waves that are (more than thirteen)-partial, if it is necessary to include magnetic boundary conditions and / or those for semiconductors.

It is also possible that "non-dispersive" leaky Zakharenko type waves could exist in layered systems in the low-symmetric propagation directions. Also, the "non-dispersive" Zakharenko type waves could exist in multi-layered systems; for example, in those studied in [23, 24, 25, 26, 27, 28], where dispersive Floquet waves [23] can also propagate. Also, "non-dispersive" ZTW-waves exist in layered structures [29], where dispersive Bleustein-Gulyaev waves can propagate.

Finally, non-dispersive Zakharenko waves (the Zakharenko condensation) can be found in different plasma waveguides that are already intensively studied, see for example, reference [30].

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